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Research Article

Electroelastic Wave Scattering in a Cracked Dielectric Polymer under a Uniform Electric Field

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We investigate the scattering of plane harmonic compression and shear waves by a Griffith crack in an infinite isotropic dielectric polymer. The dielectric polymer is permeated by a uniform electric field normal to the crack face, and the incoming wave is applied in an arbitrary direction. By the use of Fourier transforms, we reduce the problem to that of solving two simultaneous dual integral equations. The solution of the dual integral equations is then expressed in terms of a pair of coupled Fredholm integral equations of the second kind having the kernel that is a finite integral. The dynamic stress intensity factor and energy release rate for mode I and mode II are computed for different wave frequencies and angles of incidence, and the influence of the electric field on the normalized values is displayed graphically.

1. Introduction

Elastic dielectrics such as insulating materials have been reported to have poor mechanical properties. Mechanical failure of insulators is also a well-known phenomenon. Therefore, understanding the fracture behavior of the elastic dielectrics will provide useful information to the insulation designers. Toupin [1] considered the isotropic elastic dielectric material and obtained the form of the constitutive relations for the stress and electric fields. Kurlandzka [2] investigated a crack problem of an elastic dielectric material subjected to an electrostatic field. Pak and Herrmann [3, 4] also derived a material force in the form of a path-independent integral for the elastic dielectric medium, which is related to the energy release rate. Recently, Shindo and Narita [5] considered the planar problem for an infinite dielectric polymer containing a crack under a uniform electric field, and discussed the stress intensity factor and energy release rate under mode I and mode II loadings.

This paper investigates the scattering of in-plane compressional (P) and shear (SV) waves by a Griffith crack in an infinite dielectric polymer permeated by a uniform electric field. The electric field is normal to the crack surface. Fourier transforms are used to reduce the problem to the solution of two simultaneous dual integral equations. The solution of the integral equations is then expressed in terms of a pair of coupled Fredholm integral equations of the second kind. In literature, there are two derivations of dual integral equations. One is the one mentioned in this paper. The other one is for the dual boundary element methods (BEM) [6, 7]. Numerical calculations are carried out for the dynamic stress intensity factor and energy release rate under mode I and mode II, and the results are shown graphically to demonstrate the effect of the electric field.

2. Basic Equations

Consider the rectangular Cartesian coordinate system with axes x_1, x_2, and x_3. We decompose the electric field intensity vector E_i, the polarization vector P_i, and the electric displacement vector D_i into those representing the rigid body state, indicated by overbars, and those for the deformed state, denoted by lower case letters:

$$E_i = \overline{E}_i + e_i, \qquad P_i = \overline{P}_i + p_i, \qquad D_i = \overline{D}_i + d_i. \tag{2.1}$$

We assume that the deformation will be small even with large electric fields, and the second terms will have only a minor influence on the total fields. The formulations will then be linearized with respect to these unknown deformed state quantities.

The linearized field equations are obtained as

$$\begin{gathered} \sigma^L_{ji,j} + \overline{E}_{i,j}p_j + \overline{P}_j e_{i,j} = \rho u_{i,tt}, \\ \overline{D}_{i,i} = 0, \\ d_{i,i} = 0, \end{gathered} \tag{2.2}$$

where u_i is the displacement vector, σ^L_{ij} is the local stress tensor, ρ is the mass density, a comma followed by an index denotes partial differentiation with respect to the space coordinate x_i or the time t, and the summation convention for repeated indices is applied.

The linearized constitutive equations can be written as

$$\begin{gathered} \sigma^L_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + A_1\left(\overline{E}_k\overline{E}_k + 2\overline{E}_k e_k\right)\delta_{ij} + A_2\left(\overline{E}_i\overline{E}_j + \overline{E}_i e_j + \overline{E}_j e_i\right), \\ \sigma^M_{ij} = \varepsilon_0\varepsilon_r\left(\overline{E}_i\overline{E}_j + \overline{E}_i e_j + \overline{E}_j e_i\right) - \frac{1}{2}\varepsilon_0\left(\overline{E}_k\overline{E}_k + 2\overline{E}_k e_k\right)\delta_{ij}, \\ \overline{D}_i = \varepsilon_0\overline{E}_i + \overline{P}_i = \varepsilon_0\varepsilon_r\overline{E}_i, \qquad d_i = \varepsilon_0 e_i + p_i = \varepsilon_0\varepsilon_r e_i, \\ \overline{E}_i = \frac{1}{\varepsilon_0\eta}\overline{P}_i, \qquad e_i = \frac{1}{\varepsilon_0\eta}p_i, \end{gathered} \tag{2.3}$$

where σ^M_{ij} is the Maxwell stress tensor, λ and μ are the Lamé constants, A_1 and A_2 are the electrostrictive coefficients, ε_0 is the permittivity of free space, $\varepsilon_r = 1 + \eta$ is the specific permittivity, η is the electric susceptibility, and δ_{ij} is the Kronecker delta.

The linearized boundary conditions are found as

$$
\begin{gathered}
\left[\left|\sigma_{ji}^{L}\right|\right]n_j + \frac{1}{2\varepsilon_0}\left[\left(\overline{P}_k n_k\right)^2 + 2\overline{P}_k p_l n_k n_l\right]n_i = 0, \\
\left[\left|\overline{D}_i\right|\right]n_i = 0, \\
e_{ijk} n_j \left[\left|\overline{E}_i\right|\right] = 0, \\
[|d_i|]n_i - \left[\left|\overline{D}_i\right|\right]u_{i,j}n_j = 0, \\
e_{ijk}\left\{n_j[|e_i|] - n_l n_{l,j}\left[\left|\overline{E}_i\right|\right]\right\} = 0,
\end{gathered}
\tag{2.4}
$$

where n_i is an outer unit vector normal to an undeformed body, e_{ijk} is the permutation symbol, and $[|f_i|]$ means the jump in any field quantity f_i across the discontinuity surface.

3. Problem Statement

Let a Griffith crack be located in the interior of an infinite elastic dielectric. We consider a rectangular Cartesian coordinate system (x, y, z) such that the crack is placed on the x-axis from $-a$ to a as shown in Figure 1, and assume that plane strain is perpendicular to the z-axis. A uniform electric field E_0 is applied perpendicular to the crack surface. For convenience, all electric quantities outside the solid will be denoted by the superscript +. The solution for the rigid body state is

$$
\begin{gathered}
\overline{E}_y^{+} = \varepsilon_r E_0, \qquad \overline{D}_y^{+} = \varepsilon_0\varepsilon_r E_0, \qquad \overline{P}_y^{+} = 0, \\
\overline{E}_y = E_0, \qquad \overline{D}_y = \varepsilon_0\varepsilon_r E_0, \qquad \overline{P}_y = \varepsilon_0 \eta E_0.
\end{gathered}
\tag{3.1}
$$

The equations of motion are given by

$$
\begin{gathered}
\nabla_1^2 u_x + \frac{1}{1-2\nu}\left(u_{x,x} + u_{y,y}\right)_{,x} + \frac{2A_1E_0}{\mu}e_{y,x} + \frac{A_3E_0}{\mu}e_{x,y} = \frac{1}{c_2^2}u_{x,tt}, \\
\nabla_1^2 u_y + \frac{1}{1-2\nu}\left(u_{x,x} + u_{y,y}\right)_{,y} + \frac{A_2E_0}{\mu}e_{x,x} + \frac{E_0}{\mu}(2A_1 + A_2 + A_3)e_{y,y} = \frac{1}{c_2^2}u_{y,tt},
\end{gathered}
\tag{3.2}
$$

where $\nabla_1^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator in the variables x, y, ν is the Poisson's ratio, $c_2 = (\mu/\rho)^{1/2}$ is the shear wave velocity, and $A_3 = A_2 + \varepsilon_0\eta$. The electric field equations for the perturbed state are

$$
e_{x,x} + e_{y,y} = 0, \qquad e_{x,x}^{+} + e_{y,y}^{+} = 0. \tag{3.3}
$$

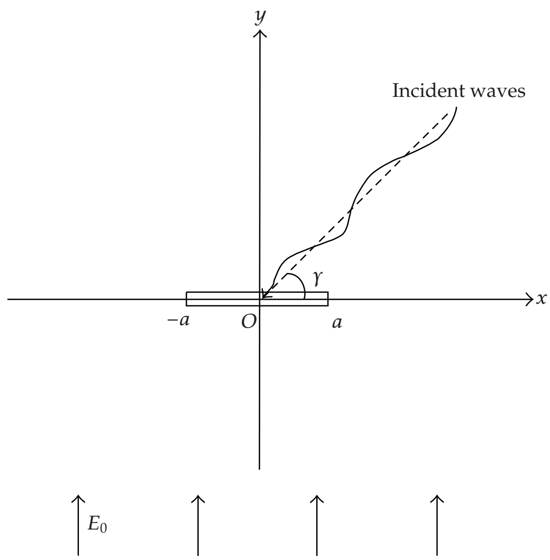

Figure 1: Scattering of waves in a dielectric medium with a Griffith crack.

The electric field equations (3.3) are satisfied by introducing an electric potential $\phi(x, y, t)$ such that

$$e_i = -\phi_{,i}, \qquad \nabla_1^2 \phi = 0,$$
$$e_i^+ = -\phi_{,i}^+, \qquad \nabla_1^2 \phi^+ = 0. \tag{3.4}$$

The displacement components can be written in terms of two scalar potentials $\varphi_e(x, y, t)$ and $\psi_e(x, y, t)$ as

$$u_x = \varphi_{e,x} + \psi_{e,y}, \qquad u_y = \varphi_{e,y} - \psi_{e,x}. \tag{3.5}$$

The equations of motion become

$$\nabla_1^2 \varphi_e - \frac{E_0}{\mu}(2A_1 + A_2 + A_3)\left(\frac{c_2}{c_1}\right)^2 \phi_{,y} = \frac{1}{c_1^2}\varphi_{e,tt},$$
$$\nabla_1^2 \psi_e + \frac{E_0}{\mu} A_2 \phi_{,x} = \frac{1}{c_2^2}\psi_{e,tt}, \tag{3.6}$$

where $c_1 = \{(\lambda + 2\mu)/\rho\}^{1/2}$ is the compression wave velocity.

Let an incident plane harmonic compression wave (P-wave) be directed at an angle γ with the x-axis so that

$$\varphi_e^i = \varphi_{e0} \exp\left[-i\omega\left\{t + \frac{x\cos\gamma + y\sin\gamma}{c_1}\right\}\right], \quad \psi_e^i = 0 \quad \text{(P-wave)}, \tag{3.7}$$

where φ_{e0} is the amplitude of the incident P-wave, and ω is the circular frequency. The superscript i stands for the incident component. Similarly, if an incident plane harmonic shear wave (SV-wave) impinges on the crack at an angle γ with x-axis, then

$$\varphi_e^i = 0, \quad \psi_e^i = \psi_{e0} \exp\left[-i\omega\left\{t + \frac{x\cos\gamma + y\sin\gamma}{c_2}\right\}\right] \quad \text{(SV-wave)}, \tag{3.8}$$

where ψ_{e0} is the amplitude of the incident SV-wave. In view of the harmonic time variation of the incident waves given by (3.7) and (3.8), the field quantities will all contain the time factor $\exp(-i\omega t)$ which will henceforth be dropped.

The problem may be split into two parts: one symmetric (opening mode, Mode I) and the other skew-symmetric (sliding mode, Mode II). Hence, the boundary conditions for the scattered fields are

Mode I:

$$\begin{gathered}
\sigma_{yx}^L(x,0) = 0 \quad (0 \le |x| < \infty), \\
\phi_{,x}(x,0) = -\eta E_0 u_{y,x}(x,0) + \phi_{,x}^+(x,0) \quad (0 \le |x| < a), \\
\phi(x,0) = 0 \quad (a \le |x| < \infty), \\
\sigma_{yy}^L(x,0) = -\varepsilon_0\eta^2 E_0 \phi_{,y} - p_j \exp(-i\alpha_j x\cos\gamma) \quad (j = 1,2)\ (0 \le |x| < a), \\
u_y(x,0) = 0 \quad (a \le |x| < \infty),
\end{gathered} \tag{3.9}$$

Mode II:

$$\begin{gathered}
\sigma_{yy}^L(x,0) = 0 \quad (0 \le |x| < \infty), \\
\phi_{,x}(x,0) = -\eta E_0 u_{y,x}(x,0) + \phi_{,x}^+(x,0) \quad (0 \le |x| < a), \\
\phi_{,y}(x,0) = 0 \quad (a \le |x| < \infty), \\
\sigma_{xy}^L(x,0) = -q_j \exp(-i\alpha_j x\cos\gamma) \quad (j = 1,2) \quad (0 \le |x| < a), \\
u_x(x,0) = 0, \quad (a \le |x| < \infty),
\end{gathered} \tag{3.10}$$

where the subscript $j = 1$ and 2 correspond to the incident P- and SV-waves, $p_1 = \mu\alpha_2^2\varphi_{e0}(1 - 2\sigma^2\cos^2\gamma)$, $p_2 = \mu\alpha_2^2\psi_{e0}\sin 2\gamma$, $q_1 = \mu\alpha_2^2\varphi_{e0}\sigma^2\sin 2\gamma$, $q_2 = \mu\alpha_2^2\psi_{e0}\cos 2\gamma$, $\alpha_1 = p/c_1$ and, $\alpha_2 = p/c_2$ are the compression and shear wave numbers, respectively, and $\sigma = c_2/c_1$.

4. Method of Solution

The desired solution of the original problem can be obtained by superposition of the solutions for the two cases: mode I and mode II. The problem will further be divided into two parts: (1) symmetric with respect to x and (2) antisymmetric with respect to x.

4.1. Mode I Problem

4.1.1. Symmetric Solution for Mode I Crack

The boundary conditions for symmetric scattered fields can be written as

$$\sigma^L_{yxs}(x,0) = 0 \quad (0 \le x < \infty), \tag{4.1}$$

$$\begin{aligned} \phi_{s,x}(x,0) &= -\eta E_0 u_{ys,x}(x,0) + \phi^+_{s,x}(x,0) \quad (0 \le x < a), \\ \phi_s(x,0) &= 0 \quad (a \le x < \infty), \end{aligned} \tag{4.2}$$

$$\begin{aligned} \sigma^L_{yys}(x,0) &= -\varepsilon_0 \eta^2 E_0 \phi_{s,y} - p_j \cos(\alpha_j x \cos\gamma) \quad (j = 1,2)\ (0 \le x < a), \\ u_{ys}(x,0) &= 0 \quad (a \le x < \infty), \end{aligned} \tag{4.3}$$

where the subscript s stands for the symmetric part. It can be shown that solutions ϕ_s, φ_{es}, ψ_{es}, and $\phi^+_s$of (3.4) and (3.6) for $y \ge 0$ are

$$\begin{aligned} \phi_s &= -\frac{2}{\pi}\int_0^\infty a_s(\alpha) e^{-\alpha y}\cos(\alpha x)d\alpha, \\ \varphi_{es} &= \frac{2}{\pi}\int_0^\infty \left\{A_{1s}(\alpha)e^{-\gamma_1(\alpha)y} + \left(\frac{c_2}{p}\right)^2 \frac{E_0}{\mu}(2A_1 + A_2 + A_3)\alpha a_s(\alpha)e^{-\alpha y}\right\}\cos(\alpha x)d\alpha, \end{aligned} \tag{4.4}$$

$$\psi_{es} = \frac{2}{\pi}\int_0^\infty \left\{A_{2s}(\alpha)e^{-\gamma_2(\alpha)y} - \left(\frac{c_2}{p}\right)^2 \frac{E_0}{\mu} A_2 \alpha a_s(\alpha)e^{-\alpha y}\right\}\sin(\alpha x)d\alpha, \tag{4.5}$$

$$\phi^+_s = -\frac{2}{\pi}\int_0^\infty a^+_s(\alpha)\sinh(\alpha y) cos(\alpha x)d\alpha, \tag{4.6}$$

where $a_s(\alpha)$, $A_{1s}(\alpha)$, $A_{2s}(\alpha)$, and $a^+_s(\alpha)$ are unknown functions, and $\gamma_1(\alpha)$ and $\gamma_2(\alpha)$ are

$$\gamma_1(\alpha) = \left\{\alpha^2 - \left(\frac{\mathrm{p}}{\mathrm{c}_1}\right)^2\right\}^{1/2}, \qquad \gamma_2(\alpha) = \left\{\alpha^2 - \left(\frac{\mathrm{p}}{\mathrm{c}_2}\right)^2\right\}^{1/2}. \tag{4.7}$$

The functions $\gamma_1(\alpha)$ and $\gamma_2(\alpha)$ should be restricted as

$$\mathrm{Re}\,\gamma_k(\alpha) > 0, \quad \mathrm{Im}\gamma_k(\alpha) < 0 \quad (k = 1,2) \tag{4.8}$$

in the upper half-space $y \ge 0$, because of a radiation condition at infinity and an edge condition near the crack tip. A simple calculation leads to the displacement and stress expressions:

$$
\begin{aligned}
u_{xs} &= -\frac{2}{\pi}\int_0^\infty \Bigg[\alpha A_{1s}(\alpha)e^{-\gamma_1(\alpha)y} + \gamma_2(\alpha)A_{2s}(\alpha)e^{-\gamma_2(\alpha)y} \\
&\qquad + \left(\frac{c_2}{p}\right)^2 \frac{E_0}{\mu}(2A_1 + A_3)\alpha^2 a_s(\alpha)e^{-\alpha y}\Bigg] \sin(\alpha x)d\alpha, \\
u_{ys} &= -\frac{2}{\pi}\int_0^\infty \Bigg[\gamma_1(\alpha)A_{1s}(\alpha)e^{-\gamma_1(\alpha)y} + \alpha A_{2s}(\alpha)e^{-\gamma_2(\alpha)y} \\
&\qquad + \left(\frac{c_2}{p}\right)^2 \frac{E_0}{\mu}(2A_1 + A_3)\alpha^2 a_s(\alpha)e^{-\alpha y}\Bigg] \cos(\alpha x)d\alpha, \\
\sigma_{xxs}^L &= -\frac{4}{\pi}\mu\int_0^\infty \Bigg[\left\{\frac{\lambda}{2\mu}\left(\frac{p}{c_1}\right)^2 + \alpha^2\right\} A_{1s}(\alpha)e^{-\gamma_1(\alpha)y} + \alpha\gamma_2(\alpha)A_{2s}(\alpha)e^{-\gamma_2(\alpha)y} \\
&\qquad + \frac{E_0}{\mu}\left\{\left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 + A_1\right\}\alpha a_s(\alpha)e^{-\alpha y}\Bigg] \cos(\alpha x)d\alpha + A_1 E_0^2, \\
\sigma_{xys}^L &= \frac{2}{\pi}\mu\int_0^\infty \Bigg[2\alpha\gamma_1(\alpha)A_{1s}(\alpha)e^{-\gamma_1(\alpha)y} + \left\{2\alpha^2 - \left(\frac{p}{c_2}\right)^2\right\} A_{2s}(\alpha)e^{-\gamma_2(\alpha)y} \\
&\qquad + \frac{E_0}{\mu}\left\{2\left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 - A_2\right\}\alpha a_s(\alpha)e^{-\alpha y}\Bigg] \sin(\alpha x)d\alpha, \\
\sigma_{yys}^L &= \frac{4}{\pi}\mu\int_0^\infty \Bigg[\left\{-\frac{1}{2}\left(\frac{p}{c_2}\right)^2 + \alpha^2\right\} A_{1s}(\alpha)e^{-\gamma_1(\alpha)y} + \alpha\gamma_2(\alpha)A_{2s}(\alpha)e^{-\gamma_2(\alpha)y} \\
&\qquad + \frac{E_0}{\mu}\left\{\left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 - (A_1 + A_2)\right\}\alpha a_s(\alpha)e^{-\alpha y}\Bigg] \cos(\alpha x)d\alpha + (A_1 + A_2)E_0^2, \\
\sigma_{xxs}^M &= \frac{2}{\pi}\varepsilon_0 E_0\int_0^\infty \alpha a_s(\alpha)e^{-\alpha y}\cos(\alpha x)d\alpha - \frac{\varepsilon_0 E_0^2}{2}, \\
\sigma_{xys}^M &= -\frac{2}{\pi}\varepsilon_0\varepsilon_r E_0\int_0^\infty \alpha a_s(\alpha)e^{-\alpha y}\sin(\alpha x)d\alpha, \\
\sigma_{yys}^M &= -\frac{2}{\pi}(1 + 2\eta)\varepsilon_0 E_0\int_0^\infty \alpha a_s(\alpha)e^{-\alpha y}\cos(\alpha x)d\alpha + \frac{\varepsilon_0 E_0^2(1 + 2\eta)}{2}.
\end{aligned}
\tag{4.9}
$$

The boundary condition of (4.1) leads to the following relation between unknown functions:

$$
2\alpha\gamma_1(\alpha)A_{1s}(\alpha) + \left\{2\alpha^2 - \left(\frac{p}{c_2}\right)^2\right\} A_{2s}(\alpha) + \frac{E_0}{\mu}\left\{2\left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 - A_2\right\}\alpha a_s(\alpha) = 0. \tag{4.10}
$$

The satisfaction of the two mixed boundary conditions (4.2) and (4.3) leads to two simultaneous dual integral equations of the following form:

$$\int_0^\infty \alpha\left[a_s(\alpha)+\eta E_0 A_s(\alpha)\right]\sin(\alpha x)d\alpha=0 \quad (0\le x<a),$$
$$\int_0^\infty a_s(\alpha)\cos(\alpha x)d\alpha=0 \quad (a\le x<\infty), \tag{4.11}$$

$$\int_0^\infty \alpha\left[f_e(\alpha)A_s(\alpha)+\frac{E_0}{\mu}f_m(\alpha)a_s(\alpha)\right]\cos(\alpha x)d\alpha=-\frac{\pi}{4\mu}p_j\cos(\alpha_j x\cos\gamma) \quad (0\le x<a),$$
$$\int_0^\infty A_s(\alpha)\cos(\alpha x)d\alpha=0 \quad (a\le x<\infty), \tag{4.12}$$

in which $f_e(\alpha)$ and $f_m(\alpha)$ are known functions given by

$$f_e(\alpha)=\frac{1}{\gamma_1(\alpha)(p/c_2)^2}\left[-\left\{2\alpha^2-\left(\frac{p}{c_2}\right)^2\right\}^2+4\alpha^2\gamma_1(\alpha)\gamma_2(\alpha)\right]\frac{1}{2\alpha},$$
$$f_m(\alpha)=\frac{1}{\gamma_1(\alpha)(p/c_2)^2}\left[-\left\{2\alpha^2-\left(\frac{p}{c_2}\right)^2\right\}(2A_1+\varepsilon_0\eta)+2\gamma_1(\alpha)\gamma_2(\alpha)A_2\right.$$
$$\left.+2\alpha\gamma_1(\alpha)(2A_1+A_3)-\frac{1}{\alpha}\gamma_1(\alpha)\left(\frac{p}{c_2}\right)^2\left(2A_1+2A_2-\varepsilon_0\eta^2\right)\right]\frac{\alpha}{2}, \tag{4.13}$$

and the original unknowns $A_{1s}(\alpha)$ and $A_{2s}(\alpha)$ are related to the new one $A_s(\alpha)$ through

$$A_{1s}(\alpha)=-\frac{1}{\gamma_1(\alpha)(p/c_2)^2}\left[\left\{2\alpha^2-(p/c_2)^2\right\}A_s(\alpha)+\frac{E_0}{\mu}(2A_1+\varepsilon_0\eta)\alpha^2 a_s(\alpha)\right],$$
$$A_{2s}(\alpha)=\frac{1}{(p/c_2)^2}\left[2\alpha A_s(\alpha)-\frac{E_0}{\mu}A_2\alpha a_s(\alpha)\right]. \tag{4.14}$$

The set of two simultaneous dual integral equations (4.11) and (4.12) may be solved by using a new function $\Phi_s(u)$, and the result is

$$A_s(\alpha)=\frac{\pi}{4}\left(\frac{p_j a^2}{\mu c_0 y_0}\right)\int_0^1 u^{1/2}\Phi_s(u)J_0(\alpha a u)du,$$
$$a_s(\alpha)=-\frac{\pi}{4}\left(\frac{\eta E_0 p_j a^2}{\mu c_0 y_0}\right)\int_0^1 u^{1/2}\Phi_s(u)J_0(\alpha a u)du, \tag{4.15}$$

where $J_0()$ is the zero-order Bessel function of the first kind, and c_0 and y_0 are

$$c_0 = \left(\frac{c_2}{c_1}\right)^2 - 1,$$
$$y_0 = 1 + \frac{1}{2}\left[(1-2\nu)(2A_1 + \varepsilon_0\eta) - 2(1-\nu)\left(\varepsilon_0\eta^2 + \varepsilon_0\eta - A_2\right)\right]\frac{\eta}{\mu}E_0^2. \tag{4.16}$$

The function $\Phi_s(u)$ is governed by the following Fredholm integral equation of second kind:

$$\Phi_s(u) - \int_0^1 \Phi_s(s)(su)^{1/2}K_s(u,s)ds = u^{1/2}J_0(\alpha_j au\cos\gamma), \tag{4.17}$$

where the kernel $K_s(u,s)$ is given by

$$K_s(u,s) = \int_0^\infty \left[\alpha + \frac{1}{c_0y_0P^2}\left\{f_e^*(\alpha) - \eta E_\mu^2 f_m^*(\alpha)\right\}\right]J_0(\alpha u)J_0(\alpha s)d\alpha, \tag{4.18}$$

$$f_e^*(\alpha) = \frac{1}{2\gamma_1^*(\alpha)}\left[-\left(2\alpha^2 - P^2\right)^2 + 4\alpha^2\gamma_1^*(\alpha)\gamma_2^*(\alpha)\right],$$
$$f_m^*(\alpha) = \frac{1}{2\gamma_1^*(\alpha)}\left[-\alpha^2\left(2\alpha^2 - P^2\right)(2A_{e1}+\eta) - 2\alpha^2\gamma_1^*(\alpha)\gamma_2^*(\alpha)A_{e2}\right.$$
$$\left. + 2\alpha^3\gamma_1^*(\alpha)(2A_{e1} + A_{e2} + \eta) - \alpha\gamma_1^*(\alpha)P^2\left(2A_{e1} + A_{e2} - \eta^2\right)\right], \tag{4.19}$$

$$\gamma_1^*(\alpha) = \left\{\alpha^2 - (P\sigma)^2\right\}^{1/2}, \qquad \gamma_2^*(\alpha) = \left(\alpha^2 - P^2\right)^{1/2},$$
$$E_\mu^2 = \frac{\varepsilon_0 E_0^2}{\mu}, \qquad A_{e1} = \frac{A_1}{\varepsilon_0}, \qquad A_{e2} = \frac{A_2}{\varepsilon_0}, \qquad P = \frac{ap}{c_2}, \qquad \sigma = \frac{c_2}{c_1}. \tag{4.20}$$

The kernel function $K_s(u,s)$ (4.18) is an infinite integral that has a rather slow of convergence. To improve this problem the infinite integral is converted into integrals with finite limits. Thus, for the calculation of the integral, we consider the contour integrals

$$I_{e1} = \oint_{\Gamma_1} L_e(k,\gamma_1^*,-\gamma_2^*)J_0(ks)H_0^{(1)}(ku)dk \quad (u > s),$$
$$I_{e2} = \oint_{\Gamma_2} L_e(k,\gamma_1^*,\gamma_2^*)J_0(ks)H_0^{(2)}(ku)dk \quad (u > s), \tag{4.21}$$

where the contours Γ_1,Γ_2 are defined in Figure 2, $H_0^{(1)}(), H_0^{(2)}()$ are, respectively, the zero-order Hankel functions of the first and second kinds, and

$$L_e(k,\gamma_1^*,\gamma_2^*) = k + \frac{1}{c_0y_0P}\left\{f_e^*(k) - \eta E_\mu^2 f_m^*(k)\right\}. \tag{4.22}$$

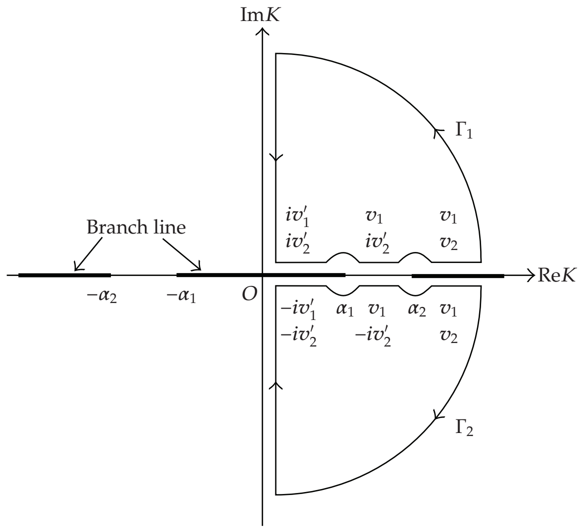

Figure 2: The counters of integration.

The integrands in (4.21) satisfy Jordan's lemma on the infinite quarter circles, so that,

$$\begin{aligned}I_{e1}+I_{e2}=&\int_0^{\alpha_1}\left\{L_e(\alpha,i\nu'_1,i\nu'_2)H_0^{(1)}(\alpha u)d\alpha+L_e(\alpha,-i\nu'_1,-i\nu'_2)H_0^{(2)}(\alpha u)\right\}J_0(\alpha s)d\alpha\\&+\int_{\alpha_1}^{\alpha_2}\left\{L_e(\alpha,\nu_1,i\nu'_2)H_0^{(1)}(\alpha u)d\alpha+L_e(\alpha,\nu_1,-i\nu'_2)H_0^{(2)}(\alpha u)\right\}J_0(\alpha s)d\alpha\\&+2\int_{\alpha_2}^{\infty}L_e(\alpha,\nu_1,\nu_2)J_0(\alpha s)J_0(\alpha u)d\alpha\\&+\int_{\infty}^{0}\{L_e(i\alpha,i\nu'_1,i\nu'_2)+L_e(-i\alpha,-i\nu'_1,-i\nu'_2)\}J_0\left(e^{i\pi/2}\alpha s\right)H_0^{(1)}\left(e^{i\pi/2}\alpha u\right)i\,d\alpha=0,\end{aligned}\tag{4.23}$$

where

$$\begin{aligned}\nu_1&=\left(\alpha^2-P^2\sigma^2\right)^{1/2}, &\nu_2&=\left(\alpha^2-P^2\right)^{1/2},\\ \nu'_1&=\left(P^2\sigma^2-\alpha^2\right)^{1/2}, &\nu'_2&=\left(P^2-\alpha^2\right)^{1/2}.\end{aligned}\tag{4.24}$$

Because of the second of (4.8), the integral in (4.18) must be taken along a path located slightly below the real k-axis as in Γ_2. Therefore $K_s(u,s)$ for $u>s$ can be finally written as

$$K_s(u,s)=iP^2\int_0^1\left\{M_1(\alpha)J_0(\alpha Ps)H_0^{(1)}(\alpha Pu)+M_2(\alpha)J_0(\alpha\sigma Ps)H_0^{(1)}(\alpha\sigma Pu)\right\}d\alpha,\quad(u>s),\tag{4.25}$$

where

$$M_1(\alpha) = -\frac{1}{c_0 y_0}\left(2+\eta E_\mu^2 A_{e2}\right)\alpha^2\left(1-\alpha^2\right)^{1/2},$$
$$M_2(\alpha) = -\frac{1}{c_0 y_0}\frac{1}{2(1-\alpha^2)^{1/2}}\left\{\left(2\alpha^2\sigma^2-1\right)^2-\eta E_\mu^2\sigma^2\left(2\alpha^2\sigma^2-1\right)\alpha^2(2A_{e1}+\eta)\right\}. \tag{4.26}$$

The kernel $K_s(u,s)$ is symmetric in u, s, and the value of this kernel for $u < s$ is obtained by interchanging u and s in (4.25).

4.1.2. Antisymmetric Solution for Mode I Crack

The boundary conditions for anti-symmetric scattered fields can be written as

$$\sigma_{yxa}^L(x,0) = 0 \quad (0 \le x < \infty), \tag{4.27}$$

$$\phi_{a,x}(x,0) = -\eta E_0 u_{ya,x}(x,0) + \phi_{a,x}^+(x,0), \quad (0 \le x < a),$$
$$\phi_a(x,0) = 0 \quad (a \le x < \infty), \tag{4.28}$$

$$\sigma_{yya}^L(x,0) = -\varepsilon_0\eta^2 E_0\phi_{a,y} - p_j\sin(\alpha_j x\cos\gamma) \quad (j=1,2)\ (0 \le x < a),$$
$$u_{ya}(x,0) = 0 \quad (a \le x < \infty), \tag{4.29}$$

where the subscript a stands for the anti-symmetric part. The solutions ϕ_a, φ_{ea}, ψ_{ea} and ϕ_a^+ are

$$\phi_a = -\frac{2}{\pi}\int_0^\infty a_a(\alpha)e^{-\alpha y}\sin(\alpha x)d\alpha,$$
$$\varphi_{ea} = \frac{2}{\pi}\int_0^\infty\left\{A_{1a}(\alpha)e^{-\gamma_1(\alpha)y}+\left(\frac{c_2}{p}\right)^2\frac{E_0}{\mu}(2A_1+A_2+A_3)\alpha a_a(\alpha)e^{-\alpha y}\right\}\sin(\alpha x)d\alpha, \tag{4.30}$$

$$\psi_{ea} = \frac{2}{\pi}\int_0^\infty\left\{A_{2a}(\alpha)e^{-\gamma_2(\alpha)y}-\left(\frac{c_2}{p}\right)^2\frac{E_0}{\mu}A_2\alpha a_a(\alpha)e^{-\alpha y}\right\}\cos(\alpha x)d\alpha, \tag{4.31}$$

$$\phi_a^+ = -\frac{2}{\pi}\int_0^\infty a_a^+(\alpha)\cosh(\alpha y)\sin(\alpha x)d\alpha, \tag{4.32}$$

where $a_a(\alpha)$, $A_{1a}(\alpha)$, $A_{2a}(\alpha)$, and $a_a^+(\alpha)$ are unknown functions. The displacements and stresses are obtained as

$$\begin{aligned} u_{xa} &= \frac{2}{\pi}\int_0^\infty \Bigg\{ \alpha A_{1a}(\alpha)e^{-\gamma_1(\alpha)y} - \gamma_2(\alpha)A_{2a}(\alpha)e^{-\gamma_2(\alpha)y} \\ &\quad + \left(\frac{c_2}{p}\right)^2 \frac{E_0}{\mu}(2A_1 + A_3)\alpha^2 a_a(\alpha)e^{-\alpha y} \Bigg\} \cos(\alpha x)d\alpha, \\ u_{ya} &= -\frac{2}{\pi}\int_0^\infty \Bigg\{ \gamma_1(\alpha)A_{1a}(\alpha)e^{-\gamma_1(\alpha)y} - \alpha A_{2a}(\alpha)e^{-\gamma_2(\alpha)y} \\ &\quad + \left(\frac{c_2}{p}\right)^2 \frac{E_0}{\mu}(2A_1 + A_3)\alpha^2 a_a(\alpha)e^{-\alpha y} \Bigg\} \sin(\alpha x)d\alpha, \end{aligned} \tag{4.33}$$

$$\begin{aligned} \sigma_{xxa}^L &= -\frac{4}{\pi}\mu\int_0^\infty \Bigg[\left\{ \frac{\lambda}{2\mu}\left(\frac{p}{c_1}\right)^2 + \alpha^2 \right\} A_{1a}(\alpha)e^{-\gamma_1(\alpha)y} - \alpha\gamma_2(\alpha)A_{2a}(\alpha)e^{-\gamma_2(\alpha)y} \\ &\quad + \frac{E_0}{\mu}\left\{ \left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 + A_1 \right\} \alpha a_a(\alpha)e^{-\alpha y} \Bigg] \sin(\alpha x)d\alpha, \\ \sigma_{xya}^L &= -\frac{2}{\pi}\mu\int_0^\infty \Bigg[2\alpha\gamma_1(\alpha)A_{1a}e^{-\gamma_1(\alpha)y} - \left\{ 2\alpha^2 - \left(\frac{p}{c_2}\right)^2 \right\} A_{2a}(\alpha)e^{-\gamma_2(\alpha)y} \\ &\quad + \frac{E_0}{\mu}\left\{ 2\left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 - A_2 \right\} \alpha a_a(\alpha)e^{-\alpha y} \Bigg] \cos(\alpha x)d\alpha, \\ \sigma_{yya}^L &= \frac{4}{\pi}\mu\int_0^\infty \Bigg[\left\{ -\frac{1}{2}\left(\frac{p}{c_2}\right)^2 + \alpha^2 \right\} A_{1a}(\alpha)e^{-\gamma_1(\alpha)y} - \alpha\gamma_2(\alpha)A_{2a}(\alpha)e^{-\gamma_2(\alpha)y} \\ &\quad + \frac{E_0}{\mu}\left\{ \left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 - (A_1 + A_2) \right\} \alpha a_a(\alpha)e^{-\alpha y} \Bigg] \sin(\alpha x)d\alpha, \end{aligned} \tag{4.34}$$

$$\begin{aligned} \sigma_{xxa}^M &= \frac{2}{\pi}\varepsilon_0 E_0\int_0^\infty \alpha a_a(\alpha)e^{-\alpha y}\sin(\alpha x)d\alpha, \\ \sigma_{xya}^M &= \frac{2}{\pi}\varepsilon_0\varepsilon_r E_0\int_0^\infty \alpha a_a(\alpha)e^{-\alpha y}\cos(\alpha x)d\alpha, \\ \sigma_{yya}^M &= -\frac{2}{\pi}(1+2\eta)\varepsilon_0 E_0\int_0^\infty \alpha a_a(\alpha)e^{-\alpha y}\sin(\alpha x)d\alpha. \end{aligned} \tag{4.35}$$

The relation between unknown functions can be found by the same procedure as in the symmetric case. The boundary condition of (4.27) leads to the following relation:

$$2\alpha\gamma_1(\alpha)A_{1a}(\alpha) - \left\{ 2\alpha^2 - \left(\frac{p}{c_2}\right)^2 \right\} A_{2a}(\alpha) + \frac{E_0}{\mu}\left\{ 2\left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 - A_2 \right\} \alpha a_a(\alpha) = 0. \tag{4.36}$$

The boundary conditions in (4.28) and (4.29) lead to two simultaneous dual integral equations of the following form:

$$\int_0^\infty \alpha\{a_a(\alpha) + \eta E_0 A_a(\alpha)\}\cos(\alpha x)d\alpha = 0 \quad (0 \le x < a),$$
$$\int_0^\infty a_a(\alpha)\sin(\alpha x)d\alpha = 0 \quad (a \le x < \infty), \tag{4.37}$$

$$\int_0^\infty \alpha\left\{f_e(\alpha)A_a(\alpha) + \frac{E_0}{\mu}f_m(\alpha)a_a(\alpha)\right\}\sin(\alpha x)d\alpha = -\frac{\pi}{4\mu}p_j\sin(\alpha_j x\cos\gamma) \quad (0 \le x < a),$$
$$\int_0^\infty A_a(\alpha)\sin(\alpha x)d\alpha = 0 \quad (a \le x < \infty), \tag{4.38}$$

in which the original unknowns $A_{1a}(\alpha), A_{2a}(\alpha)$ are related to the new one $A_a(\alpha)$ through

$$A_{1a}(\alpha) = -\frac{1}{\gamma_1(\alpha)(p/c_2)^2}\left[\left\{2\alpha^2 - \left(\frac{p}{c_2}\right)^2\right\}A_a(\alpha) + \frac{E_0}{\mu}(2A_1 + \varepsilon_0\eta)\alpha^2 a_a(\alpha)\right],$$
$$A_{2a}(\alpha) = -\frac{1}{(p/c_2)^2}\left\{2\alpha A_a(\alpha) - \frac{E_0}{\mu}A_2\alpha a_a(\alpha)\right\}. \tag{4.39}$$

The unknowns $A_a(\alpha)$ and $a_a(\alpha)$ can be found by the same method of approach as in the symmetric case. The results are

$$A_a(\alpha) = \frac{\pi}{4}\left(\frac{p_j a^2}{\mu c_0 y_0}\right)\int_0^1 u^{1/2}\Phi_a(u)J_1(\alpha a u)du,$$
$$a_a(\alpha) = -\frac{\pi}{4}\left(\frac{\eta E_0 p_j a^2}{\mu c_0 y_0}\right)\int_0^1 u^{1/2}\Phi_a(u)J_1(\alpha a u)du, \tag{4.40}$$

where $J_1()$ is the first-order Bessel function of the first kind, and $\Phi_a(u)$ in (4.40) is the solution of the following Fredholm integral equation of the second kind:

$$\Phi_a(u) - \int_0^1 \Phi_a(s)(su)^{1/2}K_a(u,s)ds = u^{1/2}J_1(\alpha_j a u\cos\gamma), \tag{4.41}$$

where

$$K_a(u,s) = \int_0^\infty \left[\alpha + \frac{1}{c_0 y_0 P^2}\left\{f_e^*(\alpha) - \eta E_\mu^2 f_m^*(\alpha)\right\}\right]J_1(\alpha u)J_1(\alpha s)d\alpha \quad (u > s). \tag{4.42}$$

By using the contours of integration in Figure 2, the kernel $K_a(u,s)$ for $u > s$ can be rewritten in the form

$$K_a(u,s) = iP^2 \int_0^1 \left\{ M_1(\alpha) J_1(\alpha P s) H_1^{(1)}(\alpha P u) + M_2(\alpha) J_1(\alpha\sigma P s) H_1^{(1)}(\alpha\sigma P u) \right\} d\alpha \quad (u > s), \tag{4.43}$$

where $H_1^{(1)}()$ is the first-order Hankel function of the first kind. The value of $K_a(u,s)$ for $u < s$ is obtained by interchanging u and s in (4.43).

4.1.3. Mode I Dynamic Singular Stresses Near the Crack Tip

The mode I dynamic electric stress intensity factor K_{ID} is

$$\begin{aligned} K_{ID} &= \lim_{x \to a^+} \{2\pi(x-a)\}^{1/2} \left\{ \sigma^L_{yys} + \sigma^L_{yya} + \sigma^M_{yys} + \sigma^M_{yya} \right\}_{y=0} \\ &= p_j(\pi a)^{1/2} \frac{z_0}{y_0} [\Phi_s(1) - i\Phi_a(1)], \end{aligned} \tag{4.44}$$

where

$$z_0 = 1 + \frac{1}{2}\{(1-2\nu)(2A_{e1} + \eta) + 2(1-\nu)(A_{e2} + \eta + 1)\}\eta E_\mu^2. \tag{4.45}$$

Next, we examine the static electroelastic crack problem. The boundary conditions may be written as

$$\sigma^L_{yx}(x,0) = 0 \quad (0 \le x < \infty), \tag{4.46}$$

$$\begin{aligned} \phi_{,x}(x,0) = -\eta E_0 u_{y,x}(x,0) + \phi^+_{,x}(x,0), \quad (0 \le x < a), \\ \phi(x,0) = 0 \quad (a \le x < \infty), \end{aligned} \tag{4.47}$$

$$\begin{aligned} \sigma^L_{yy}(x,0) &= \varepsilon_0 \eta^2 \left\{ \frac{E_0^2}{2} - E_0 \phi_{,y} \right\} \quad (0 \le x < a), \\ u_y(x,0) &= 0 \quad (a \le x < \infty). \end{aligned} \tag{4.48}$$

The electric stress intensity factor K_{IS} may be obtained as

$$K_{IS} = \mu E_\mu^2 (\pi a)^{1/2} \left(\frac{z_0}{y_0} \right) \frac{2A_{e1} + 2A_{e2} - \eta^2}{2}. \tag{4.49}$$

The dynamic stress intensity factor K_I can be found as

$$K_I = |K_{ID}| + K_{IS}. \tag{4.50}$$

The dynamic electroelastic stress is given by

$$\sigma_{ij}^{c} = \sigma_{ij}^{L(i)} + \sigma_{ij}^{L(s)} + \sigma_{ij}^{M(i)} + \sigma_{ij}^{M(s)}. \tag{4.51}$$

The singular parts of the dynamic local stresses and Mexwell stresses near the crack tip can be expressed as

$$\begin{aligned}
\sigma_{xx}^{L} &\sim \frac{K_I}{2z_0}\left\{\left[2+\left\{2(1-2\nu)A_{e1}+2(1-\nu)A_{e2}-\eta\right\}E_\mu^2\eta\right]\right.\\
&\qquad\left.-\left[2+\left\{(1-2\nu)(2A_{e1}+\eta)+2(1-\nu)A_{e2}\right\}E_\mu^2\eta\right]\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right\}\cos\frac{\theta}{2}\frac{1}{(2\pi r)^{1/2}},\\
\sigma_{xy}^{L} &\sim \frac{K_I}{2z_0}\left[2+\left\{2(1-2\nu)(2A_{e1}+\eta)+2(1-\nu)A_{e2}\right\}E_\mu^2\eta\right]\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\frac{1}{(2\pi r)^{1/2}},\\
\sigma_{yy}^{L} &\sim \frac{K_I}{2z_0}\left\{\left[2+\left\{2(1-2\nu)A_{e1}+2(1-\nu)A_{e2}-\eta\right\}E_\mu^2\eta\right]\right.\\
&\qquad\left.+\left[2+\left\{(1-2\nu)(2A_{e1}+\eta)+2(1-\nu)A_{e2}\right\}E_\mu^2\eta\right]\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right\}\cos\frac{\theta}{2}\frac{1}{(2\pi r)^{1/2}},
\end{aligned} \tag{4.52}$$

$$\begin{aligned}
\sigma_{xx}^{M} &\sim -\frac{K_I}{z_0}(1-\nu)\eta E_\mu^2\cos\frac{\theta}{2}\frac{1}{(2\pi r)^{1/2}},\\
\sigma_{xy}^{M} &\sim -\frac{K_I}{z_0}(1-\nu)\eta\varepsilon_r E_\mu^2\sin\frac{\theta}{2}\frac{1}{(2\pi r)^{1/2}},\\
\sigma_{yy}^{M} &\sim \frac{K_I}{z_0}(1-\nu)(1+2\eta)\eta E_\mu^2\cos\frac{\theta}{2}\frac{1}{(2\pi r)^{1/2}},
\end{aligned} \tag{4.53}$$

where $r = \{(x-a)^2 + y^2\}^{1/2}$ and $\theta = \tan^{-1}(y/(x-a))$ are the polar coordinates. Also, the singular parts of the displacements and electric fields near the crack tip are

$$\begin{aligned}
u_x &\sim \frac{K_I}{2z_0\mu}\left(\frac{r}{2\pi}\right)^{1/2}\left\{2(1-2\nu)-\left\{(1-2\nu)(A_{e1}+\eta)-2(1-\nu)A_{e2}\right\}E_\mu^2\eta\right.\\
&\qquad\left.+\left[2+\left\{(1-2\nu)(A_{e1}+\eta)+2(1-\nu)A_{e2}\right\}E_\mu^2\eta\right]\sin^2\frac{\theta}{2}\right\}\cos\frac{\theta}{2},\\
u_y &\sim \frac{K_I}{2z_0\mu}\left(\frac{r}{2\pi}\right)^{1/2}\left\{4(1-\nu)+\left[2+\left\{(1-2\nu)(A_{e1}+\eta)+2(1-\nu)A_{e2}\right\}E_\mu^2\eta\right]\cos^2\frac{\theta}{2}\right\}\sin\frac{\theta}{2},
\end{aligned} \tag{4.54}$$

$$\begin{aligned}
E_x &\sim -\frac{K_I}{z_0\mu}\frac{1}{(2\pi r)^{1/2}}(1-\nu)\eta E_0\sin\frac{\theta}{2},\\
E_y &\sim \frac{K_I}{z_0\mu}\frac{1}{(2\pi r)^{1/2}}(1-\nu)\eta E_0\cos\frac{\theta}{2}.
\end{aligned} \tag{4.55}$$

4.2. Mode II Problem

Since the mode II problem may also be reduced to the solution of two simultaneous dual integral equations in the same way as the mode I, many of the details of solution procedure will be omitted and only the essential steps will be provided.

4.2.1. Symmetric Solution for Mode II Crack

The boundary conditions for symmetric scattered fields are

$$\sigma_{yys}^{L}(x,0) = 0 \quad (0 \le x < \infty), \tag{4.56}$$

$$\begin{aligned} \phi_{s,x}(x,0) = -\eta E_0 u_{ys,x}(x,0) + \phi_{s,x}^{+}(x,0) \quad (0 \le x < a), \\ \phi_{s,y}(x,0) = 0 \quad (a \le x < \infty), \end{aligned} \tag{4.57}$$

$$\begin{aligned} \sigma_{xys}^{L}(x,0) = -q_j \cos(\alpha_j x \cos\gamma) \quad (j = 1,2)\ (0 \le x < a), \\ u_{xs}(x,0) = 0 \quad (a \le x < \infty). \end{aligned} \tag{4.58}$$

Replace the subscript a by s, $a_a(\alpha)$, $A_{1a}(\alpha)$, $A_{2a}(\alpha)$, and $a_a^{+}(\alpha)$ by $b_s(\alpha)$, $B_{1s}(\alpha)$, $B_{2s}(\alpha)$, and $b_s^{+}(\alpha)$, respectively, in (4.30)–(4.35). The boundary condition of (4.56) leads to

$$\left\{2\alpha^2 - \left(\frac{p}{c_2}\right)^2\right\} B_{1s}(\alpha) - 2\alpha\gamma_2(\alpha) B_{2s}(\alpha) + 2\frac{E_0}{\mu}\left\{\left(\frac{c_2}{p}\right)^2 (2A_1 + A_3)\alpha^2 - (A_1 + A_2)\right\}\alpha b_s(\alpha) = 0. \tag{4.59}$$

Introducing the abbreviation

$$B_s(\alpha) = \alpha B_{1s}(\alpha) - \gamma_2(\alpha) B_{2s}(\alpha) + \frac{1}{(p/c_2)^2}\frac{E_0}{\mu}(2A_1 + A_3)\alpha^2 b_s(\alpha), \tag{4.60}$$

and in view of two mixed boundary conditions (4.57) and (4.58), together with (4.59) and (4.60), we have the following two simultaneous dual integral equations for the determination of the function $B_s(\alpha)$:

$$\begin{aligned} \int_0^\infty \alpha\{\eta E_0 f_1(\alpha) B_s(\alpha) + f_2(\alpha) b_s(\alpha)\} \cos(\alpha x)\, d\alpha = 0 \quad (0 \le x < a), \\ \int_0^\infty \alpha b_s(\alpha) \sin(\alpha x)\, d\alpha = 0 \quad (a \le x < \infty), \end{aligned} \tag{4.61}$$

$$\int_0^\infty \alpha\left\{f_3(\alpha)B_s(\alpha)+\frac{E_0}{\mu}f_4(\alpha)b_s(\alpha)\right\}\cos(\alpha x)d\alpha=-\frac{\pi q_j}{2\mu}\cos\left(\alpha_j x\cos\gamma\right)\quad(0\le x<a),$$
$$\int_0^\infty B_s(\alpha)\cos(\alpha x)d\alpha=0\quad(a\le x<\infty),\tag{4.62}$$

where

$$f_1(\alpha)=\frac{\alpha}{\gamma_2(\alpha)(p/c_2)^2}\left\{\left(\frac{p}{c_2}\right)^2+2\gamma_1(\alpha)\gamma_2(\alpha)-2\alpha^2\right\},$$
$$f_2(\alpha)=1+\frac{\alpha}{\gamma_2(\alpha)(p/c_2)^2}\frac{\eta E_0^2}{\mu}\left\{-2\gamma_1(\alpha)\gamma_2(\alpha)(A_1+A_2)+\gamma_2(\alpha)\alpha(2A_1+A_3)+(2A_2-A_3)\alpha^2\right\}$$
$$f_3(\alpha)=\frac{1}{\gamma_2(\alpha)(p/c_2)^2}\left[-\left\{\left(\frac{p}{c_2}\right)^2-2\alpha^2\right\}^2+4\gamma_1(\alpha)\gamma_2(\alpha)\alpha^2\right]\frac{1}{\alpha},$$
$$f_4(\alpha)=\frac{\alpha}{\gamma_2(\alpha)(p/c_2)^2}\left\{\left(2\alpha^2-\frac{p}{c_2^2}\right)(2A_2-A_3)-4\gamma_1(\alpha)\gamma_2(\alpha)(A_1+A_2)\right.$$
$$\left.+\gamma_2(\alpha)\alpha(2\mathrm{A}_1+\mathrm{A}_3)-\frac{1}{\alpha}\gamma_2(\alpha)\left(\frac{p}{c_2}\right)^2\mathrm{A}_2\right\},\tag{4.63}$$

The solution of (4.61) and (4.62) are obtained by using two new functions $g_s(u)$ and $h_s(u)$, and the results are

$$B_s(\alpha)=\frac{\pi}{2}\left(\frac{q_j a^2}{\mu}\right)\int_0^1 u^{1/2}g_s(u)J_0(\alpha a u)du,$$
$$b_s(\alpha)=\frac{\pi}{2}\left(\frac{\eta E_0 q_j a^2}{\mu}\right)\int_0^1 u^{1/2}h_s(u)J_0(\alpha a u)du,\tag{4.64}$$

where $g_s(u)$ and $h_s(u)$ are the solutions of the following Fredholm integral equations of the second kind:

$$F_1 g_s(u)+F_2 h_s(u)-\int_0^1(su)^{1/2}\left\{g_s(s)K_{1s}(u,s)+h_s(s)K_{2s}(u,s)\right\}ds=0,\tag{4.65}$$

$$F_3 g_s(u)+\eta E_\mu^2 F_4 h_s(u)-\int_0^1(su)^{1/2}\left\{g_s(s)K_{3s}(u,s)+h_s(s)K_{4s}(u,s)\right\}ds=-u^{1/2}J_0\left(\alpha_j a u\cos\gamma\right).\tag{4.66}$$

The kernels are given by

$$K_{1s}(u,s)=iP^2\int_0^1\left\{M_{11}(\alpha)J_0(\alpha Ps)H_0^{(1)}(\alpha Pu)+M_{12}(\alpha)J_0(\alpha\sigma Ps)H_0^{(1)}(\alpha\sigma Pu)\right\}d\alpha\quad(u>s),$$
$$K_{2s}(u,s)=iP^2\eta E_\mu^2\int_0^1\left\{M_{21}(\alpha)J_0(\alpha Ps)H_0^{(1)}(\alpha Pu)+M_{22}(\alpha)J_0(\alpha\sigma Ps)H_0^{(1)}(\alpha\sigma Pu)\right\}d\alpha\quad(u>s),$$
$$K_{3s}(u,s)=iP^2\int_0^1\left\{M_{31}(\alpha)J_0(\alpha Ps)H_0^{(1)}(\alpha Pu)+M_{32}(\alpha)J_0(\alpha\sigma Ps)H_0^{(1)}(\alpha\sigma Pu)\right\}d\alpha\quad(u>s),$$
$$K_{4s}(u,s)=iP^2\eta E_\mu^2\int_0^1\left\{M_{41}(\alpha)J_0(\alpha Ps)H_0^{(1)}(\alpha Pu)+M_{42}(\alpha)J_0(\alpha\sigma Ps)H_0^{(1)}(\alpha\sigma Pu)\right\}d\alpha\quad(u>s),\tag{4.67}$$

where

$$\begin{gathered}M_{11}(\alpha)=-\frac{\alpha^2-2\alpha^4}{(1-\alpha^2)^{1/2}},\\ M_{12}(\alpha)=2\sigma^4\alpha^2\left(1-\alpha^2\right)^{1/2},\\ M_{21}(\alpha)=-\frac{\alpha^4}{(1-\alpha^2)^{1/2}}(A_{e2}-\eta),\\ M_{22}(\alpha)=-2\sigma^4\alpha^2\left(1-\alpha^2\right)^{1/2}(A_{e1}+A_{e2}),\\ M_{31}(\alpha)=\frac{(1-2\alpha^2)^2}{(1-\alpha^2)^{1/2}},\\ M_{32}(\alpha)=4\sigma^4\alpha^2\left(1-\alpha^2\right)^{1/2},\\ M_{41}(\alpha)=-\frac{\alpha^2(2\alpha^2-1)}{(1-\alpha^2)^{1/2}}(A_{e2}-\eta),\\ M_{42}(\alpha)=-4\sigma^4\alpha^2\left(1-\alpha^2\right)^{1/2}(A_{e1}+A_{e2}),\end{gathered}\tag{4.68}$$

and $F_i=\lim_{\alpha\to\infty}f_i(\alpha)$ $(i=1,\ldots,4)$. The kernels $K_{is}(u,s)$ $(i=1,\ldots,4)$ are symmetric in u and s.

4.2.2. Antisymmetric Solution for Mode II Crack

The boundary conditions for anti-symmetric scattered fields are

$$\sigma_{yya}^L(x,0)=0\quad(0\le x<\infty),\tag{4.69}$$

$$\begin{gathered}\phi_{a,x}(x,0)=-\eta E_0u_{ya,x}(x,0)+\phi_{a,x}^+(x,0)\quad(0\le x<a),\\ \phi_{a,y}(x,0)=0\quad(a\le x<\infty),\end{gathered}\tag{4.70}$$

$$\sigma^L_{xya}(x,0) = -q_j \sin(\alpha_j x \cos\gamma) \quad (j=1,2)\ (0 \le x < a),$$
$$u_{xa}(x,0) = 0, \quad (a \le x < \infty). \tag{4.71}$$

Let replace the subscript s by a, $a_s(\alpha)$, $A_{1s}(\alpha)$, $A_{2s}(\alpha)$, and $a_s^+(\alpha)$ by $b_a(\alpha)$, $B_{1a}(\alpha)$, $B_{2a}(\alpha)$ and $b_a^+(\alpha)$ in (4.4)–(4.6). The boundary condition of (4.69) leads to

$$\left\{2\alpha^2 - \left(\frac{p}{c_2}\right)^2\right\}B_{1a}(\alpha) + 2\alpha\gamma_2(\alpha)B_{2a}(\alpha) + 2\frac{E_0}{\mu}\left\{\left(\frac{c_2}{p}\right)^2(2A_1+A_3)\alpha^2 - (A_1+A_2)\right\}\alpha b_a(\alpha) = 0. \tag{4.72}$$

Introducing the abbreviation

$$B_a(\alpha) = \alpha B_{1a}(\alpha) + \gamma_2(\alpha)B_{2a} + \frac{1}{(p/c_2)^2}\frac{E_0}{\mu}(2A_1+A_3)\alpha^2 b_a(\alpha), \tag{4.73}$$

and in view of boundary conditions (4.70) and (4.71), together with (4.72) and (4.73), we have the following two simultaneous dual integral equations:

$$\int_0^\infty \alpha\{\eta E_0 f_1(\alpha)B_a(\alpha) + f_2(\alpha)b_a(\alpha)\}\sin(\alpha x)d\alpha = 0 \quad (0 \le x < a),$$
$$\int_0^\infty \alpha b_s(\alpha)\cos(\alpha x)d\alpha = 0 \quad (a \le x < \infty), \tag{4.74}$$

$$\int_0^\infty \alpha\left\{f_3(\alpha)B_a(\alpha) + \frac{E_0}{\mu}f_4(\alpha)b_a(\alpha)\right\}\sin(\alpha x)d\alpha = -\frac{\pi q_j}{2\mu}\sin(\alpha_j x\cos\gamma) \quad (0 \le x < a),$$
$$\int_0^\infty B_a(\alpha)\sin(\alpha x)d\alpha = 0 \quad (a \le x < \infty). \tag{4.75}$$

Equations (4.74) and (4.75) yield the solutions

$$B_a(\alpha) = \frac{\pi q_j a^2}{2\mu}\int_0^1 u^{1/2} g_a(u) J_1(\alpha a u)du,$$
$$b_a(\alpha) = \eta E_0 \frac{\pi q_j a^2}{2\mu}\int_0^1 u^{1/2} h_a(u) J_1(\alpha a u)du, \tag{4.76}$$

$g_a(u)$ and $h_a(u)$ are the solutions of the following Fredholm integral equations of the second kind:

$$F_1 g_a(u) + F_2 h_a(u) - \int_0^1 (su)^{1/2}\{g_a(s)K_{1a}(u,s) + h_a(s)K_{2a}(u,s)\}ds = 0, \quad (4.77)$$

$$F_3 g_a(u) + \eta E_\mu^2 F_4 h_a(u) - \int_0^1 (su)^{1/2}\{g_a(s)K_{3a}(u,s) + h_a(s)K_{4a}(u,s)\}ds = -u^{1/2} J_1(\alpha_j a u \cos\gamma), \quad (4.78)$$

where

$$\begin{aligned}
K_{1a}(u,s) &= iP^2 \int_0^1 \Big\{M_{11}(\alpha)J_1(\alpha P s)H_1^{(1)}(\alpha P u) + M_{12}(\alpha)J_1(\alpha\sigma P s)H_1^{(1)}(\alpha\sigma P u)\Big\}d\alpha \quad (u > s),\\
K_{2a}(u,s) &= iP^2 \eta E_\mu^2 \int_0^1 \Big\{M_{21}(\alpha)J_1(\alpha P s)H_1^{(1)}(\alpha P u) + M_{22}(\alpha)J_1(\alpha\sigma P s)H_1^{(1)}(\alpha\sigma P u)\Big\}d\alpha \quad (u > s),\\
K_{3a}(u,s) &= iP^2 \int_0^1 \Big\{M_{31}(\alpha)J_1(\alpha P s)H_1^{(1)}(\alpha P u) + M_{32}(\alpha)J_1(\alpha\sigma P s)H_1^{(1)}(\alpha\sigma P u)\Big\}d\alpha \quad (u > s),\\
K_{4a}(u,s) &= iP^2 \eta E_\mu^2 \int_0^1 \Big\{M_{41}(\alpha)J_1(\alpha P s)H_1^{(1)}(\alpha P u) + M_{42}(\alpha)J_1(\alpha\sigma P s)H_1^{(1)}(\alpha\sigma P u)\Big\}d\alpha \quad (u > s),
\end{aligned} \quad (4.79)$$

and $K_{ia}(u,s)$ $(i = 1,\ldots,4)$ are symmetric in u and s.

4.2.3. Mode II Dynamic Singular Stresses Near the Crack Tip

The dynamic stress intensity factor K_{IID} is obtained as

$$\begin{aligned}
K_{IID} &= \lim_{x \to a^+} \{2\pi(x-a)\}^{1/2} \Big\{\sigma_{xys}^L + \sigma_{xya}^L + \sigma_{xys}^M + \sigma_{xya}^M\Big\}_{y=0}\\
&= q_j(\pi a)^{1/2}\{-F_3[g_s(1) - i g_a(1)] + z_2[h_s(1) - i h_a(1)]\}\\
&= K_{II}^g + K_{II}^h,
\end{aligned} \quad (4.80)$$

where

$$\begin{aligned}
K_{II}^g &= -q_j(\pi a)^{1/2} F_3[g_s(1) - i g_a(1)],\\
K_{II}^h &= q_j(\pi a)^{1/2} z_2[h_s(1) - i h_a(1)],\\
z_2 &= \Big[2\sigma^2(A_{e1} + A_{e2}) - (A_{e2} + \eta + 1)\Big]\eta E_\mu^2.
\end{aligned} \quad (4.81)$$

The singular parts of the dynamic local stresses and Maxwell stresses near the crack tip can be derived as follows:

$$\sigma_{xx}^{L} \sim -\frac{K_{II}^{g}}{(2\pi r)^{1/2}}\left[2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]\sin\frac{\theta}{2}-\frac{K_{II}^{h}}{z_2(2\pi r)^{1/2}}\left[2E_{\mu}^{2}\eta F_4+F_5\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]\sin\frac{\theta}{2},$$
$$\sigma_{xy}^{L} \sim \frac{K_{II}^{g}}{(2\pi r)^{1/2}}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]\cos\frac{\theta}{2}+\frac{K_{II}^{h}}{z_2(2\pi r)^{1/2}}\left[E_{\mu}^{2}\eta F_4-F_5\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]\cos\frac{\theta}{2},$$
$$\sigma_{yy}^{L} \sim \frac{K_{II}^{g}}{(2\pi r)^{1/2}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}+\frac{K_{II}^{h}}{z_2(2\pi r)^{1/2}}F_5\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2},$$
$$\sigma_{xx}^{M} \sim \frac{K_{II}^{h}}{z_2(2\pi r)^{1/2}}\eta E_{\mu}^{2}\sin\frac{\theta}{2},$$
$$\sigma_{xy}^{M} \sim -\frac{K_{II}^{h}}{z_2(2\pi r)^{1/2}}\eta E_{\mu}^{2}(1+\eta)\cos\frac{\theta}{2},$$
$$\sigma_{yy}^{M} \sim -\frac{K_{II}^{h}}{z_2(2\pi r)^{1/2}}\eta E_{\mu}^{2}(1+2\eta)\sin\frac{\theta}{2}. \tag{4.82}$$

The singular parts of the displacements and electric fields near the crack tip can be expressed as

$$u_x \sim \frac{K_{II}^{g}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2}\left(\frac{1}{F_3}+\cos^2\frac{\theta}{2}\right)\sin\frac{\theta}{2}+\frac{K_{II}^{h}}{\mu z_2}\left(\frac{r}{2\pi}\right)^{1/2}\left(F_5\cos^2\frac{\theta}{2}\right)\sin\frac{\theta}{2},$$
$$u_y \sim \frac{K_{II}^{g}}{\mu z_2}\left(\frac{r}{2\pi}\right)^{1/2}\left(-1+\sin^2\frac{\theta}{2}\right)\cos\frac{\theta}{2}+\frac{K_{II}^{h}}{\mu z_2}\left(\frac{r}{2\pi}\right)^{1/2}\left(-F_6+F_5\sin^2\frac{\theta}{2}\right)\cos\frac{\theta}{2},$$
$$E_x \sim \frac{K_{II}^{h}}{z_2\mu}\frac{1}{(2\pi r)^{1/2}}\eta E_0\cos\frac{\theta}{2},$$
$$E_y \sim -\frac{K_{II}^{h}}{z_0\mu}\frac{1}{(2\pi r)^{1/2}}\eta E_0\sin\frac{\theta}{2}, \tag{4.83}$$

where

$$F_5 = \left[2\sigma^2(A_{e1}+A_{e2})-(A_{e2}-\eta)\right]\eta E_{\mu}^{2},$$
$$F_6 = \left[2\sigma^2(A_{e1}+A_{e2})+(A_{e2}-\eta)\right]\eta E_{\mu}^{2}. \tag{4.84}$$

Table 1: Material properties of PMMA.

$\mu(M/m^2)$	ν	A_{e1}	A_{e2}	η	ε_r
1.1×10^9	0.4	0	3.61	2	3

5. Dynamic Energy Release Rate

The dynamic energy release rate G is obtained as

$$G=\int_S \rho u_{i,tt}u_{i,x}dS+\int_\Gamma\left\{(\rho\Sigma+\Phi)\delta_{jx}-\left(\sigma_{ij}^L+\sigma_{ij}^M\right)u_{i,x}+D_iE_x\right\}n_j d\Gamma, \tag{5.1}$$

where S is the region with the contour Γ. This expression may be thought of as an extension to the J-integral given in [3]. If all the electrical field quantities are made to vanish, then (5.1) reduces to the dynamic energy release rate for the elastic materials [8]. Writing the dynamic energy release rate expression in terms of the mode I dynamic stress intensity factor, there results

$$G=\frac{1}{(1-2\nu)}\frac{K_I^2}{128\mu z_0^2}\left\{C_1E_\mu^4+C_2E_\mu^2+64(1-\nu)(1-2\nu)\right\}, \tag{5.2}$$

where

$$\begin{gathered}C_1=2k_2^2+k_3^2+4(1-2\nu)k_1k_3+2(1-2\nu)k_1k_2+(1+4\nu)k_3k_2+4(1-\nu)(1-2\nu)\eta\left(k_2-2\eta k_3\right),\\ C_2=4(1-2\nu)\left[3k_1-4\nu k_2-3k_3+2(1-\nu)\left\{12-16\nu+(7-8\nu)\eta-8(1-\nu)\eta^2\right\}\eta\right],\\ k_1=\{2(1-2\nu)A_{e1}+2(1-\nu)A_{e2}-\eta\}\eta,\\ k_2=\{(1-2\nu)\left(2A_{e1}+\eta\right)+2(1-\nu)A_{e2}\}\eta,\\ k_3=\{(1-2\nu)\left(2A_{e1}+\eta\right)-2(1-\nu)A_{e2}\}\eta.\end{gathered} \tag{5.3}$$

6. Results and Discussion

To examine the effect of electroelastic interactions on the dynamic stress intensity factor and dynamic energy release rate, the solutions of the Fredholm integral equations of the second kind (4.17), (4.41) for Mode I and (4.65), (4.66), (4.77), (4.78) for Mode II have been computed numerically by the use of Gaussian quadrature formulas. We can consider polymethylmethacrylate (PMMA), and the engineering material constants of PMMA are listed in Table 1. The dynamic stress intensity factor K_I can be found as $K_I = |K_{ID}| + K_{IS}$.

Figure 3 exhibits the variation of the normalized mode I dynamic stress intensity factor $|K_I/p_{0P}(\pi a)^{1/2}|(p_{0P} = \mu\alpha_2^2\varphi_{e0})$ against the normalized frequency $\Omega = a\alpha_2$ subjected to P-waves for the normalized electric field $E_\mu = (\eta/\varepsilon_0)^{1/2}E_0 = 0.0, 0.1$ and the angle of incidence $\gamma = \pi/2$. The dynamic stress intensity factor drops rapidly beyond the first maximum and exhibits oscillations of approximately constant period as Ω increases. The peak value of

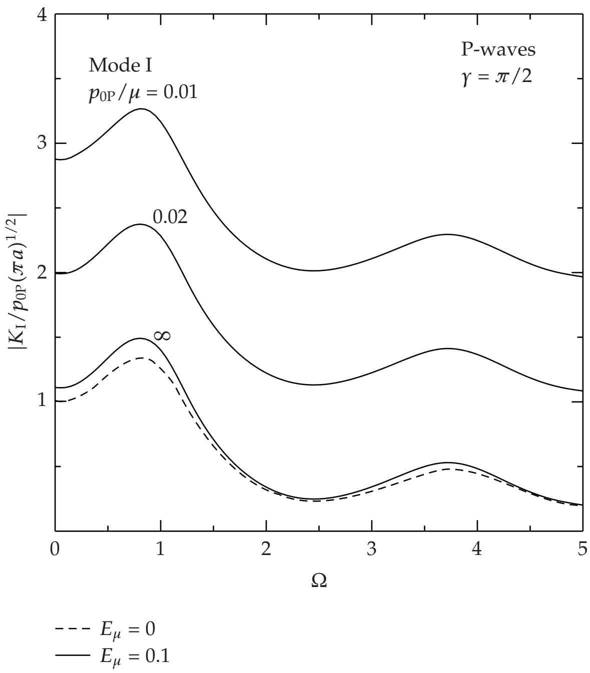

Figure 3: Mode I dynamic stress intensity factor versus frequency (P-waves, $\gamma = \pi/2$).

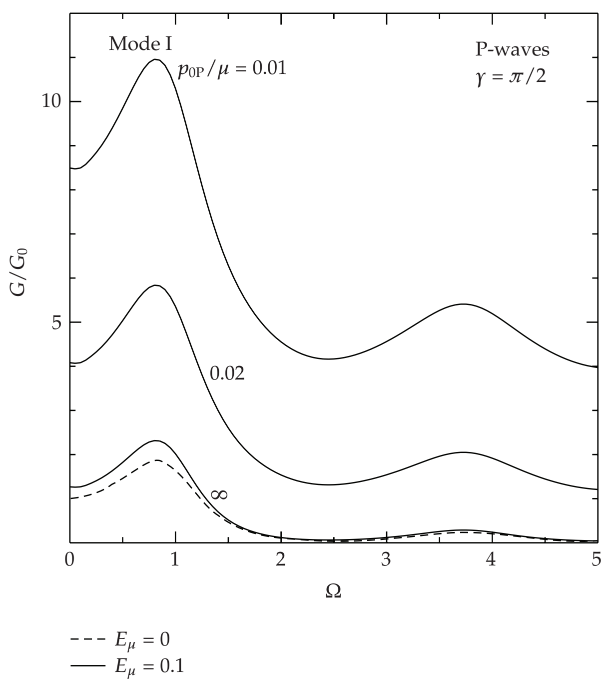

Figure 4: Mode I dynamic energy relrase rate versus frequency (P-waves, $\gamma = \pi/2$).

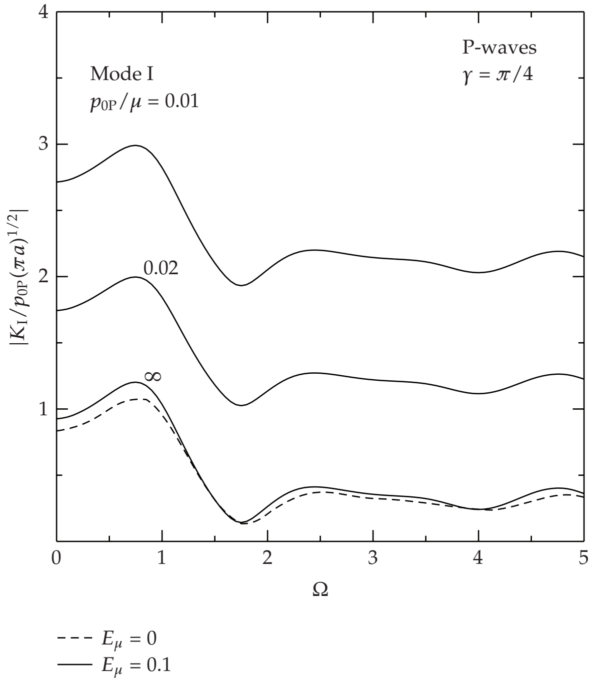

Figure 5: Mode I dynamic stress intensity factor versus frequency (P-waves, $\gamma = \pi/4$).

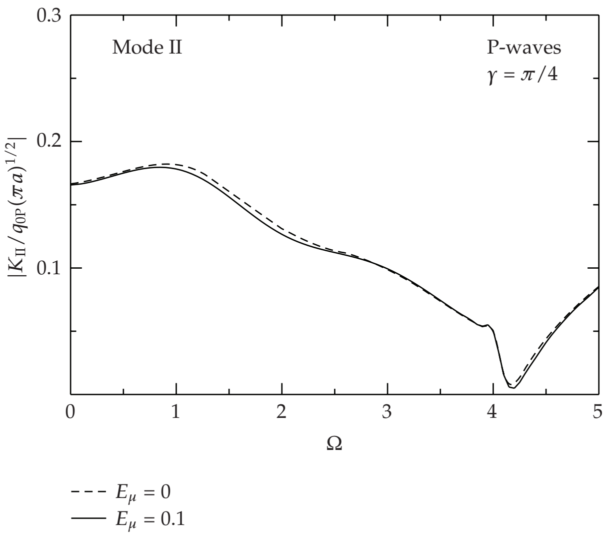

Figure 6: Mode II dynamic stress intensity factor versus frequency (P-waves, $\gamma = \pi/4$).

$|K_I/p_{0P}(\pi a)^{1/2}|$ under $E_\mu = 0.0$ is 1.364. Also, the peak values of $|K_I/p_{0P}(\pi a)^{1/2}|$ under $E_\mu = 0.1$ are 1.522, 2.416, 3.310 for $p_{0P}/\mu = \infty, 0.02, 0.01$, respectively. As $\Omega \to 0$, the dynamic stress intensity factor tends to static stress intensity factor [5]. In the absence of the electric fields, the dynamic stress intensity factor becomes the solution for the elastic solid (see e.g. [9]). Figure 4 also shows the variation of the normalized mode I dynamic

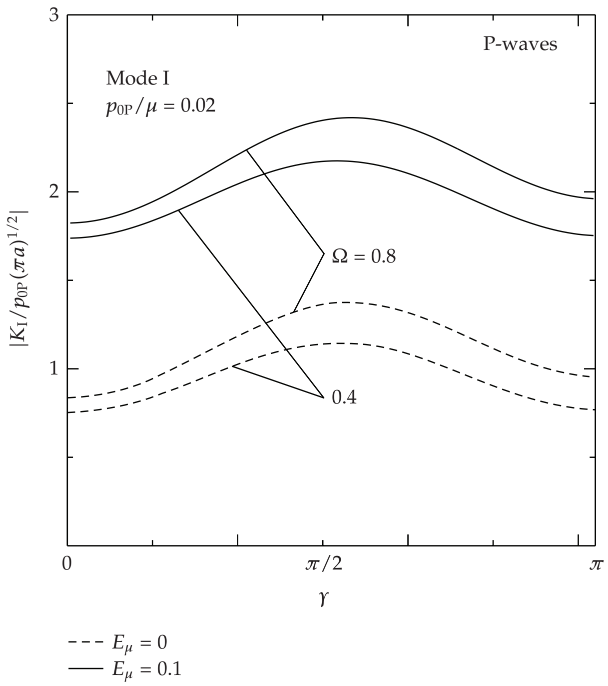

Figure 7: Mode I dynamic stress intensity factor versus angle of incidence (P-waves).

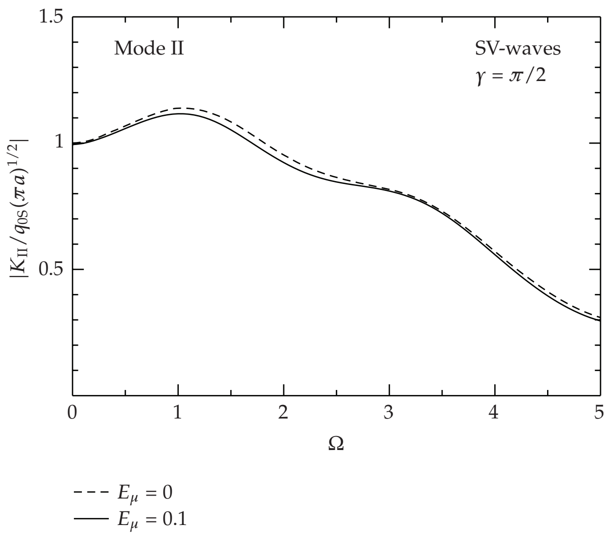

Figure 8: Mode II dynamic stress intensity factor versus frequency (SV-waves, $\gamma = \pi/2$).

energy release rate G/G_0, where $G_0 = \pi a(1-\nu)p_{0P}^2/2\mu$ is the static energy release rate. The peak values of G/G_0 under $E_\mu = 0.0, 0.1$ for $p_{0P}/\mu = \infty, 0.02, 0.01$ are 1.861, 2.361, 5.838, 10.96, respectively. Figure 5 shows the normalized mode I dynamic stress intensity factor $|K_I/p_{0P}(\pi a)^{1/2}|$ versus Ω subjected to P-waves for $E_\mu = 0.0, 0.1$ and $\gamma = \pi/4$. The peak values of $|K_I/p_{0P}(\pi a)^{1/2}|$ under $E_\mu = 0.0,\ 0.1$ are 1.078, 1.198 for $p_{0P}/\mu = \infty$, respectively. Figure 6

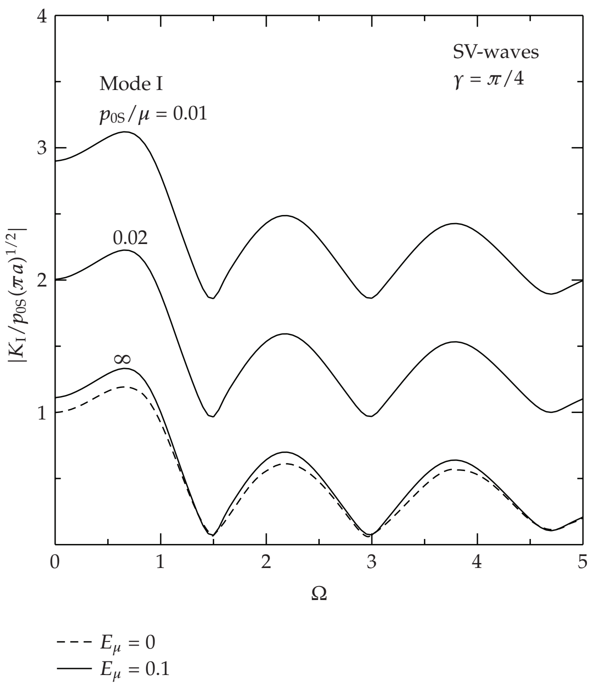

Figure 9: Mode I dynamic stress intensity factor versus frequency (SV-waves, $\gamma = \pi/4$).

shows the normalized mode II dynamic stress intensity factor $|K_{II}/q_{0P}(\pi a)^{1/2}|(q_{0P} = \mu\alpha_2^2\varphi_{e0})$ versus Ω subjected to P-waves for $E_\mu = 0.0, 0.1$ and $\gamma = \pi/4$. The effect of electric fields on the mode II dynamic stress intensity factor is small. Figure 7 displays the normalized mode I dynamic stress intensity factor $|K_I/p_{0P}(\pi a)^{1/2}|$ against the angle of incidence γ subjected to P-waves for $E_\mu = 0.0, 0.1$ and $\Omega = 0.4, 0.8$ $(p_{0P}/\mu = 0.02)$. The mode I dynamic stress intensity factors for $\Omega = 0.4$ and 0.8 attain its maximum values at an incident angle of approximately $\pi/2$.

Figure 8 shows the variation of the normalized mode II dynamic stress intensity factor $|K_{II}/q_{0S}(\pi a)^{1/2}|(q_{0S} = \mu\alpha_2^2\psi_{e0})$ versus Ω subjected to SV-waves for $E_\mu = 0.0, 0.1$ and $\gamma = \pi/2$. The electric fields have small effect on the mode II dynamic stress intensity factor. Figure 9 shows the normalized mode I dynamic stress intensity factor $|K_I/p_{0S}(\pi a)^{1/2}|$ $(p_{0S} = \mu\alpha_2^2\psi_{e0})$ against Ω subjected to SV-waves for $E_\mu = 0.0, 0.1$ and $\gamma = \pi/4$. Similar trend to the case under P-waves is observed.

7. Conclusions

The dynamic electroelastic problem for a dielectric polymer having a finite crack has been analyzed theoretically. The results are expressed in terms of the dynamic stress intensity factor and dynamic energy release rate. It is found that the dynamic stress intensity factor and dynamic energy release rate tend to increase with frequency reaching a peak and then decrease in magnitude. These peaks depend on the angle of incidence. Also, applied electric fields increase the mode I dynamic stress intensity factor and dynamic energy release rate, whereas the mode II dynamic stress intensity factor is less dependent on the electric field.

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