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Remarks on the regularity criteria for the 3D MHD equations in the multiplier spaces

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Abstract
In this paper, we consider the regularity criteria for the 3D MHD equations. It is proved that if

$$\partial_3(\mathbf{u}+\mathbf{b}) \in L^{\frac{2}{1-r}}(0,T;\dot{X}_r) \quad (0 \le r \le 1),$$

or

$$\partial_3(\mathbf{u}-\mathbf{b}) \in L^{\frac{2}{1-r}}(0,T;\dot{X}_r) \quad (0 \le r \le 1),$$

then the solution actually is smooth. This extends the previous results given by Guo and Gala (Anal. Appl. 10:373-380, 2013), Gala (Math. Methods Appl. Sci. 33:1496-1503, 2010).
MSC: 35B65; 35Q35; 76D03

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1 Introduction

In this paper, we consider the following three-dimensional (3D) magnetohydrodynamic (MHD) equations:

$$\begin{cases} \mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} - (\mathbf{b}\cdot\nabla)\mathbf{b} - \triangle\mathbf{u} + \nabla\pi = \mathbf{0}, \\ \mathbf{b}_t + (\mathbf{u}\cdot\nabla)\mathbf{b} - (\mathbf{b}\cdot\nabla)\mathbf{u} - \triangle\mathbf{b} = \mathbf{0}, \\ \nabla\cdot\mathbf{u} = 0, \\ \nabla\cdot\mathbf{b} = 0, \\ \mathbf{u}(0) = \mathbf{u}_0, \qquad \mathbf{b}(0) = \mathbf{b}_0, \end{cases} \tag{1}$$

where $\mathbf{u} = (u_1, u_2, u_3)$ is the fluid velocity field, $\mathbf{b} = (b_1, b_2, b_3)$ is the magnetic field, π is a scalar pressure, and $(\mathbf{u}_0, \mathbf{b}_0)$ are the prescribed initial data satisfying $\nabla\cdot\mathbf{u}_0 = \nabla\cdot\mathbf{b}_0 = 0$ in the distributional sense. Physically, (1) governs the dynamics of the velocity and magnetic fields in electrically conducting fluids such as plasmas, liquid metals, and salt water. Moreover, $(1)_1$ reflects the conservation of momentum, $(1)_2$ is the induction equation, and $(1)_3$ specifies the conservation of mass.

Besides its physical applications, MHD system (1) is also mathematically significant. Duvaut and Lions [1] constructed a global weak solution to (1) for initial data with finite energy. However, the issue of regularity and uniqueness of such a given weak solution remains a challenging open problem. Many sufficient conditions (see, *e.g.*, [2–16] and the references therein) were derived to guarantee the regularity of the weak solution. Among these results, we are interested in regularity criteria involving only partial components of the velocity $\mathbf{u}$, the magnetic field $\mathbf{b}$, the pressure gradient $\nabla\pi$, *etc.*

In [8], Jia and Zhou used an intricate decomposition technique and delicate inequalities to obtain the following regularity criterion:

$$\partial_3\pi \in L^p\big(0,T;L^q\big(\mathbb{R}^3\big)\big), \quad \frac{2}{p}+\frac{3}{q}=2, 3\leq q<\infty, \tag{2}$$

that is, if (2) holds, then the solution of (1) is smooth. Applying a more subtle decomposition technique (see [13, Remark 3]), Zhang, Li, and Yu [13] could be able to prove smoothness condition (2) in the case $3/2\leq q\leq 3$. Noticeably, Zhang [17] treated the range $\frac{3}{2}\leq q\leq\infty$ in a unified approach.

As far as one directional derivative of the velocity field is concerned, Cao and Wu [2] proved the following regularity criterion:

$$\partial_3\mathbf{u} \in L^p\big(0,T;L^q\big(\mathbb{R}^3\big)\big), \quad \frac{2}{p}+\frac{3}{q}=1, 3\leq q\leq\infty;$$

Jia and Zhou [10] showed that if

$$\partial_3\mathbf{u} \in L^p\big(0,T;L^q\big(\mathbb{R}^3\big)\big), \quad \frac{2}{p}+\frac{3}{q}=\frac{3}{4}+\frac{3}{2q}, q>2,$$

then the solution is regular. These results were improved by Zhang [18] to be

$$\partial_3\mathbf{u} \in L^p\big(0,T;L^q\big(\mathbb{R}^3\big)\big), \quad \frac{2}{p}+\frac{3}{q}=1+\frac{1}{q}, q>2.$$

We would like to give another contribution in this direction and prove that if

$$\partial_3(\mathbf{u}+\mathbf{b}) \in L^{\frac{2}{1-r}}(0,T;\dot{X}_r) \quad (0\leq r\leq 1),$$

or

$$\partial_3(\mathbf{u}-\mathbf{b}) \in L^{\frac{2}{1-r}}(0,T;\dot{X}_r) \quad (0\leq r\leq 1),$$

then the solution actually is smooth. Here, $\dot{X}_r$ is the multiplier spaces, which is strictly larger than $L^{\frac{3}{r}}(\mathbb{R}^3)$ (see Section 2 for details).

Notice that our result extends

$$\partial_3\mathbf{u} \in L^{\frac{2}{1-r}}(0,T;\dot{X}_r) \quad (0\leq r\leq 1)$$

which is in [19] for the Navier-Stokes equations. At this moment, for MHD equations (1), we could not be able to add the regularity condition on one directional derivative of the velocity field only, since we need to convert (1) into a symmetric form. We also improve the result

$$\nabla(u+b) \in L^{\frac{2}{2-r}}(0,T;\dot{X}_r) \quad (0 \le r \le 1)$$

of [20] in the sense that we need only one directional derivative of $\mathbf{u}+\mathbf{b}$ or $\mathbf{u}-\mathbf{b}$ to ensure the smoothness of the solution.

Before stating the precise result, let us recall the weak formulation of MHD equations (1).

Definition 1 Let $(\mathbf{u}_0, \mathbf{b}_0) \in L^2(\mathbb{R}^3)$ with $\nabla \cdot \mathbf{u}_0 = \nabla \cdot \mathbf{b}_0 = 0$, and $T > 0$. A measurable $\mathbb{R}^3$-valued pair $(\mathbf{u}, \mathbf{b})$ is called a weak solution to (1) with initial data $(\mathbf{u}_0, \mathbf{b}_0)$ provided that the following three conditions hold:

(1) $\mathbf{u} \in L^\infty(0,T;L^2(\mathbb{R}^3)) \cap L^2(0,T;H^1(\mathbb{R}^3))$, $\mathbf{b} \in L^\infty(0,T;L^2(\mathbb{R}^3)) \cap L^2(0,T;H^1(\mathbb{R}^3))$;

(2) $(1)_{1,2,3,4}$ are satisfied in the distributional sense;

(3) the energy inequality

$$\left\|(\mathbf{u},\mathbf{b})\right\|_{L^2}^2(t) + 2\int_0^t \left\|\nabla(\mathbf{u},\mathbf{b})\right\|_{L^2}^2 \,\mathrm{d}s \le \left\|(\mathbf{u}_0,\mathbf{b}_0)\right\|_{L^2}^2.$$

Now, our main results read as follows.

Theorem 2 *Let* $(\mathbf{u}_0, \mathbf{b}_0) \in H^1(\mathbb{R}^3)$ *with* $\nabla \cdot \mathbf{u}_0 = \nabla \cdot \mathbf{b}_0 = 0$, *and* $T > 0$. *Assume that* $(\mathbf{u}, \mathbf{b})$ *is a given weak solution pair of MHD system* (1) *with initial data* $(\mathbf{u}_0, \mathbf{b}_0)$ *on* $(0,T)$. *If*

$$\partial_3(\mathbf{u}+\mathbf{b}) \in L^{\frac{2}{1-r}}(0,T;\dot{X}_r) \quad (0 \le r \le 1),$$

or

$$\partial_3(\mathbf{u}-\mathbf{b}) \in L^{\frac{2}{1-r}}(0,T;\dot{X}_r) \quad (0 \le r \le 1),$$

then the solution is smooth on $(0,T)$.

The proof of Theorem 2 will be given in Section 3. In Section 2, we introduce the multiplier spaces $\dot{X}_r$ and recall their fine properties.

2 Preliminaries

In this section, we recall the definition and fine properties of the multiplier spaces $\dot{X}_r$ (see [21, 22] for example).

Definition 3 For $0 \le r < 3/2$, the homogeneous space $\dot{X}_r$ is defined as the space of $f \in L^2_{loc}(\mathbb{R}^3)$ such that

$$\|f\|_{\dot{X}_r} \equiv \sup_{\|g\|_{\dot{H}^r} \le 1} \|fg\|_{L^2} < \infty,$$

where $\dot{H}^r(\mathbb{R}^3)$ is the space of distributions u such that

$$\|u\|_{\dot{H}^r} \equiv \left\|(-\triangle)^{\frac{r}{2}} u\right\|_{L^2} < \infty.$$

We have the following scaling properties:

$$\begin{aligned}
&\left\|f(\cdot + x_0)\right\|_{\dot{X}_r} = \|f\|_{\dot{X}_r}, \quad \forall x_0 \in \mathbb{R}^3,\\
&\left\|f(\lambda \cdot)\right\|_{\dot{X}_r} = \frac{1}{\lambda^r}\|f\|_{\dot{X}_r}, \quad \forall \lambda > 0.
\end{aligned}$$

When $r = 0$, we have

$$\dot{X}_0 \cong BMO,$$

where BMO is the homogeneous space of bounded mean oscillations associated with the semi-norm

$$\|f\|_{BMO} = \sup_{x\in\mathbb{R}^3, r>0} \frac{1}{|B_r(x)|}\int_{B_r(x)}\left|f(x) - \frac{1}{B_r(y)}\int_{B_r(y)} f(z)\,\mathrm{d}z\right|\mathrm{d}y.$$

Furthermore, we have the following strict imbeddings:

$$L^{\frac{3}{r}}\left(\mathbb{R}^3\right) \subsetneq \dot{X}_r\left(\mathbb{R}^3\right),$$

which could be justified simply as

$$\begin{aligned}
\|fg\|_{L^2} &\le \|f\|_{L^{\frac{3}{r}}}\|g\|_{L^{\frac{6}{3-2r}}} \quad \text{(Hölder inequality)}\\
&\le C\|f\|_{L^{\frac{3}{r}}}\|g\|_{\dot{H}^r} \quad \text{(Sobolev imbeddings)}\\
&\le C\|f\|_{L^{\frac{3}{r}}} \quad \left(\forall g \in \dot{H}^r\left(\mathbb{R}^3\right) \text{ with } \|g\|_{\dot{H}^r} \le 1\right).
\end{aligned}$$

3 Proof of Theorem 2

In this section, we shall prove Theorem 2. As we will see in the proof below, we need only to consider the case that

$$\partial_3(\mathbf{u} + \mathbf{b}) \in L^{\frac{2}{1-r}}(0, T; \dot{X}_r) \quad (0 \le r \le 1). \tag{3}$$

First, let us convert (1) into a symmetric form. Writing

$$\boldsymbol{\omega}^{\pm} = \mathbf{u} \pm \mathbf{b},$$

we find by adding and subtracting $(1)_1$ with $(1)_2$,

$$\begin{cases}
\boldsymbol{\omega}_t^+ + (\boldsymbol{\omega}^- \cdot \nabla)\boldsymbol{\omega}^+ - \triangle\boldsymbol{\omega}^+ + \nabla\pi = \mathbf{0},\\
\boldsymbol{\omega}_t^- + (\boldsymbol{\omega}^+ \cdot \nabla)\boldsymbol{\omega}^- - \triangle\boldsymbol{\omega}^- + \nabla\pi = \mathbf{0},\\
\nabla\cdot\boldsymbol{\omega}^+ = \nabla\cdot\boldsymbol{\omega}^- = 0,\\
\boldsymbol{\omega}^+(0) = \boldsymbol{\omega}_0^+ \equiv \mathbf{u}_0 + \mathbf{b}_0, \qquad \boldsymbol{\omega}^-(0) = \boldsymbol{\omega}_0^- \equiv \mathbf{u}_0 - \mathbf{b}_0.
\end{cases} \tag{4}$$

The rest of the proof is organized as follows. In the first step, we dominate $\|\partial_3\boldsymbol{\omega}^{\pm}(t)\|_{L^2}$ and the time integral of $\|\nabla\partial_3\boldsymbol{\omega}^{\pm}(t)\|_{L^2}$, while the second step is devoted to controlling $\|\nabla\boldsymbol{\omega}^{\pm}(t)\|_{L^2}$.

Step 1. $\|\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2}$ estimates.

Taking the inner product of $(4)_1$ with $-\partial_3\partial_3\boldsymbol{\omega}^+$, $(4)_2$ with $-\partial_3\partial_3\boldsymbol{\omega}^-$ in $L^2(\mathbb{R}^3)$ respectively, adding them together, and noticing that $\nabla\cdot\boldsymbol{\omega}^{\pm}=0$, we obtain

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2}^2+\|\nabla\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2}^2\\
&\quad=-\int_{\mathbb{R}^3}\bigl[(\boldsymbol{\omega}^-\cdot\nabla)\boldsymbol{\omega}^+\bigr]\cdot\partial_3\partial_3\boldsymbol{\omega}^+\,\mathrm{d}x-\int_{\mathbb{R}^3}\bigl[(\boldsymbol{\omega}^+\cdot\nabla)\boldsymbol{\omega}^-\bigr]\cdot\partial_3\partial_3\boldsymbol{\omega}^-\,\mathrm{d}x\\
&\quad=\int_{\mathbb{R}^3}\bigl[(\partial_3\boldsymbol{\omega}^-\cdot\nabla)\boldsymbol{\omega}^+\bigr]\cdot\partial_3\boldsymbol{\omega}^+\,\mathrm{d}x+\int_{\mathbb{R}^3}\bigl[(\partial_3\boldsymbol{\omega}^+\cdot\nabla)\boldsymbol{\omega}^-\bigr]\cdot\partial_3\boldsymbol{\omega}^-\,\mathrm{d}x\\
&\quad\equiv I.
\end{aligned}\tag{5}$$

We now estimate I as

$$\begin{aligned}
I&\le\|\partial_3\boldsymbol{\omega}^+\cdot\partial_3\boldsymbol{\omega}^-\|_{L^2}\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}\quad\text{(Hölder inequality)}\\
&\le\|\partial_3\boldsymbol{\omega}^+\|_{\dot{X}_r}\|\partial_3\boldsymbol{\omega}^-\|_{\dot{H}^r}\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}\quad\text{(definition of multiplier spaces)}\\
&\le\|\partial_3\boldsymbol{\omega}^+\|_{\dot{X}_r}\|\partial_3\boldsymbol{\omega}^-\|_{L^2}^{1-r}\|\nabla\partial_3\boldsymbol{\omega}^-\|_{L^2}^{r}\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}\quad\text{(interpolation inequalities)}\\
&\le C\|\partial_3\boldsymbol{\omega}^+\|_{\dot{X}_r}^{\frac{2}{2-r}}\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^{\frac{2}{2-r}}\|\partial_3\boldsymbol{\omega}^-\|_{L^2}^{\frac{2(1-r)}{2-r}}+\frac{1}{2}\|\nabla\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2}^2\quad\text{(Young inequality)}\\
&\le C\Bigl[\|\partial_3\boldsymbol{\omega}^+\|_{\dot{X}_r}^{\frac{2}{1-r}}+\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^2\Bigr]\cdot\Bigl[\|\partial_3\boldsymbol{\omega}^-\|_{L^2}^2+1\Bigr]\\
&\quad+\frac{1}{2}\|\nabla\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2}^2\quad\text{(Young inequality).}
\end{aligned}\tag{6}$$

Plugging (6) into (5), invoking the Gronwall inequality, and noticing (3), we deduce

$$\sup_{0\le t\le T}\|\partial_3\boldsymbol{\omega}^{\pm}(t)\|_{L^2}\le C<\infty,\quad\text{and}\quad\int_0^T\|\nabla\partial_3\boldsymbol{\omega}^{\pm}(t)\|_{L^2}^2\,\mathrm{d}t\le C<\infty.\tag{7}$$

Step 2. $\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}$ estimates.

Taking the inner product of $(4)_1$ with $-\triangle\boldsymbol{\omega}^+$, $(4)_2$ with $-\triangle\boldsymbol{\omega}^-$ in $L^2(\mathbb{R}^3)$ respectively, adding them together, and noticing that $\nabla\cdot\boldsymbol{\omega}^{\pm}=0$, we obtain

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^2+\|\triangle\boldsymbol{\omega}^{\pm}\|_{L^2}^2\\
&\quad=-\int_{\mathbb{R}^3}\bigl[(\boldsymbol{\omega}^-\cdot\nabla)\boldsymbol{\omega}^+\bigr]\cdot\triangle\boldsymbol{\omega}^+\,\mathrm{d}x-\int_{\mathbb{R}^3}\bigl[(\boldsymbol{\omega}^+\cdot\nabla)\boldsymbol{\omega}^-\bigr]\cdot\triangle\boldsymbol{\omega}^-\,\mathrm{d}x\\
&\quad=\int_{\mathbb{R}^3}\bigl[(\nabla\boldsymbol{\omega}^-\cdot\nabla)\boldsymbol{\omega}^+\bigr]\cdot\nabla\boldsymbol{\omega}^+\,\mathrm{d}x+\int_{\mathbb{R}^3}\bigl[(\nabla\boldsymbol{\omega}^+\cdot\nabla)\boldsymbol{\omega}^-\bigr]\cdot\nabla\boldsymbol{\omega}^-\,\mathrm{d}x\\
&\quad\equiv J.
\end{aligned}\tag{8}$$

Utilizing interpolation inequalities and the following multiplicative Gagliardo-Nireberg inequalities

$$\|f\|_{L^6}\le C\|\partial_1 f\|_{L^2}^{\frac{1}{3}}\|\partial_2 f\|_{L^2}^{\frac{1}{3}}\|\partial_3 f\|_{L^2}^{\frac{1}{3}},\tag{9}$$

it follows that

$$\begin{aligned}
J &\le \|\nabla\boldsymbol{\omega}^{\pm}\|_{L^3}^{3} \\
&\le \|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^{\frac{3}{2}}\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^6}^{\frac{3}{2}} \quad \text{(interpolation inequalities)} \\
&\le C\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^{\frac{3}{2}}\|\nabla(\partial_1,\partial_2)\boldsymbol{\omega}^{\pm}\|_{L^2}\|\nabla\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2}^{\frac{1}{2}} \quad \text{(by (9))} \\
&\le C\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^{3}\|\nabla\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2} + \frac{1}{2}\|\triangle\boldsymbol{\omega}^{\pm}\|_{L^2}^{2} \quad \text{(Young inequality)} \\
&\le C\bigl[\|\nabla\partial_3\boldsymbol{\omega}^{\pm}\|_{L^2}^{2} + \|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^{2}\bigr]\cdot\|\nabla\boldsymbol{\omega}^{\pm}\|_{L^2}^{2} + \frac{1}{2}\|\triangle\boldsymbol{\omega}^{\pm}\|_{L^2}^{2} \quad \text{(Young inequality).}
\end{aligned} \tag{10}$$

Putting (10) into (8), utilizing the Gronwall inequality, and noticing (7), we deduce

$$\sup_{0\le t\le T}\|\nabla\boldsymbol{\omega}^{\pm}(t)\|_{L^2} \le C < \infty, \quad \text{and} \quad \int_0^T \|\triangle\boldsymbol{\omega}^{\pm}(t)\|_{L^2}^{2}\,\mathrm{d}t \le C < \infty,$$

and consequently,

$$\mathbf{u} = \frac{1}{2}(\boldsymbol{\omega}^{+} + \boldsymbol{\omega}^{-}) \in L^{\infty}(0,T;L^{6}(\mathbb{R}^3)).$$

The classical Serrin-type regularity criterion [6] then concludes the proof of Theorem 2.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
We declare that all authors collaborated and dedicated the same amount of time in order to perform this article.

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