EXISTENCE AND NONEXISTENCE OF POSITIVE SOLUTIONS FOR QUASILINEAR SYSTEMS

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The paper deals with the existence and nonexistence of positive solutions for a class of p-Laplacian systems. We investigate the effect of the size of the domain on the existence of positive solution for the problem in sublinear cases. We will use fixed point theorems in a cone.

1. Introduction

In this paper we consider the existence and nonexistence of positive solutions to the boundary value problem of the p-Laplacian system

$$
\begin{aligned}
&\left(t^{N-1}\left|u_i'(t)\right|^{p-2}u_i'(t)\right)' + t^{N-1}f_i(u_1,\ldots,u_n) = 0, \quad 0 < t < R,\ i = 1,\ldots,n, \\
&u_i'(0) = u_i(R) = 0, \quad i = 1,\ldots,n,
\end{aligned}
\tag{1.1}
$$

where $p > 1$, $N \geq 1$, $R > 0$, and f_i is nonnegative continuous, $i = 1,\ldots,n$.

Such a problem arises when we seek the radial solutions of the following elliptic system:

$$
\begin{aligned}
&-\Delta_p u_i = f_i(u_1,\ldots,u_n) \quad \text{in } B,\ i = 1,\ldots,n, \\
&u_i = 0 \quad \text{on } \partial\Omega,\ i = 1,\ldots,n,
\end{aligned}
\tag{1.2}
$$

where $\Delta_p u_i = div(|\nabla u_i|^{p-2}\nabla u_i)$, $i = 1,\ldots,n$, $p > 1$, $B = \{x \in \mathbb{R}^N : |x| < R\}$, $R > 0$.

Equation (1.2) covers several important cases. When $p = 2$, (1.2) becomes the elliptic system

$$
\begin{aligned}
&-\Delta u_i = f_i(u_1,\ldots,u_n) \quad \text{in } B,\ i = 1,\ldots,n, \\
&u_i = 0 \quad \text{on } \partial B,\ i = 1,\ldots,n.
\end{aligned}
\tag{1.3}
$$

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When $n = 1$, (1.2) becomes the usual p-Laplacian

$$-\Delta_p u = f(u) \quad \text{in } B,$$
$$u = 0 \quad \text{on } \partial B. \tag{1.4}$$

When $n = 1$ and $p = 2$, (1.2) becomes the usual Laplacian

$$-\Delta u = f(u) \quad \text{in } B$$
$$u = 0 \quad \text{on } \partial B. \tag{1.5}$$

In several papers [6, 8], Wang studied the existence of nontrivial solutions of (1.1) for a fixed $R > 0$. It was shown that (1.1), for a fixed $R > 0$, has a nontrivial solution for sublinear nonlinearities. Related results can also be found in [1].

In this paper we investigate the effect of the size of the domain on the existence and nonexistence of positive solutions of the quasilinear elliptic system (1.1) in sublinear cases.

Let $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}_+ = [0, \infty)$, and $\mathbb{R}_+^n = \prod_{i=1}^n \mathbb{R}^+$. Also, for $\mathbf{u} = (u_1, \ldots, u_n) \in \mathbb{R}_+^n$, let $\|\mathbf{u}\| = \sum_{i=1}^n |u_i|$ and

$$\mathbf{f}(\mathbf{u}) = (f_1(\mathbf{u}), \ldots, f_n(\mathbf{u})) = (f_1(u_1, \ldots, u_n), \ldots, f_n(u_1, \ldots, u_n)). \tag{1.6}$$

We now turn to the general assumptions for this paper.

(H1) $f_i : \mathbb{R}_+^n \to \mathbb{R}_+$ is continuous, $i = 1, \ldots, n$.

(H2) There exists an $i \in \{1, \ldots, n\}$ such that

$$\lim_{\|\mathbf{u}\| \to 0} \frac{f_i(\mathbf{u})}{\|\mathbf{u}\|^{p-1}} = \infty \tag{1.7}$$

for $\mathbf{u} = (u_1, \ldots, u_n) \in \mathbb{R}_+^n$.

(H3) For all $i \in \{1, \ldots, n\}$,

$$\lim_{\|\mathbf{u}\| \to \infty} \frac{f_i(\mathbf{u})}{\|\mathbf{u}\|^{p-1}} = 0, \tag{1.8}$$

where $\mathbf{u} = (u_1, \ldots, u_n) \in \mathbb{R}_+^n$.

The main results of this paper are Theorems 1.1, 1.2, and 1.3.

THEOREM 1.1. *Assume (H1) and (H2) hold. Then there is an $R_0 > 0$ such that (1.1) has a positive solution for $0 < R < R_0$.*

THEOREM 1.2. *Assume (H1), (H2), and (H3) hold. Then (1.1) has a positive solution for all $R > 0$.*

The following assumption will allow us to establish a nonexistence theorem.

(H4) For all $i \in \{1, \ldots, n\}$,

$$\limsup_{\|\mathbf{u}\| \to 0} \frac{f_i(\mathbf{u})}{\|\mathbf{u}\|^{p-1}} < \infty, \qquad \limsup_{\|\mathbf{u}\| \to \infty} \frac{f_i(\mathbf{u})}{\|\mathbf{u}\|^{p-1}} < \infty, \tag{1.9}$$

where $\mathbf{u} = (u_1, \ldots, u_n) \in \mathbb{R}_+^n$.

THEOREM 1.3. *Assume (H1) and (H4) hold. Then there is an $R_0 > 0$ such that (1.1) has no positive solution for $0 < R < R_0$.*

We now give two examples to demonstrate the theorems.

Example 1.4.

$$\text{div}\left(|\nabla u_1|^{p-2}\nabla u_1\right) + e^{(u_1+\cdots+u_n)} = 0 \quad \text{in } B,$$
$$\text{div}\left(|\nabla u_i|^{p-2}\nabla u_i\right) + f_i(u_1,\ldots,u_n) \quad \text{in } B, \ i = 2,\ldots,n, \tag{1.10}$$
$$u_i = 0 \quad \text{on } \partial B, \ i = 1,\ldots,n,$$

where $p > 1$, $B = \{x \in \mathbb{R}^N : |x| < R\}$, $R > 0$, f_i are any nonnegative continuous functions. Then (1.10) has a positive solution for sufficiently small $R > 0$ according to Theorem 1.1.

Example 1.5.

$$\text{div}\left(|\nabla u_i|^{p-2}\nabla u_i\right) + (u_1 + \cdots + u_n)^{p_i} = 0 \quad \text{in } B \ i = 1,\ldots,n,$$
$$u_i = 0 \quad \text{on } \partial B, \ i = 1,\ldots,n, \tag{1.11}$$

where $p > 1$, $0 < p_1, p_2,\ldots,p_n < p - 1$, $B = \{x \in \mathbb{R}^N : |x| < R\}$, $R > 0$. Then (1.11) has a nontrivial solution for all $R > 0$ according to Theorem 1.2.

2. Preliminaries

Let $\varphi(t) = |t|^{p-2}t$, then, for $t > 0$, $\varphi(t) = t^{p-1}$ and $\varphi^{-1}(t) = t^{1/(p-1)}$. It is easy to see that $\varphi^{-1}(\sigma\varphi(t)) = \varphi^{-1}(\sigma)t$ for $t > 0$ and $\sigma > 0$.

We will deal with classical solutions of (1.1), namely a vector-valued function $\mathbf{u} = (u_1(t),\ldots,u_n(t))$ with $u_i \in C^1[0,R]$, and $\varphi(u_i') \in C^1(0,R)$, $i = 1,\ldots,n$, which satisfies (1.1). A solution $\mathbf{u}(t) = (u_1(t),\ldots,u_n(t))$ is positive if $u_i(t) \geq 0$, $i = 1,\ldots,n$, for all $t \in (0,R)$ and there is at least one nontrivial component of $\mathbf{u}$. In fact, it is easy to prove that such a nontrivial component of $\mathbf{u}$ is positive on $(0,R)$.

Applying the change of variables, $t = Rr$, we can transform (1.1) into the form

$$\left(r^{N-1}\varphi\left(\frac{u_i'(r)}{R}\right)\right)' + Rr^{N-1}f_i(\mathbf{u}) = 0, \quad 0 < r < 1, \ i = 1,\ldots,n, \tag{2.1}$$
$$\mathbf{u}'(0) = \mathbf{u}(1) = 0.$$

Note that we still use $u_i(r)$ and $v_i(r)$ for the new functions, $u_i(Rr)$ and $v_i(Rr)$. Thus $du_i(t)/dt = (du_i(Rr)/dr)(dr/dt) = (du_i(Rr)/dr)(1/R) = (du_i(r)/dr)(1/R)$.

We now recall some concepts and conclusions on the fixed point index in a cone in [2, 3]. Let X be a Banach space and let K be a closed, nonempty subset of X. K is said to be a cone if (i) $\alpha u + \beta v \in K$ for all $u, v \in K$ and all $\alpha, \beta > 0$ and (ii) $u, -u \in K$ imply $u = 0$. Assume Ω is a bounded open subset in X with the boundary $\partial\Omega$, and let $T : K \cap \overline{\Omega} \to K$ be completely continuous such that $Tx \neq x$ for $x \in \partial\Omega \cap K$, then the fixed point index $i(T, K \cap \Omega, K)$ is defined. If $i(T, K \cap \Omega, K) \neq 0$, then T has a fixed point in $K \cap \Omega$. The following well-known result of the fixed point index is crucial in our arguments.

LEMMA 2.1 [2, 3]. *Let E be a Banach space and K a cone in E. Further let $r > 0$, $K_r = \{u \in K : \|x\| < r\}$, and $\partial K_r = \{u \in K : \|x\| = r\}$. Assume that $T : \bar{K}_r \to K$ is completely continuous.*

(i) *If there exists an $x_0 \in K \setminus \{0\}$ such that*

$$x - Tx \neq tx_0 \quad \forall x \in \partial K_r,\ t \geq 0, \tag{2.2}$$

then

$$i(T, K_r, K) = 0. \tag{2.3}$$

(ii) *If $\|Tx\| \leq \|x\|$ for $x \in \partial K_r$ and $Tx \neq x$ for $x \in \partial K_r$, then*

$$i(T, K_r, K) = 1. \tag{2.4}$$

In order to apply Lemma 2.1 to (1.1), let X be the Banach space $\underbrace{C[0,1] \times \cdots \times C[0,1]}_{n}$ and, for $\mathbf{u} = (u_1, \dots, u_n) \in X$,

$$\|\mathbf{u}\| = \sum_{i=1}^{n} \sup_{t \in [0,1]} |u_i(t)|. \tag{2.5}$$

For $\mathbf{u} \in X$ or $\mathbb{R}_+^n$, $\|\mathbf{u}\|$ denotes the norm of $\mathbf{u}$ in X or $\mathbb{R}_+^n$, respectively.

Define K to be a cone in X defined by

$$K = \{(u_1, \dots, u_n) \in X : u_i(t) \geq 0,\ t \in [0,1],\ i = 1, \dots, n\}. \tag{2.6}$$

Also, for each r positive number, define Ω_r by

$$\Omega_r = \{\mathbf{u} \in K : \|\mathbf{u}\| < r\}. \tag{2.7}$$

Note that $\partial \Omega_r = \{\mathbf{u} \in K : \|\mathbf{u}\| = r\}$.

Let $\mathbf{T} : K \to X$ be a map with components $(T^1, \dots, T^n)$. We define T^i, $i = 1, \dots, n$, by

$$T^i \mathbf{u}(t) = R \int_t^1 \varphi^{-1}\left(\frac{R}{s^{N-1}} \int_0^s \tau^{N-1} f_i(\mathbf{u}(\tau)) d\tau\right) ds, \quad t \in [0,1]. \tag{2.8}$$

It is straightforward to verify that the problem of finding positive solutions to (1.1) is equivalent to the fixed point equation

$$\mathbf{Tu} = \mathbf{u} \quad \text{in } K. \tag{2.9}$$

It is easy to show that $\mathbf{T}(K) \subset K$ and is completely continuous. In particular, we have the following assertion.

LEMMA 2.2. *Assume (H1) holds. Then $\mathbf{T}(K) \subset K$ and $\mathbf{T} : K \to K$ is completely continuous.*

For each $i = 1, \dots, n$, define new function $\hat{f}_i(t) : \mathbb{R}_+ \to \mathbb{R}_+$ by

$$\hat{f}_i(t) = \max\{f_i(\mathbf{u}) : \mathbf{u} \in \mathbb{R}_+^n \text{ and } \|\mathbf{u}\| \leq t\}. \tag{2.10}$$

LEMMA 2.3 [7, Lemma 2.8]. *Let (H1) hold and assume* $\lim_{\|\mathbf{u}\| \to \infty}(f_i(\mathbf{u})/\|\mathbf{u}\|^{p-1}) = f_\infty^i$
and $\lim_{\|\mathbf{u}\| \to 0}(f_i(\mathbf{u})/\|\mathbf{u}\|^{p-1}) = f_0^i$, $\mathbf{u} \in \mathbb{R}_+^n$, $f_0^i, f_\infty^i \in [0, \infty]$ *for some* $i \in \{1, \dots, n\}$.
    Then $\lim_{t \to 0^+}(\hat{f}_i(t)/\varphi(t)) = f_0^i$ *and* $\lim_{t \to \infty}(\hat{f}_i(t)/\varphi(t)) = f_\infty^i$.

LEMMA 2.4. *Assume (H1) holds and let* $r > 0$. *If there exists an* $\varepsilon > 0$ *such that*

$$\hat{f}_i(r) \le \varphi(\varepsilon)\varphi(r), \quad i = 1, \dots, n, \tag{2.11}$$

then

$$\|\mathbf{Tu}\| \le nR\varphi^{-1}\left[\frac{R}{N}\right]\varepsilon\|\mathbf{u}\| \quad \text{for } \mathbf{u} \in \partial\Omega_r. \tag{2.12}$$

Proof. From the definition of T, for $\mathbf{u} \in \partial\Omega_r$, we have

$$\|\mathbf{Tu}\| = \sum_{i=1}^n \sup_{t \in [0,1]} |T^i\mathbf{u}(t)| = R\sum_{i=1}^n \int_0^1 \varphi^{-1}\left[\frac{R}{s^{N-1}}\int_0^s \tau^{N-1}f_i(\mathbf{u}(\tau))d\tau\right]ds$$

$$\le R\sum_{i=1}^n \int_0^1 \varphi^{-1}\left[\frac{R}{s^{N-1}}\int_0^s \tau^{N-1}d\tau\,\hat{f}_i(r)\right]ds \le nR\varphi^{-1}\left[\frac{R}{N}\varphi(\varepsilon)\varphi(r)\right] \tag{2.13}$$

$$= nR\varphi^{-1}\left[\frac{R}{N}\varphi(\varepsilon r)\right] = nR\varphi^{-1}\left(\frac{R}{N}\right)\varepsilon\|\mathbf{u}\|. \qquad \square$$

LEMMA 2.5. *Assume (H1) holds and* $r > 0$. *Then*

$$\|\mathbf{Tu}\| \le nR\varphi^{-1}\left(\frac{R}{N}\right)\varphi^{-1}(\widehat{M}_r) \quad \text{holds } \forall \mathbf{u} \in \partial\Omega_r, \tag{2.14}$$

where $\widehat{M}_r = 1 + \max\{f_i(\mathbf{u}) : \mathbf{u} \in \mathbb{R}_+^n \text{ and } \|\mathbf{u}\| \le r, i = 1, \dots, n\} > 0$.

Proof. Since $f_i(\mathbf{u}(t)) \le \widehat{M}_r = \varphi(\varphi^{-1}(\widehat{M}_r))$ for $t \in [0, 1]$, $i = 1, \dots, n$, it is easy to see that this lemma can be shown in a similar manner as Lemma 2.4. $\qquad \square$

3. Proof of Theorem 1.1

Fix a number $r_2 > 0$. Lemma 2.5 implies that there exists an $R_0 > 0$ such that

$$\|\mathbf{Tu}\| < \|\mathbf{u}\| \quad \text{for } \mathbf{u} \in \partial\Omega_{r_2}, \qquad 0 < R < R_0. \tag{3.1}$$

Now let $0 < R < R_0$ and $\eta > 0$ be such that

$$R\frac{\eta}{2}\varphi^{-1}\left(\frac{R}{N4^N}\right) \ge 1. \tag{3.2}$$

Since

$$\lim_{\|\mathbf{u}\|\to 0} \frac{f_i(\mathbf{u})}{\|\mathbf{u}\|^{p-1}} = \infty, \tag{3.3}$$

there is $0 < r_1 < r_2$ such that

$$f_i(\mathbf{u}) \geq \varphi(\eta)\varphi(\|\mathbf{u}\|) \tag{3.4}$$

for $\mathbf{u} = (u_1,\dots,u_n) \in \mathbb{R}_+^n$ and $\|\mathbf{u}\| \leq r_1$.

If $\mathbf{u} - \mathbf{Tu} = 0$ for some $\mathbf{u} \in \partial\Omega_{r_1}$, we already find the desired solution of (1.1). Therefore we assume that

$$\mathbf{u} - \mathbf{Tu} \neq 0 \quad \forall \mathbf{u} \in \partial\Omega_{r_1}, \tag{3.5}$$

we now claim that

$$\mathbf{u} - \mathbf{Tu} \neq t\mathbf{v} \quad \forall \mathbf{u} \in \partial\Omega_{r_1}, \ t \geq 0, \tag{3.6}$$

where $\mathbf{v} = (\theta(r),\dots,\theta(r))$, and $\theta \in C[0,1]$ such that $0 \leq \theta(r) \leq 1$ on $[0,1]$, $\theta(r) \equiv 1$ on $[0,1/4]$ and $\theta(r) \equiv 0$ on $[1/2,1]$. Thus, $\mathbf{v} \in K \setminus \{0\}$. If there exists $\mathbf{u}^* = (u_1^*,\dots,u_n^*) \in \partial\Omega_{r_1}$ and $t_0 \geq 0$ such that $\mathbf{u}^* - \mathbf{Tu}^* = t_0\mathbf{v}$, we will show that this leads to a contradiction. Since (3.5) is true, we have $t_0 > 0$. Since $\mathbf{T}(K) \subset K$, we obtain $u_i^*(r) \geq t_0\theta(r)$ for all $r \in [0,1]$. Let

$$t^* = \sup\{t : u_i^*(r) \geq t\theta(r) \ \forall r \in [0,1]\}. \tag{3.7}$$

It follows that $t_0 \leq t^* < \infty$ and $u_i^*(r) \geq t^*\theta(r)$ for all $r \in [0,1]$. Now, for $r \in [0,1]$, we have

$$u_i^*(r) = \mathbf{T}^i\mathbf{u}^*(r) + t_0\theta(r)$$
$$= R\int_r^1 \varphi^{-1}\left(\frac{R}{s^{N-1}}\int_0^s \tau^{N-1} f_i(\mathbf{u}^*(\tau))d\tau\right)ds + t_0\theta(r). \tag{3.8}$$

Note that $\sum_{j=1}^n u_j^*(r) \leq r_1$ for $r \in [0,1]$. Formula (3.4) implies that, for $r \in [0,1/2]$,

$$u_i^*(r) \geq R\int_{1/2}^1 \varphi^{-1}\left(\frac{R}{s^{N-1}}\int_0^s \tau^{N-1}\varphi(\eta)\varphi\left(\sum_{j=1}^n u_j^*(\tau)\right)d\tau\right)ds + t_0\theta(r)$$

$$\geq R\int_{1/2}^1 \varphi^{-1}\left(R\int_0^s \tau^{N-1}\varphi(\eta)\varphi(u_i^*(\tau))d\tau\right)ds + t_0\theta(r)$$

$$\geq \frac{R}{2}\varphi^{-1}\left(R\int_0^{1/4} \tau^{N-1}\varphi(\eta)\varphi(t^*\theta(\tau))d\tau\right) + t_0\theta(r) \tag{3.9}$$

$$= \frac{R}{2}\varphi^{-1}\left(R\int_0^{1/4} \tau^{N-1}d\tau\varphi(\eta)\varphi(t^*)\right) + t_0\theta(r)$$

$$= \frac{R}{2}\varphi^{-1}\left(\frac{R}{N4^N}\varphi(\eta t^*)\right) + t_0\theta(r).$$

Now, in view of the fact that $\varphi^{-1}(\sigma\varphi(t)) = \varphi^{-1}(\sigma)t$, we have, for $r \in [0, 1/2]$,

$$u_i^*(r) \geq t^* \frac{\eta R}{2} \varphi^{-1}\left(\frac{R}{N4^N}\right) + t_0 \theta(r) \geq t^* + t_0 \theta(r) \geq (t^* + t_0)\theta(r), \qquad (3.10)$$

and hence

$$u_i^*(r) \geq (t^* + t_0)\theta(r), \quad r \in [0, 1], \qquad (3.11)$$

which is a contradiction to the definition of t^*. Thus, in view of Lemma 2.1,

$$i(\mathbf{T}, \Omega_{r_1}, K) = 0, \qquad i(\mathbf{T}, \Omega_{r_2}, K) = 1. \qquad (3.12)$$

It follows from the additivity of the fixed point index that $i(\mathbf{T}, \Omega_{r_2} \setminus \bar{\Omega}_{r_1}, K) = 1$. Thus, $\mathbf{T}$ has a fixed point in $\Omega_{r_2} \setminus \bar{\Omega}_{r_1}$, which is the desired positive solution of (1.1).

4. Proof of Theorem 1.2

Let R be an arbitrary positive number. Since (H3) is true, it follows from Lemma 2.3 that $\lim_{t \to \infty}(\hat{f}_i(t)/\varphi(t)) = 0$, $i = 1, \ldots, n$. Hence, there is an $r_2 > 0$ such that

$$\hat{f}_i(r_2) \leq \varphi(\varepsilon)\varphi(r_2), \quad i = 1, \ldots, n, \qquad (4.1)$$

where the constant $\varepsilon > 0$ satisfies

$$nR\varphi^{-1}\left(\frac{R}{N}\right)\varepsilon < 1. \qquad (4.2)$$

Thus, we have by Lemma 2.4 that

$$\|\mathbf{T}(\mathbf{u})\| \leq nR\varphi^{-1}\left(\frac{R}{N}\right)\varepsilon\|\mathbf{u}\| < \|\mathbf{u}\| \quad \text{for } \mathbf{u} \in \partial\Omega_{r_2}. \qquad (4.3)$$

By Lemma 2.1,

$$i(\mathbf{T}, \Omega_{r_2}, K) = 1. \qquad (4.4)$$

Next using exactly the same argument as in Theorem 1.1, we can determine a $0 < r_1 < r_2$ from (H2) such that (3.6) holds. Note that R can be any positive number for Theorem 1.2. Thus it follows from Lemma 2.1 that

$$i(\mathbf{T}, \Omega_{r_1}, K) = 0, \qquad i(\mathbf{T}, \Omega_{r_2}, K) = 1, \qquad (4.5)$$

and hence, $i(\mathbf{T}, \Omega_{r_2} \setminus \bar{\Omega}_{r_1}, K) = 1$. Thus, $\mathbf{T}$ has a fixed point in $\Omega_{r_2} \setminus \bar{\Omega}_{r_1}$. Consequently, (1.1) has a positive solution for all $R > 0$.

5. Proof of Theorem 1.3

Since (H4) is true, for each $i = 1,\ldots,n$, there exist positive numbers ε_1^i, ε_2^i, r_1^i, and r_2^i such that $r_1^i < r_2^i$,

$$
\begin{aligned}
f_i(\mathbf{u}) &\leq \varepsilon_1^i \varphi(\|\mathbf{u}\|) \quad \text{for } \mathbf{u} \in \mathbb{R}_+^n, \; \|\mathbf{u}\| \leq r_1^i, \\
f_i(\mathbf{u}) &\leq \varepsilon_2^i \varphi(\|\mathbf{u}\|) \quad \text{for } \mathbf{u} \in \mathbb{R}_+^n, \; \|\mathbf{u}\| \geq r_2^i.
\end{aligned}
\tag{5.1}
$$

Let

$$
\varepsilon^i = \max\left\{\varepsilon_1^i, \varepsilon_2^i, \max\left\{\frac{f_i(\mathbf{u})}{\varphi(\|\mathbf{u}\|)} : \mathbf{u} \in \mathbb{R}_+^n, \; r_1^i \leq \|\mathbf{u}\| \leq r_2^i\right\}\right\} > 0
\tag{5.2}
$$

and $\varepsilon = \max_{i=1,\ldots,n}\{\varepsilon^i\} > 0$. Thus, we have

$$
f_i(\mathbf{u}) \leq \varepsilon\varphi(\|\mathbf{u}\|) \quad \text{for } \mathbf{u} \in \mathbb{R}_+^n, \; i = 1,\ldots,n.
\tag{5.3}
$$

Assume $\mathbf{v}(t)$ is a positive solution of (1.1). We will show that this leads to a contradiction for $0 < R < R_0$, where

$$
nR_0\varphi^{-1}\left(\frac{R_0\varepsilon}{N}\right) < 1.
\tag{5.4}
$$

In fact, for $0 < R < R_0$, since $\mathbf{Tv}(t) = \mathbf{v}(t)$ for $t \in [0,1]$, we find

$$
\begin{aligned}
\|\mathbf{v}\| = \|\mathbf{Tv}\| = \sum_{i=1}^n \sup_{t\in[0,1]} |T^i\mathbf{v}(t)| &\leq R\sum_{i=1}^n \int_0^1 \varphi^{-1}\left[\frac{R}{s^{N-1}}\int_0^s \tau^{N-1}d\tau\varepsilon\varphi(\|\mathbf{v}\|)\right]ds \\
&\leq nR\varphi^{-1}\left(\frac{R\varepsilon}{N}\varphi(\|\mathbf{v}\|)\right) = nR\varphi^{-1}\left(\frac{R\varepsilon}{N}\right)\|\mathbf{v}\| < \|\mathbf{v}\|,
\end{aligned}
\tag{5.5}
$$

which is a contradiction.

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