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Existence of weak solutions for viscoelastic hyperbolic equations with variable exponents

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Abstract

The authors of this paper study a nonlinear viscoelastic equation with variable exponents. By using the Faedo-Galerkin method and embedding theory, the existence of weak solutions is given to the initial and boundary value problem under suitable assumptions.

Keywords: existence; weak solutions; viscoelastic; variable exponents

1 Introduction

Let $\Omega \subset \mathbb{R}^N$ ($N \geq 2$) be a bounded Lipschitz domain and $0 < T < \infty$. Consider the following nonlinear viscoelastic hyperbolic problem:

$$\begin{cases} u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-\tau) \Delta u(\tau) d\tau + |u_t|^{m(x)-2} u_t = |u|^{p(x)-2} u, & (x, t) \in Q_T, \\ u(x, t) = 0, & (x, t) \in S_T, \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), & x \in \Omega, \end{cases} \quad (1.1)$$

where $Q_T = \Omega \times (0, T]$, S_T denotes the lateral boundary of the cylinder Q_T .

It will be assumed throughout the paper that the exponents $m(x)$, $p(x)$ are continuous in Ω with logarithmic module of continuity:

$$1 < m^- = \inf_{x \in \Omega} m(x) \leq m(x) \leq m^+ = \sup_{x \in \Omega} m(x) < \infty, \quad (1.2)$$

$$1 < p^- = \inf_{x \in \Omega} p(x) \leq p(x) \leq p^+ = \sup_{x \in \Omega} p(x) < \infty, \quad (1.3)$$

$$\forall z, \xi \in \Omega, |z - \xi| < 1, \quad |m(z) - m(\xi)| + |p(z) - p(\xi)| \leq \omega(|z - \xi|), \quad (1.4)$$

where

$$\limsup_{\tau \rightarrow 0^+} \omega(\tau) \ln \frac{1}{\tau} = C < +\infty.$$

And we also assume that

(H1) $g: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is C^1 function and satisfies

$$g(0) > 0, \quad 1 - \int_0^\infty g(s) ds = l > 0;$$

(H2) there exists $\eta > 0$ such that

$$g'(t) < -\eta g(t), \quad t \geq 0.$$

In the case when m, p are constants, there have been many results about the existence and blow-up properties of the solutions, we refer the readers to the bibliography given in [1–6].

In recent years, much attention has been paid to the study of mathematical models of electro-rheological fluids. These models include hyperbolic, parabolic or elliptic equations which are nonlinear with respect to gradient of the thought solution and with variable exponents of nonlinearity; see [7–10] and references therein. Besides, another important application is the image processing where the anisotropy and nonlinearity of the diffusion operator and convection terms are used to underline the borders of the distorted image and to eliminate the noise [11, 12].

To the best of our knowledge, there are only a few works about viscoelastic hyperbolic equations with variable exponents of nonlinearity. In [13] the authors studied the finite time blow-up of solutions for viscoelastic hyperbolic equations, and in [1] the authors discussed only the viscoelastic hyperbolic problem with constant exponents. Motivated by the works of [1, 13], we shall study the existence and energy decay of the solutions to Problem (1.1) and state some properties to the solutions.

The outline of this paper is the following. In Section 2, we introduce the function spaces of Orlicz-Sobolev type, give the definition of the weak solution to the problem and prove the existence of weak solutions for Problem (1.1) with Galerkin’s method.

2 Existence of weak solutions

In this section, the existence of weak solutions is studied. Firstly, we introduce some Banach spaces

$$L^{p(x)}(\Omega) = \left\{ u(x) : u \text{ is measurable in } \Omega, A_{p(\cdot)}(u) = \int_{\Omega} |u|^{p(x)} dx < \infty \right\},$$

$$\|u\|_{p(\cdot)} = \inf \{ \lambda > 0, A_{p(\cdot)}(u/\lambda) \leq 1 \}.$$

Lemma 2.1 [14] *For $u \in L^{p(x)}(\Omega)$, the following relations hold:*

- (1) $\|u\|_{p(\cdot)} < 1$ ($= 1; > 1$) $\Leftrightarrow A_{p(\cdot)}(u) < 1$ ($= 1; > 1$);
- (2) $\|u\|_{p(\cdot)} < 1 \Rightarrow \|u\|_{p(\cdot)}^{p^+} \leq A_{p(\cdot)}(u) \leq \|u\|_{p(\cdot)}^{p^-}$; $\|u\|_{p(\cdot)} > 1 \Rightarrow \|u\|_{p(\cdot)}^{p^+} \geq A_{p(\cdot)}(u) \geq \|u\|_{p(\cdot)}^{p^-}$;
- (3) $\|u\|_{p(\cdot)} \rightarrow 0 \Leftrightarrow A_{p(\cdot)}(u) \rightarrow 0$; $\|u\|_{p(\cdot)} \rightarrow \infty \Leftrightarrow A_{p(\cdot)}(u) \rightarrow \infty$.

Lemma 2.2 [15, 16] *For $u \in W_0^{1,p(\cdot)}(\Omega)$, if p satisfies condition (1.2), the $p(\cdot)$ -Poincaré inequality*

$$\|u\|_{p(x)} \leq C \|\nabla u\|_{p(x)}$$

holds, where the positive constant C depends on p and Ω .

Remark 2.1 *Note that the following inequality*

$$\int_{\Omega} |u|^{p(x)} dx \leq C \int_{\Omega} |\nabla u|^{p(x)} dx$$

does not in general hold.

Lemma 2.3 [17] *Let Ω be an open domain (that may be unbounded) in \mathbb{R}^N with cone property. If $p(x) : \overline{\Omega} \rightarrow \mathbb{R}$ is a Lipschitz continuous function satisfying $1 < p^- \leq p^+ < \frac{N}{k}$ and $r(x) : \overline{\Omega} \rightarrow \mathbb{R}$ is measurable and satisfies*

$$p(x) \leq r(x) \leq p^*(x) = \frac{Np(x)}{N - kp(x)} \quad \text{a.e. } x \in \overline{\Omega},$$

then there is a continuous embedding $W^{k,p(x)}(\Omega) \hookrightarrow L^{r(x)}(\Omega)$.

The main theorem in this section is the following.

Theorem 2.1 *Let $u_0, u_1 \in H_0^1(\Omega)$, the exponents $m(x), p(x)$ satisfy conditions (1.2)-(1.4). Then Problem (1.1) has at least one weak solution $u : \Omega \times (0, \infty) \rightarrow \mathbb{R}$ in the class*

$$u \in L^\infty(0, \infty; H_0^1(\Omega)), \quad u' \in L^\infty(0, \infty; H_0^1(\Omega)), \quad u'' \in L^2(0, \infty; H_0^1(\Omega)).$$

And one of the following conditions holds:

- (A1) $2 < p^- < p^+ < \max\{N, \frac{Np^-}{N-p^-}\}, 2 < m^- < m^+ < p^-;$
- (A2) $\max\{1, \frac{2N}{N+2}\} < p^- < p^+ < 2, 1 < m^- < m^+ < \frac{3p^- - 2}{p^-} < 2.$

Proof Let $\{w_j\}_{j=1}^\infty$ be an orthogonal basis of $H_0^1(\Omega)$ with w_j

$$-\Delta w_j = \lambda_j w_j, \quad x \in \Omega, \quad w_j = 0, \quad x \in \partial\Omega.$$

$V_k = \text{span}\{w_1, \dots, w_k\}$ is the subspace generated by the first k vectors of the basis $\{w_j\}_{j=1}^\infty$. By normalization, we have $\|w_j\|_2 = 1$. Let us define the operator

$$\begin{aligned} \langle Lu, \Phi \rangle = & \int_{\Omega} \left[u_{tt} \Phi + \nabla u \nabla \Phi - \int_0^t g(t - \tau) \nabla u \nabla \Phi \, d\tau + |u_t|^{m(x)-2} u_t \Phi \right. \\ & \left. - \alpha |u|^{p(x)-2} u \Phi \right] dx, \quad \Phi \in V_k. \end{aligned}$$

For any given integer k , we consider the approximate solution

$$u_k = \sum_{i=1}^k c_i^k(t) w_i,$$

which satisfies

$$\begin{cases} \langle Lu_k, w_i \rangle = 0, & i = 1, 2, \dots, k, \\ u_k(0) = u_{0k}, & u_{kt}(0) = u_{1k}, \end{cases} \quad (2.1)$$

here $u_{0k} = \sum_{i=1}^k (u_0, w_i) w_i$, $u_{1k} = \sum_{i=1}^k (u_1, w_i) w_i$ and $u_{0k} \rightarrow u_0$, $u_{1k} \rightarrow u_1$ in $H_0^1(\Omega)$.

Here we denote by (\cdot, \cdot) the inner product in $L^2(\Omega)$.

Problem (1.1) generates the system of k ordinary differential equations

$$\begin{cases} (c_i^k(t))'' = -\lambda_i c_i^k(t) + \lambda_i \int_0^t g(t-\tau) c_i^k(\tau) d\tau \\ \quad + |(\sum_{i=1}^k (c_i^k(t))', w_i)|^{m(x)-2} (\sum_{i=1}^k (c_i^k(t))', w_i) \\ \quad - \alpha |(\sum_{i=1}^k c_i^k(t), w_i)|^{p(x)-2} (\sum_{i=1}^k c_i^k(t), w_i), \\ (c_i^k(0)) = (u_0, w_i), \quad (c_i^k(0))' = (u_1, w_i), \quad i = 1, 2, \dots, k. \end{cases} \quad (2.2)$$

By the standard theory of the ODE system, we infer that problem (2.2) admits a unique solution $c_i^k(t)$ in $[0, t_k]$, where $t_k > 0$. Then we can obtain an approximate solution $u_k(t)$ for (1.1), in V_k , over $[0, t_k)$. And the solution can be extended to $[0, T]$, for any given $T > 0$, by the estimate below. Multiplying (2.1) $(c_i^k(t))'$ and summing with respect to i , we conclude that

$$\begin{aligned} & \frac{d}{dt} \left(\frac{1}{2} \|u_k'\|_2^2 + \frac{1}{2} \|\nabla u_k'\|_2^2 - \int_0^t g(t-\tau) \int_{\Omega} (\nabla u_k(\tau) \nabla u_k'(t)) dx d\tau \right) + \int_{\Omega} |u_k'|^{m(x)} dx \\ & - \alpha \frac{d}{dt} \left(\int_{\Omega} \frac{1}{p(x)} |u_k|^{p(x)} dx \right) = 0. \end{aligned} \quad (2.3)$$

By simple calculation, we have

$$\begin{aligned} & - \int_0^t g(t-\tau) \int_{\Omega} (\nabla u_k(\tau), \nabla u_k'(t)) dx d\tau \\ & = \frac{1}{2} \frac{d}{dt} (g \diamond \nabla u_k)(t) - \frac{1}{2} (g' \diamond \nabla u_k)(t) - \frac{1}{2} \frac{d}{dt} \int_0^t g(s) ds \|\nabla u_k\|_2^2 \\ & \quad + \frac{1}{2} g(t) \|\nabla u_k\|_2^2, \end{aligned} \quad (2.4)$$

here

$$(\varphi \diamond \nabla \psi)(t) \triangleq \int_0^t \varphi(t-\tau) \|\nabla \psi(t) - \nabla \psi(\tau)\|_2^2 d\tau.$$

Combining (2.3)-(2.4) and (H1)-(H2), we get

$$\begin{aligned} & \frac{d}{dt} \left(\frac{1}{2} \|u_k'\|_2^2 + \frac{1}{2} \|\nabla u_k'\|_2^2 \right. \\ & \quad \left. + \frac{1}{2} \left(1 - \int_0^t g(s) ds \right) \|\nabla u_k\|_2^2 + \frac{1}{2} (g \diamond \nabla u_k)(t) - \alpha \int_{\Omega} \frac{1}{p(x)} |u_k|^{p(x)} dx \right) \\ & = \frac{1}{2} (g' \diamond \nabla u_k)(t) - \frac{1}{2} g(t) \|\nabla u_k\|_2^2 - \int_{\Omega} |u_k'|^{m(x)} dx. \end{aligned} \quad (2.5)$$

Integrating (2.5) over $(0, t)$, and using assumptions (1.2)-(1.4), we have

$$\begin{aligned} & \frac{1}{2} \|u_k'\|_2^2 + \frac{1}{2} \|\nabla u_k'\|_2^2 + \frac{1}{2} \left(1 - \int_0^t g(s) ds \right) \|\nabla u_k\|_2^2 \\ & \quad + \frac{1}{2} (g \diamond \nabla u_k)(t) - \alpha \frac{1}{p(x)} |u_k|^{p(x)} \leq C1, \end{aligned}$$

where $C1$ is a positive constant depending only on $\|u_0\|_{H_0^1}, \|u_1\|_{H_0^1}$.

Hence, by Lemma 2.1, we also have

$$\begin{aligned} & \frac{1}{2} \|u'_k\|_2^2 + \frac{1}{2} \|\nabla u'_k\|_2^2 + \frac{1}{2} \left(1 - \int_0^t g(s) ds\right) \|\nabla u_k\|_2^2 + \frac{1}{2} (g \diamond \nabla u_k)(t) \\ & - \max \left\{ \alpha \frac{1}{p^-} \|u_k\|_{p(x)}^{p^-}, \alpha \frac{1}{p^+} \|u_k\|_{p(x)}^{p^+} \right\} \leq C1. \end{aligned} \tag{2.6}$$

In view of (H1)-(H2) and (A1)-(A2), we also have

$$\|u'_k\|_2^2 + \|\nabla u'_k\|_2^2 + \|\nabla u_k\|_2^2 + (g \diamond \nabla u_k)(t) \leq C2, \tag{2.7}$$

where C2 is a positive constant depending only on $\|u_0\|_{H_0^1}$, $\|u_1\|_{H_0^1}$, l , p^- , p^+ . It follows from (2.7) that

$$u_k \text{ is uniformly bounded in } L^\infty(0, T; H_0^1(\Omega)), \tag{2.8}$$

$$u'_k \text{ is uniformly bounded in } L^\infty(0, T; H_0^1(\Omega)). \tag{2.9}$$

Next, multiplying (1.1) by $(c_i^k(t))''$ and then summing with respect to i , we get that the following holds:

$$\begin{aligned} & \int_\Omega |u''_k|_2^2 dx + \|\nabla u''_k\|_2^2 + \frac{d}{dt} \left(\frac{1}{m(x)} |u'_k|^{m(x)} \right) \\ & = - \int_\Omega \nabla u_k \nabla u''_k dx + \int_0^t g(t-\tau) \int_\Omega \nabla u_k(\tau) \nabla u''_k(t) dx d\tau \\ & \quad + \alpha \int_\Omega |u_k|^{p(x)-2} u_k u''_k dx. \end{aligned} \tag{2.10}$$

Note that

$$\left| - \int_\Omega \nabla u_k \nabla u''_k dx \right| \leq \varepsilon \|\nabla u''_k\|_2^2 + \frac{1}{4\varepsilon} \|\nabla u_k\|_2^2, \quad \varepsilon > 0, \tag{2.11}$$

$$\begin{aligned} & \left| \int_0^t g(t-\tau) \int_\Omega \nabla u_k(\tau) \nabla u''_k(t) dx d\tau \right| \\ & \leq \varepsilon \|\nabla u''_k\|_2^2 + \frac{1}{4\varepsilon} \int_\Omega \left(\int_0^t g(t-\tau) \nabla u_k(\tau) d\tau \right)^2 dx \\ & \leq \varepsilon \|\nabla u''_k\|_2^2 + \frac{1}{4\varepsilon} \int_0^t g(s) ds \int_0^t g(t-\tau) \int_\Omega |\nabla u_k(\tau)|^2 dx d\tau \\ & \leq \varepsilon \|\nabla u''_k\|_2^2 + \frac{(1-l)g(0)}{4\varepsilon} \int_0^t \|\nabla u_k(\tau)\|_2^2 d\tau, \end{aligned} \tag{2.12}$$

$$\begin{aligned} \alpha \int_\Omega |u_k|^{p(x)-2} u_k u''_k dx & \leq \alpha \varepsilon \|u''_k\|_2^2 + \frac{\alpha}{4\varepsilon} \int_\Omega |u_k|^{p(x)-2} u_k^2 dx \\ & \leq \alpha \varepsilon \|u''_k\|_2^2 + \frac{\alpha}{4\varepsilon} \int_\Omega (|u_k|^{p(x)-2} u_k)^2 dx. \end{aligned}$$

From Lemma 2.2, we have

$$\|u''_k\|_2^2 \leq C^2 \|\nabla u''_k\|_2^2, \tag{2.13}$$

$$\begin{aligned} \int_{\Omega} (|u_k|^{p(x)-2} u_k)^2 dx &= \int_{\Omega} |u_k|^{2(p(x)-1)} dx \\ &\leq \max \left\{ \int_{\Omega} |u_k|^{2(p^- - 1)} dx, \int_{\Omega} |u_k|^{2(p^+ - 1)} dx \right\} \\ &\leq \max \left\{ C^{* \frac{1}{2(p^- - 1)}} \|\nabla u_k\|_{\frac{2}{2(p^- - 1)}}, C^{* \frac{1}{2(p^+ - 1)}} \|\nabla u_k\|_{\frac{2}{2(p^+ - 1)}} \right\}, \end{aligned} \quad (2.14)$$

where C, C^* are embedding constants. From (2.10)-(2.14), we obtain that

$$\begin{aligned} &\int_{\Omega} |u_k''|^2 dx + (1 - 2\varepsilon - \alpha\varepsilon C) \|\nabla u_k''\|_2^2 + \frac{d}{dt} \left(\frac{1}{m(x)} |u_k'|^{m(x)} \right) \\ &\leq \frac{1}{4\varepsilon} \|\nabla u_k\|_2^2 + \frac{(1-l)g(0)}{4\varepsilon} \int_0^t \|\nabla u_k(\tau)\|_2^2 d\tau \\ &\quad + \max \left\{ C^{* \frac{1}{2(p^- - 1)}} \|\nabla u_k\|_{\frac{1}{p^- - 1}}, C^{* \frac{1}{2(p^+ - 1)}} \|\nabla u_k\|_{\frac{1}{p^+ - 1}} \right\}. \end{aligned} \quad (2.15)$$

Integrating (2.15) over $(0, t)$ and using (2.7), Lemma 2.3, we get

$$\begin{aligned} &\int_0^t \|u_k''\|_2^2 d\tau + (1 - 2\varepsilon - \alpha\varepsilon C) \int_0^t \|\nabla u_k''\|_2^2 d\tau + \int_{\Omega} \frac{1}{m(x)} |u_k'|^{m(x)} dx \\ &\leq \frac{1}{4\varepsilon} (C_2 + (1-l)g(0)T) + C_3, \end{aligned} \quad (2.16)$$

where C_3 is a positive constant depending only on $\|u_1\|_{H_0^1}$.

Taking α, ε small enough in (2.16), we obtain the estimate

$$\int_0^t \|u_k''\|_2^2 d\tau + \int_{\Omega} \frac{1}{m(x)} |u_k'|^{m(x)} dx \leq C_4.$$

Hence, by Lemma 2.1, we have

$$\int_0^t \|u_k''\|_2^2 d\tau + \min \left\{ \frac{1}{m^+} \|u_k'\|_{m(x)}^{m^-}, \frac{1}{m^+} \|u_k'\|_{m(x)}^{m^+} \right\} \leq C_4, \quad (2.17)$$

where C_4 is a positive constant depending only on $\|u_0\|_{H_0^1}, \|u_1\|_{H_0^1}, l, g(0), T$.

From estimate (2.17), we get

$$u_k'' \text{ is uniformly bounded in } L^2(0, T; H_0^1(\Omega)). \quad (2.18)$$

By (2.7)-(2.9) and (2.18), we infer that there exist a subsequence $\{u_i\}$ of $\{u_k\}$ and a function u such that

$$u_i \rightharpoonup u \text{ weakly star in } L^\infty(0, T; H_0^1(\Omega)), \quad (2.19)$$

$$u_i \rightharpoonup u \text{ weakly in } L^{p^-}(0, T; W^{1,p(x)}(\Omega)), \quad (2.20)$$

$$u_i' \rightharpoonup u' \text{ weakly star in } L^\infty(0, T; H_0^1(\Omega)), \quad (2.21)$$

$$u_i'' \rightharpoonup u'' \text{ weakly in } L^2(0, T; H_0^1(\Omega)). \quad (2.22)$$

Next, we will deal with the nonlinear term. From the Aubin-Lions theorem, see Lions [18, pp.57-58], it follows from (2.21) and (2.22) that there exists a subsequence of $\{u_i\}$, still

represented by the same notation, such that

$$u'_i \rightarrow u' \quad \text{strongly in } L^2(0, T; L^2(\Omega)),$$

which implies $u'_i \rightarrow u'$ almost everywhere in $\Omega \times (0, T)$. Hence, by (2.19)-(2.22),

$$|u_i|^{p(x)-2} u_i \rightharpoonup |u|^{p(x)-2} u \quad \text{weakly in } \Omega \times (0, T), \tag{2.23}$$

$$|u'_i|^{m(x)-2} u'_i \rightarrow |u'|^{m(x)-2} u' \quad \text{almost everywhere in } \Omega \times (0, T). \tag{2.24}$$

Multiplying (2.2) by $\phi(t) \in \mathcal{C}(0, T)$ (which $\mathcal{C}(0, T)$ is the space of C^∞ function with compact support in $(0, T)$) and integrating the obtained result over $(0, T)$, we obtain that

$$\langle Lu_k, w_i \phi(t) \rangle = 0, \quad i = 1, 2, \dots, k. \tag{2.25}$$

Note that $\{w_i\}_{i=1}^\infty$ is a basis of $H_0^1(\Omega)$. Convergence (2.19)-(2.24) is sufficient to pass to the limit in (2.25) in order to get

$$u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t - \tau) \Delta u(\tau) d\tau + |u_t|^{m(x)-2} u_t = |u|^{p(x)-2} u, \text{ in } L^2(0, T; H^{-1}(\Omega))$$

for arbitrary $T > 0$. In view of (2.19)-(2.22) and Lemma 3.3.17 in [19], we obtain

$$u_k(0) \rightharpoonup u(0) \quad \text{weakly in } H_0^1(\Omega), \quad u'_k(0) \rightharpoonup u'(0) \quad \text{weakly in } H_0^1(\Omega).$$

Hence, we get $u(0) = u_0, u_1(0) = u_1$. Then, the existence of weak solutions is established. □

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

YG carried out the study of existence of weak solutions for viscoelastic hyperbolic equations with variable exponents and drafted the manuscript. WG participated in the discussion of existence of weak solutions for viscoelastic hyperbolic equations with variable exponents.

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