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Uniqueness results for the Dirichlet problem for higher order elliptic equations in polyhedral angles

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Abstract

We consider the Dirichlet boundary value problem for higher order elliptic equations in divergence form with discontinuous coefficients in polyhedral angles. Some uniqueness results are proved.

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1 Introduction

The Dirichlet problem for the polyharmonic equation in a bounded domain of $\mathbb{R}^n$ has been studied by Sobolev in [1]. Later on, different problems (Dirichlet, Neumann and Riquier problems) for harmonic, biharmonic and meta-harmonic functions have been considered by Vekua in [2] and [3], both in the cases of bounded and unbounded domains of $\mathbb{R}^n$. Successively, many authors studied analogous problems in more general cases and with different methods (see, for instance, [4–10], the general survey on this subject [11] and the references quoted therein). In particular, in [10], the author obtains the uniqueness of the solution of the Dirichlet problem,

$$\begin{cases} \Delta^m u(x) = 0 & \text{in } \mathbb{R}^n_l, \\ \frac{\partial^j u(x)}{\partial \vec{\nu}^j}|_{\partial \mathbb{R}^n_l} = 0, & j = 0, \dots, m-1, \end{cases} \tag{1.1}$$

where Δ^m denotes the polyharmonic operator of order m, Δ is the Laplace operator and $\mathbb{R}^n_l$ is a polyhedral angle of $\mathbb{R}^n$, defined in Section 2. We explicitly observe that for $l = 0$ the above mentioned definition gives the half-space $\mathbb{R}^n_+$. We note that, due to the tools used in the proof, some restrictions on the dimension n of the space are required.

Our aim, in this paper, is to generalize the uniqueness result of [10]. More precisely, we are concerned with the following Dirichlet problem for a homogeneous equation in divergence form of order $2m$:

$$\begin{cases} \sum_{|\alpha|=|\beta|=m} D^\alpha(a_{\alpha\beta}(x) D^\beta u(x)) = 0 & \text{in } \mathbb{R}^n_l, \\ \frac{\partial^j u(x)}{\partial \vec{\nu}^j}|_{\partial \mathbb{R}^n_l} = 0, & j = 0, \dots, m-1, \end{cases} \tag{1.2}$$

where the discontinuous coefficients $a_{\alpha\beta}$ are bounded and measurable functions satisfying the uniform ellipticity condition.

Let us remark that if we take $\alpha = \beta$ and if the coefficients of the equation are constants $a_{\alpha\beta}(x) = \frac{m!}{\alpha!}$, then the left-hand side of the equation in (1.2) is exactly the polyharmonic operator Δ^m in (1.1).

Our main results consist in two uniqueness theorems obtained for some particular cases of problem (1.2). More precisely, in Section 4 we consider problem (1.2) in the case $m = 1$ and in Section 5 we assume that $\alpha = \beta$. The main tool in our analysis is a generalization of the Hardy inequality proved by Kondrat'ev and Oleinik in [5] (see Section 3).

2 Notation

Throughout this work we use the following notation:

- $n \in \mathbb{N}$ is the dimension of the considered space;
- Greek letters denote n-dimensional multi-indices, for instance $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$, where $\alpha_i \in \mathbb{N} \cup \{0\}$, $i = 1, \dots, n$;
- $|\alpha| = \alpha_1 + \cdots + \alpha_n$ is the module of the multi-index α;
- $\alpha! = \alpha_1! \cdots \alpha_n!$ is the factorial of the multi-index α;
- $\varphi_{,i}(x) = \frac{\partial \varphi(x)}{\partial x_i}$, $i = 1, \dots, n$;
- $D_i^{\alpha_i} = \frac{\partial^{\alpha_i}}{(\partial x_i)^{\alpha_i}}$, $i = 1, \dots, n$;
- $D^\alpha = D_1^{\alpha_1} \cdots D_n^{\alpha_n}$;
- for $\xi = (\xi_1, \dots, \xi_n) \in \mathbb{R}^n$ we set $\xi^\alpha = \xi_1^{\alpha_1} \cdots \xi_n^{\alpha_n}$;
- for every $l \in \{0, \dots, n-1\}$,

$$\mathbb{R}_l^n = \left\{ x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n : x_i > 0, i = n-l, \dots, n \right\}$$

  is the 'polyhedral angle' with vertex in the origin;
- for $l = 0$ the above definition gives the half-space $\mathbb{R}_+^n$;
- for $\rho > 0$ we denote by $Q_\rho = \{x \in \mathbb{R}_l^n : |x| < \rho\}$.

3 Setting of the problem

We want to consider the following differential equation in divergence form of order $2m$, $m \in \mathbb{N}$, in certain unbounded domains of $\mathbb{R}^n$, $n > 2$:

$$\sum_{|\alpha|=|\beta|=m} D^\beta \left(a_{\alpha\beta}(x) D^\alpha u(x) \right) = f(x), \tag{3.1}$$

where $f(x)$ is a given datum and the coefficients $a_{\alpha\beta}(x)$ are bounded measurable functions satisfying the uniform ellipticity condition, *i.e.* there exist two positive constants λ_1 and λ_2 such that for each nonzero vector $\xi \in \mathbb{R}^n$ one has

$$\lambda_1 |\xi|^{2m} \le \sum_{|\alpha|=|\beta|=m} a_{\alpha\beta}(x) \xi^\alpha \xi^\beta \le \lambda_2 |\xi|^{2m} \quad \text{a.e.} \tag{3.2}$$

Let us mention that if we take $\alpha = \beta$ in (3.1) and if the coefficients of the equation are constants $a_\alpha(x) = \frac{m!}{\alpha!}$, then left-hand side of this equation is the polyharmonic operator Δ^m, where Δ denotes, as usual, the Laplace operator.

For every sufficiently differentiable functions u and v let us set

$$E_m^a(u,v) = \sum_{|\alpha|=|\beta|=m} a_{\alpha\beta}(x) D^\alpha u D^\beta v,$$

$$E_m(u,v) = \sum_{|\alpha|=m} \frac{m!}{\alpha!} D^\alpha u D^\alpha v,$$

$$E_m^a(u) = E_m^a(u,u),$$

$$E_m(u) = E_m(u,u).$$

Definition 3.1 We say that the function u is a generalized solution of (3.1) in $\mathbb{R}_l^n$ with homogeneous Dirichlet boundary conditions if $u \in W^{m,2}(\mathbb{R}_l^n)$ and it satisfies the integral identity

$$(-1)^m \int_{Q_\rho} E_m^a(u,v)\,dx = \int_{Q_\rho} f(x)v(x)\,dx, \tag{3.3}$$

for any $\rho > 0$ and any function $v \in W_0^{m,2}(Q_\rho)$, where $f \in L^2(\mathbb{R}_l^n)$.

To prove our main results, consisting in two uniqueness theorems, we will essentially use the following generalized Hardy inequality.

Lemma 3.2 (Generalized Hardy inequality) *Let $p > 1$, j, and n be such that $j + n - p \neq 0$. Assume that for a sufficiently smooth function g the following condition is fulfilled in a cone $V \subset \mathbb{R}^n$ with vertex in the origin of coordinates*:

$$\int_V |x|^j \left|\nabla g(x)\right|^p dx < \infty, \tag{3.4}$$

where $\nabla g = (\frac{\partial g}{\partial x_1}, \dots, \frac{\partial g}{\partial x_n})$ is the gradient of the function g. Then there exist two constants $M, K > 0$ such that

$$\int_V |x|^{j-p} \left|g(x) - M\right|^p dx < K \int_V |x|^j \left|\nabla g(x)\right|^p dx, \tag{3.5}$$

where the constant K does not depend on the function g. If, in addition, $g(0) = 0$ then $M = 0$.

Remark 3.3 The previous lemma, which was proved by Kondrat'ev and Oleinik in [5], holds also if we replace (3.5) with the following inequality:

$$\int_{V_{R_2}\setminus V_{R_1}} |x|^{j-p} \left|g(x) - M\right|^p dx < K \int_{V_{R_2}\setminus V_{R_1}} |x|^j \left|\nabla g(x)\right|^p dx, \tag{3.6}$$

with $0 < R_1 < R_2$, where V_R, $R > 0$, denotes the intersection between the cone V and the open ball of center in the origin and radius R.

This result can be deduced by the proof of Lemma 3.2, with slight modifications. We point out that in this proof it is also well rendered that the constant K does not depend on R_1 and R_2.

Remark 3.4 As evidenced in many works about different variants of Hardy or Caffarelli-Kohn-Nirenberg type inequalities (see for instance [5, 12–16]), there are always very important restrictions on the dimension of the space n, the order of 'singularity' j and the order of the integral norm p.

4 Dirichlet problem for second order elliptic equations

In this section we consider, for $m = 1$, the homogeneous equation (3.1) in the polyhedral angle $\mathbb{R}^n_l$, $0 \le l \le n-1$, with the homogeneous Dirichlet boundary condition, namely

$$\begin{cases} \sum_{i,j=1}^n \frac{\partial}{\partial x_j}(a_{ij}(x)\frac{\partial u}{\partial x_i}) = 0 & \text{in } \mathbb{R}^n_l, \\ u|_{\partial \mathbb{R}^n_l} = 0. \end{cases} \tag{4.1}$$

Let us observe that by Definition 3.1 it follows that every generalized solution is such that

$$\int_{\mathbb{R}^n_l} E_1(u)\,dx < \infty. \tag{4.2}$$

Now we prove our first uniqueness result.

Theorem 4.1 *Let $n > 2$. Assume that* (3.2) *is satisfied, with $m = 1$. If u is a generalized solution of problem* (4.1), *then $u \equiv 0$ in $\mathbb{R}^n_l$.*

Proof Let $\Theta(s)$ be an auxiliary function in $C_0^\infty([0,\infty[)$ defined by

$$\Theta(s) \equiv \begin{cases} 1 & 0 \le s \le 1, \\ \theta(s) & 1 \le s \le 2, \\ 0 & s \ge 2, \end{cases} \tag{4.3}$$

with $0 \le \theta(s) \le 1$, and such that there exists a positive constant K_0 such that

$$\left|\Theta'(s)\right|^2 \le K_0 \Theta(s). \tag{4.4}$$

We note that in order to obtain a cut-off function $\Theta(s)$ of the above mentioned type one can consider a classical mollifier and modify it suitably near to $s = 1$ and $s = 2$.

Set, for any $R > 0$,

$$\Theta_R(x) = \Theta\left(\frac{|x|}{R}\right). \tag{4.5}$$

Note that the function Θ_R is such that, for any $j = 1, \dots, n$, one has

$$\Theta_{R,j}(x) = \Theta'\left(\frac{|x|}{R}\right) \cdot \frac{x_j}{R|x|}. \tag{4.6}$$

Let us now consider the function

$$v(x) = u(x) \cdot \Theta_R(x).$$

Clearly, by definition of Θ_R and as a consequence of our boundary condition, one has $v(x) \in W_0^{1,2}(Q_{2R})$. Thus, substituting this function in the integral identity (3.3), we get

$$\int_{Q_{2R}} \sum_{i,j=1}^{n} a_{ij}(x) u_{,i} u_{,j} \Theta_R(x)\, dx + \int_{Q_{2R}\setminus Q_R} \sum_{i,j=1}^{n} a_{ij}(x) u_{,i} u \Theta_{R,j}(x)\, dx = 0.$$

Hence, by (3.2) we have

$$\int_{Q_{2R}} E_1^a(u) \cdot \Theta_R(x)\, dx = \left| \int_{Q_{2R}\setminus Q_R} \sum_{i,j=1}^{n} a_{ij}(x) u_{,i} u \Theta'\left(\frac{|x|}{R}\right) \cdot \frac{x_j}{R|x|}\, dx \right|.$$

Recalling that for any $\varepsilon > 0$ one has

$$ab \le \frac{\varepsilon \cdot a^2}{2} + \frac{b^2}{2\varepsilon}, \quad a, b \ge 0, \tag{4.7}$$

in view of the boundedness of the coefficients and of (4.4), one gets

$$\begin{aligned} &\int_{Q_{2R}} E_1^a(u) \cdot \Theta_R(x)\, dx \\ &\quad \le \frac{\varepsilon}{2} K_1 K_0 \int_{Q_{2R}\setminus Q_R} E_1(u) \cdot \Theta_R\, dx + \frac{K_1}{2\varepsilon} \int_{Q_{2R}\setminus Q_R} \frac{u^2}{|x|^2}\, dx, \end{aligned}$$

where K_0 is the constant in (4.4) and $K_1 = K_1(n, \lambda_2)$. Thus, taking into account the ellipticity of the coefficients, we obtain

$$\begin{aligned} &\int_{Q_{2R}} E_1(u) \cdot \Theta_R(x)\, dx \\ &\quad \le \frac{\varepsilon}{2} K_2 K_0 \int_{Q_{2R}\setminus Q_R} E_1(u) \cdot \Theta_R\, dx + \frac{K_2}{2\varepsilon} \int_{Q_{2R}\setminus Q_R} \frac{u^2}{|x|^2}\, dx, \end{aligned} \tag{4.8}$$

with $K_2 = K_2(n, \lambda_1, \lambda_2)$.

Therefore, if we choose $\varepsilon = \frac{1}{K_2 K_0}$ and we apply the generalized Hardy inequality (3.6) (with $p = 2$ and $j = 0$) to the second term in the right-hand side of (4.8), we deduce that

$$\int_{Q_{2R}} E_1(u) \cdot \Theta_R(x)\, dx \le K_3 \int_{Q_{2R}\setminus Q_R} E_1(u)\, dx,$$

where the constant K_3 does not depend on the radius R and on the function u (see Remark 3.3).

Now, observe that clearly for any $P > 0$ there exists $R > P$ such that $Q_P \subset Q_R$ and therefore, in view of the definition of Θ_R, by the former inequality we obtain

$$\int_{Q_P} E_1(u)\, dx \le K_3 \int_{Q_{2R}\setminus Q_R} E_1(u)\, dx. \tag{4.9}$$

Condition (4.2) being satisfied, the right-hand side of (4.9) tends to zero when $R \longrightarrow \infty$. Now, since the left-hand side of (4.9) is independent of the radius R, we have, for any $P > 0$,

$$\int_{Q_P} E_1(u)\,dx = \int_{Q_P} \sum_{i=1}^{n} u_{,i}u_{,i}\,dx = 0.$$

This means that the function $u(x)$ is a constant and, according to the boundary condition in (4.1), this constant is zero. This concludes our proof. □

Remark 4.2 Note that our proof do not provide any uniqueness result for $n = 2$, since in this case the generalized Hardy inequality in Lemma 3.2 does not apply, as a consequence of our choice of p and j.

5 Dirichlet problem for higher order elliptic equations

Here, we consider the following homogeneous equation of order $2m$ with homogeneous Dirichlet boundary conditions in the polyhedral angle $\mathbb{R}^n_l$, $l \in \{0,\dots,n-1\}$:

$$\begin{cases} \sum_{|\alpha|=m} D^\alpha(a_\alpha(x)D^\alpha u(x)) = 0 & \text{in } \mathbb{R}^n_l, \\ \frac{\partial^j u(x)}{\partial \vec{\nu}^j}\big|_{\partial\mathbb{R}^n_l} = 0, & j = 0,\dots,m-1. \end{cases} \tag{5.1}$$

Note that, again, in view of Definition 3.1 one finds that every generalized solution of problem (5.1) is such that

$$\int_{\mathbb{R}^n_l} E_m(u)\,dx < \infty. \tag{5.2}$$

Theorem 5.1 *Let $n > 2m$ or $n = 2k + 1$, with $k \in \mathbb{N}$. Assume that* (3.2) *is satisfied. If u is a generalized solution of problem* (5.1), *then $u(x) \equiv 0$ in $\mathbb{R}^n_l$.*

Proof Let us use again the function Θ_R introduced in the proof of Theorem 4.1.

It is easy to check that

$$D^\alpha \Theta_R(x) = \sum_{i=1}^{|\alpha|} \Theta^{(i)}\left(\frac{|x|}{R}\right) \cdot \frac{P_{|\alpha|}(x)}{R^i |x|^{(2|\alpha|-i)}},$$

where $\Theta^{(i)}$ denotes the derivative of order i of the function Θ and $P_{|\alpha|}(x)$ is a polynomial of order $|\alpha|$.

Moreover, if we assume that there exist some positive constants K_i, such that

$$\left|\Theta^{(i)}(s)\right|^2 \le K_i \Theta(s), \quad i = 1,\dots,|\alpha|,$$

then, for $R < |x| < 2R$, one has

$$\left|D^\alpha \Theta_R(x)\right|^2 \le \frac{K_\alpha \Theta_R(x)}{|x|^{2|\alpha|}}, \tag{5.3}$$

where the constant K_α depends only on α.

Note that function $v(x) = u(x) \cdot \Theta_R(x) \in W_0^{m,2}(Q_{2R})$, thus, substituting it in the integral identity (3.3), we deduce

$$\int_{Q_{2R}} E_m^a(u, u\Theta_R)\,dx = \int_{Q_{2R}} E_m^a(u)\Theta_R\,dx + \int_{Q_{2R}\setminus Q_R} \sum_{|\alpha|=m} a_\alpha(x) D^\alpha u \left[\sum_{\gamma+\iota=\alpha,\iota\neq 0} \frac{(|\gamma|+|\iota|)!}{\gamma!\iota!} D^\gamma u D^\iota \Theta_R\right] dx = 0.$$

Therefore

$$\int_{Q_{2R}} E_m^a(u)\Theta_R\,dx = \left|\int_{Q_{2R}\setminus Q_R} \sum_{|\alpha|=m} a_\alpha(x) D^\alpha u \left[\sum_{\gamma+\iota=\alpha,\iota\neq 0} \frac{(|\gamma|+|\iota|)!}{\gamma!\iota!} D^\gamma u D^\iota \Theta_R\right] dx\right|.$$

From (4.7) and (5.3), arguing as in the proof of Theorem 4.1, we deduce that

$$\int_{Q_{2R}} E_m(u)\Theta_R(x)\,dx \leq \sum_{|\alpha|=m} K'_\alpha \varepsilon_\alpha \int_{Q_{2R}\setminus Q_R} E_m(u)\Theta_R(x)\,dx + \sum_{|\alpha|=m} \frac{K''_\alpha}{\varepsilon_\alpha} \int_{Q_{2R}\setminus Q_R} \left[\sum_{|\gamma|=0}^{m-1} E_{|\gamma|}(u) \cdot \frac{1}{|x|^{2(m-|\gamma|)}}\right] dx, \tag{5.4}$$

with $K'_\alpha = K'_\alpha(n, \alpha, \lambda_1, \lambda_2, K_\alpha)$ and $K''_\alpha = K''_\alpha(n, \alpha, \lambda_1, \lambda_2)$.

Now, we apply repeatedly the Hardy inequality (3.6) to the single summands of the second term in the right-hand side of (5.4) until the order of the partial derivatives achieves m. Thus, after an appropriate selection of ε_α, we get

$$\int_{Q_{2R}} E_m(u)\Theta_R(x)\,dx \leq \widetilde{K} \int_{Q_{2R}\setminus Q_R} E_m(u)\,dx,$$

where the constant $\widetilde{K}$ is independent of the radius R and of the function u.

Finally, following the same argument used in Theorem 4.1, for any $P > 0$ we find $R > P$ such that $Q_P \subset Q_R$ and therefore, taking into account the definition of Θ_R, we obtain

$$\int_{Q_P} E_m(u)\,dx \leq \widetilde{K} \int_{Q_{2R}\setminus Q_R} E_m(u)\,dx. \tag{5.5}$$

In view of condition (5.2), the right-hand side of (5.5) goes to zero when $R \longrightarrow \infty$, and, since the left-hand side of (5.5) is independent of R, we have, for any $P > 0$,

$$\int_{Q_P} E_m(u)\,dx = \int_{Q_P} \left[\sum_{|\alpha|=m} \frac{m!}{\alpha!} D^\alpha u D^\alpha u\right] dx = 0.$$

Therefore, the partial derivatives of any order of the solution are equal to zero, thus, as a consequence of the boundary conditions in (5.1), we deduce that $u(x) \equiv 0$ in $\mathbb{R}^n_l$. □

Remark 5.2 Clearly also in this case the repeated application of the Hardy inequality yields the restrictions $n > 2m$ or $n = 2k + 1$, $k \in \mathbb{N}$, on the space dimension.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors conceived and wrote this article in collaboration and with same responsibility. All of them read and approved the final manuscript.

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References

1. Sobolev, SL: Some Applications of Functional Analysis in Mathematical Physics. Izdat. Leningrad. Gos. Univ., Leningrad (1950). Translations of Mathematical Monographs, vol. 7. Am. Math. Soc., Providence (1963)
2. Vekua, IN: On metaharmonic functions. Lect. Notes TICMI **14**, 62 pp. (2013). Translated from the 1943 Russian original Trav. Inst. Math. Tbilissi **12**, 105-174 (1943)
3. Vekua, IN: New Methods for Solving Elliptic Equations. OGIZ, Moscow (1948) (in Russian)
4. Cassisa, C, Ricci, P, Tavkhelidze, I: Analogue of Saint-Venant's principle for the one special type 4-th order elliptic equation and its applications. Appl. Math. Inform. **4**, 11-29 (1999)
5. Kondrat'ev, VA, Oleinik, OA: Boundary value problems for the system of elasticity theory in unbounded domains. Korn's inequalities. Usp. Mat. Nauk **43**(5), 55-98 (1988) (in Russian)
6. Kozlov, V, Maz'ya, V: Asymptotics of a singular solution to the Dirichlet problem for an elliptic equation with discontinuous coefficients near the boundary. In: Function Spaces, Differential Operators and Nonlinear Analysis (Teistungen, 2001), pp. 75-115. Birkhäuser, Basel (2003)
7. Monsurrò, S, Transirico, M: A L^p-estimate for weak solutions of elliptic equations. Abstr. Appl. Anal. **2012**, Article ID 376179 (2012)
8. Monsurrò, S, Transirico, M: Dirichlet problem for divergence form elliptic equations with discontinuous coefficients. Bound. Value Probl. **2012**, Article ID 67 (2012)
9. Monsurrò, S, Transirico, M: A priori bounds in L^p for solutions of elliptic equations in divergence form. Bull. Sci. Math. **137**, 851-866 (2013)
10. Tavkhelidze, I: On some properties of solutions of polyharmonic equation in polyhedral angles. Georgian Math. J. **14**, 565-580 (2007)
11. Gazzola, F, Grunau, H, Sweers, G: Polyharmonic Boundary Value Problems. Positivity Preserving and Nonlinear Higher Order Elliptic Equations in Bounded Domains. Lecture Notes in Mathematics, vol. 1991. Springer, Berlin (2010)
12. Bojarski, B, Hajlasz, P: Pointwise inequalities for Sobolev functions and some applications. Stud. Math. **106**, 77-92 (1993)
13. Brezis, H, Marcus, M: Hardy's inequalities revisited. Dedicated to Ennio De Giorgi. Ann. Sc. Norm. Super. Pisa, Cl. Sci. **25**, 217-237 (1997)
14. Catrina, F, Wang, ZQ: On the Caffarelli-Kohn-Nirenberg inequalities: sharp constants, existence (and nonexistence), and symmetry of extremal functions. Commun. Pure Appl. Math. **54**, 229-258 (2001)
15. Dávila, J, Dupaigne, L: Hardy-type inequalities. J. Eur. Math. Soc. **6**, 335-365 (2004)
16. Filippas, S, Tertikas, A, Tidblom, J: On the structure of Hardy-Sobolev-Maz'ya inequalities. J. Eur. Math. Soc. **11**, 1165-1185 (2009)

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