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Exponential energy decay and blow-up of solutions for a system of nonlinear viscoelastic wave equations with strong damping

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Abstract

In this paper, we consider the system of nonlinear viscoelastic equations

$$\begin{cases} u_{tt} - \Delta u + \int\limits_0^t g_1(t-\tau)\Delta u(\tau)d\tau - \Delta u_t = f_1(u,v), & (x,t) \in \Omega \times (0,T), \\ v_{tt} - \Delta v + \int\limits_0^t g_2(t-\tau)\Delta v(\tau)d\tau - \Delta v_t = f_2(u,v), & (x,t) \in \Omega \times (0,T) \end{cases}$$

with initial and Dirichlet boundary conditions. We prove that, under suitable assumptions on the functions g_i, f_i ($i = 1, 2$) and certain initial data in the stable set, the decay rate of the solution energy is exponential. Conversely, for certain initial data in the unstable set, there are solutions with positive initial energy that blow up in finite time.

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Keywords: decay, blow-up, positive initial energy, viscoelastic wave equations

1. Introduction

In this article, we study the following system of viscoelastic equations:

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau - \Delta u_t = f_1(u,v), & (x,t) \in \Omega \times (0,T), \\ v_{tt} - \Delta v + \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau - \Delta v_t = f_2(u,v), & (x,t) \in \Omega \times (0,T), \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in \Omega, \\ v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$, and $g_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$, $f_i(\cdot, \cdot)$: $\mathbb{R}^2 \to \mathbb{R}$ ($i = 1, 2$) are given functions to be specified later. Here, u and v denote the transverse displacements of waves. This problem arises in the theory of viscoelastic and describes the interaction of two scalar fields, we can refer to Cavalcanti et al. [1], Messaoudi and Tatar [2], Renardy et al. [3].

To motivate this study, let us recall some results regarding single viscoelastic wave equation. Cavalcanti et al. [4] studied the following equation:

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + a(x)u_t + |u|^\gamma u = 0, \quad \text{in } \Omega \times (0,\infty)$$

for $a : \Omega \to \mathbb{R}^+$, a function, which may be null on a part of the domain Ω. Under the conditions that $a(x) \geq a_0 > 0$ on $\Omega_1 \subset \Omega$, with Ω_1 satisfying some geometry restrictions and

$$-\xi_1 g(t) \leq g'(t) \leq -\xi_2 g(t), \quad t \geq 0,$$

the authors established an exponential rate of decay. This latter result has been improved by Cavalcanti and Oquendo [5] and Berrimi and Messaoudi [6]. In their study, Cavalcanti and Oquendo [5] considered the situation where the internal dissipation acts on a part of Ω and the viscoelastic dissipation acts on the other part. They established both exponential and polynomial decay results under the conditions on g and its derivatives up to the third order, whereas Berrimi and Messaoudi [6] allowed the internal dissipation to be nonlinear. They also showed that the dissipation induced by the integral term is strong enough to stabilize the system and established an exponential decay for the solution energy provided that g satisfies a relation of the form

$$g'(t) \leq -\xi g(t), \quad t \geq 0.$$

Cavalcanti et al. [1] also studied, in a bounded domain, the following equation:

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau - \gamma\Delta u_t = 0,$$

$\rho > 0$, and proved a global existence result for $\gamma \geq 0$ and an exponential decay for $\gamma > 0$. This result has been extended by Messaoudi and Tatar [2,7] to the situation where $\gamma = 0$ and exponential and polynomial decay results in the absence, as well as in the presence, of a source term have been established. Recently, Messaoudi [8,9] considered

$$u_{tt} - \Delta u + \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau = b|u|^\gamma u, \quad (x,t) \in \Omega \times (0,\infty),$$

for $b = 0$ and $b = 1$ and for a wider class of relaxation functions. He established a more general decay result, for which the usual exponential and polynomial decay results are just special cases.

For the finite time blow-up of a solution, the single viscoelastic wave equation of the form

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + h(u_t) = f(u) \tag{1.2}$$

in $\Omega \times (0, \infty)$ with initial and boundary conditions has extensively been studied. See in this regard, Kafini and Messaoudi [10], Messaoudi [11,12], Song and Zhong [13], Wang [14]. For instance, Messaoudi [11] studied (1.2) for $h(u_t) = a|u_t|^{m-2}u_t$ and $f(u) = b|u|^{p-2}u$ and proved a blow-up result for solutions with negative initial energy if $p > m \geq 2$ and a global result for $2 \leq p \leq m$. This result has been later improved by Messaoudi [12] to accommodate certain solutions with positive initial energy. Song and Zhong [13] considered (1.2) for $h(u_t) = -\Delta u_t$ and $f(u) = |u|^{p-2}u$ and proved a blow-up result for solutions with positive initial energy using the ideas of the "potential well" theory introduced by Payne and Sattinger [15].

This study is also motivated by the research of the well-known Klein-Gordon system

$$\begin{cases} u_{tt} - \Delta u + m_1 u + k_1 u v^2 = 0, \\ v_{tt} - \Delta v + m_2 v + k_2 u^2 v = 0, \end{cases}$$

which arises in the study of quantum field theory [16]. See also Medeiros and Miranda [17], Zhang [18] for some generalizations of this system and references therein. As far as we know, the problem (1.1) with the viscoelastic effect described by the memory terms has not been well studied. Recently, Han and Wang [19] considered the following problem

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau + |u_t|^{m-1}u_t = f_1(u,v), & (x,t) \in \Omega \times (0,T), \\ v_{tt} - \Delta v + \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau + |v_t|^{r-1}v_t = f_2(u,v), & (x,t) \in \Omega \times (0,T), \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in \Omega, \\ v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), & x \in \Omega, \end{cases}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$ in $\mathbb{R}^n$, $n = 1, 2, 3$. Under suitable assumptions on the functions g_i, f_i ($i = 1, 2$), the initial data and the parameters in the equations, they established several results concerning local existence, global existence, uniqueness, and finite time blow-up (the initial energy $E(0) < 0$) property. This latter blow-up result has been improved by Messaoudi and Said-Houari [20], to certain solutions with positive initial energy. Liu [21] studied the following system

$$\begin{cases} |u_t|^\rho u_{tt} - \Delta u - \gamma_1 \Delta u_{tt} + \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau + f(u,v) = 0, & (x,t) \in \Omega \times (0,T), \\ |v_t|^\rho v_{tt} - \Delta v - \gamma_2 \Delta v_{tt} + \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau + k(u,v) = 0, & (x,t) \in \Omega \times (0,T), \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in \Omega, \\ v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), & x \in \Omega, \end{cases}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$ in $\mathbb{R}^n$, $\gamma_1, \gamma_2 \geq 0$ are constants and ρ is a real number such that $0 < \rho \leq 2/(n - 2)$ if $n \geq 3$ or $\rho > 0$ if $n = 1, 2$. Under suitable assumptions on the functions $g(s)$, $h(s)$, $f(u, v)$, $k(u, v)$, they used the perturbed energy method to show that the dissipations given by the viscoelastic terms are strong enough to ensure exponential or polynomial decay of the solutions energy, depending on the decay rate of the relaxation functions $g(s)$ and $h(s)$. For the problem (1.1) in $\mathbb{R}^n$, we mention the work of Kafini and Messaoudi [10].

Motivated by the above research, we consider in this study the coupled system (1.1). We prove that, under suitable assumptions on the functions g_i, f_i ($i = 1, 2$) and certain initial data in the stable set, the decay rate of the solution energy is exponential. Conversely, for certain initial data in the unstable set, there are solutions with positive initial energy that blow up in finite time.

This article is organized as follows. In Section 2, we present some assumptions and definitions needed for this study. Section 3 is devoted to the proof of the uniform decay result. In Section 4, we prove the blow-up result.

2. Preliminaries

First, let us introduce some notation used throughout this article. We denote by $||\cdot||_q$ the $L^q(\Omega)$ norm for $1 \leq q \leq \infty$ and by $||\nabla\cdot||_2$ the Dirichlet norm in $H_0^1(\Omega)$ which is equivalent to the $H^1(\Omega)$norm. Moreover, we set

$$(\varphi, \psi) = \int_\Omega \varphi(x)\psi(x)dx$$

as the usual $L^2(\Omega)$ inner product.

Concerning the functions $f_1(u, v)$ and $f_2(u, v)$, we take

$$\begin{aligned} f_1(u, v) &= [a|u + v|^{2(p+1)}(u + v) + b|u|^p u|v|^{(p+2)}], \\ f_2(u, v) &= [a|u + v|^{2(p+1)}(u + v) + b|u|^{(p+2)}|v|^p v], \end{aligned}$$

where a, $b > 0$ are constants and p satisfies

$$\begin{cases} p > -1, & \text{if } n = 1, 2, \\ -1 < p \le 1, & \text{if } n = 3. \end{cases} \tag{2.1}$$

One can easily verify that

$$uf_1(u, v) + vf_2(u, v) = 2(p + 2)F(u, v), \quad \forall (u, v) \in \mathbb{R}^2,$$

where

$$F(u, v) = \frac{1}{2(p + 2)}[a|u + v|^{2(p+2)} + 2b|uv|^{p+2}].$$

For the relaxation functions $g_i(t)$ ($i = 1, 2$), we assume

(G1) $g_i(t) : \mathbb{R}_+ \to \mathbb{R}_+$ belong to $C^1(\mathbb{R}_+)$ and satisfy

$$g_i(t) \ge 0, \quad g_i'(t) \le 0, \quad \text{for } t \ge 0$$

and

$$1 - \int_0^\infty g_i(s)ds = k_i > 0.$$

(G2) $\max\left\{\int_0^\infty g_1(s)ds, \int_0^\infty g_2(s)ds\right\} < \frac{4(p + 1)(p + 2)}{4(p + 1)(p + 2) + 1}$.

We next state the local existence and the uniqueness of the solution of problem (1.1), whose proof can be found in Han and Wang [19] (Theorem 2.1) with slight modification, so we will omit its proof. In the proof, the authors adopted the technique of Agre and Rammaha [22] which consists of constructing approximations by the Faedo-Galerkin procedure without imposing the usual smallness conditions on the initial data to handle the source terms. Unfortunately, due to the strong nonlinearities on f_1 and f_2, the techniques used by Han and Wang [19] and Agre and Rammaha [22] allowed them to prove the local existence result only for $n \le 3$. We note that the local existence result in the case of $n > 3$ is still open. For related results, we also refer the reader to Said-Houari and Messaoudi [23] and Messaoudi and Said-Houari [20]. So throughout this article, we have assumed that $n \le 3$.

Theorem 2.1. *Assume that (2.1) and* (G1) *hold, and that* $(u_0, u_1) \in H_0^1(\Omega) \times L^2(\Omega)$, $(v_0, v_1) \in H_0^1(\Omega) \times L^2(\Omega)$. *Then problem (1.1) has a unique local solution*

$$u,\ v \in C([0, T]; H_0^1(\Omega)), \quad u_t,\ v_t \in C([0, T]; L^2(\Omega)) \cap L^2([0, T]; H_0^1(\Omega))$$

for some $T > 0$. *If* $T < \infty$, *then*

$$\lim_{t\to T}(k_1||\nabla u(t)||_2^2 + ||u_t(t)||_2^2 + k_2||\nabla v(t)||_2^2 + ||v_t(t)||_2^2) = \infty. \tag{2.2}$$

Finally, we define

$$\begin{aligned} I(t) = (1 - \int_0^t g_1(\tau)d\tau)||\nabla u(t)||_2^2 + \left(1 - \int_0^t g_2(\tau)d\tau\right)||\nabla v(t)||_2^2 \\ + [(g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)] - 2(p+2)\int_\Omega F(u,v)dx, \end{aligned} \tag{2.3}$$

$$\begin{aligned} J(t) = \frac{1}{2}\left[\left(1 - \int_0^t g_1(\tau)d\tau\right)||\nabla u(t)||_2^2 + \left(1 - \int_0^t g_2(\tau)d\tau\right)||\nabla v(t)||_2^2\right] \\ + \frac{1}{2}[(g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)] - \int_\Omega F(u,v)dx, \end{aligned} \tag{2.4}$$

such functionals we could refer to Muñoz Rivera [24,25]. We also define the energy function as follows

$$E(t) = \frac{1}{2}\left(||u_t(t)||_2^2 + ||v_t(t)||_2^2\right) + J(t), \tag{2.5}$$

where

$$(g_i \circ w)(t) = \int_0^t g_i(t-\tau)||w(t) - w(\tau)||_2^2 d\tau.$$

3. Global existence and energy decay

In this section, we deal with the uniform exponential decay of the energy for system (1.1) by using the perturbed energy method. Before we state and prove our main result, we need the following lemmas.

Lemma 3.1. *Assume (2.1) and* (*G*1) *hold. Let* (*u*, *v*) *be the solution of the system (1.1), then the energy functional is a decreasing function, that is*

$$\begin{aligned} E'(t) = -||\nabla u_t(t)||_2^2 - ||\nabla v_t(t)||_2^2 + \frac{1}{2}(g'_1 \circ u)(t) + \frac{1}{2}(g'_2 \circ v)(t) \\ - \frac{1}{2}g_1(t)||\nabla u(t)||_2^2 - \frac{1}{2}g_2(t)||\nabla v(t)||_2^2 \le 0. \end{aligned} \tag{3.1}$$

Moreover, the following energy inequality holds:

$$E(t) + \int_s^t (||\nabla u_t(\tau)||_2^2 + ||\nabla v_t(\tau)||_2^2)d\tau \le E(s), \quad \text{for } 0 \le s \le t < T. \tag{3.2}$$

Lemma 3.2. *Let (2.1) hold. Then, there exists* $\eta > 0$ *such that for any* $(u, v) \in H_0^1(\Omega) \times H_0^1(\Omega)$, *we have*

$$||u+v||_{2(p+2)}^{2(p+2)} + 2||uv||_{p+2}^{p+2} \le \eta(k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2)^{p+2}. \tag{3.3}$$

Proof. The proof is almost the same that of Said-Houari [26], so we omit it here. □

To prove our result and for the sake of simplicity, we take $a = b = 1$ and introduce the following:

$$B = \eta^{\frac{1}{2(p+2)}}, \quad \alpha^* = B^{-\frac{p+2}{P+1}}, \quad E_1 = \left(\frac{1}{2} - \frac{1}{2(p+2)}\right)\alpha^{*2}, \tag{3.4}$$

where η is the optimal constant in (3.3). The following lemma will play an essential role in the proof of our main result, and it is similar to a lemma used first by Vitillaro [27], to study a class of a single wave equation, which introduces a potential well.

Lemma 3.3. *Let (2.1) and (G1) hold. Let* (u, v) *be the solution of the system (1.1). Assume further that* $E(0) < E_1$ *and*

$$(k_1||\nabla u_0||_2^2 + k_2||\nabla v_0||_2^2)^{1/2} < \alpha^*, \tag{3.5}$$

Then

$$(k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 + (g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t))^{1/2} < \alpha^*, \quad \text{for } t \in [0, T). \tag{3.6}$$

Proof. We first note that, by (2.5), (3.3) and the definition of B, we have

$$\begin{aligned} E(t) &\geq \frac{1}{2}(k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 + (g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)) \\ &- \frac{1}{2(p+2)}(||u+v||_{2(p+2)}^{2(p+2)} + 2||uv||_{p+2}^{p+2}) \\ &\geq \frac{1}{2}(k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 + (g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)) \\ &- \frac{B^{2(p+2)}}{2(p+2)}(k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2)^{p+2} \\ &\geq \frac{1}{2}\alpha^2 - \frac{B^{2(p+2)}}{2(p+2)}\alpha^{2(p+2)} = g(\alpha), \end{aligned} \tag{3.7}$$

where $\alpha = (k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 + (g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t))^{1/2}$. It is not hard to verify that g is increasing for $0 < \alpha < \alpha^*$, decreasing for $\alpha > \alpha^*$, $g(\alpha) \to -\infty$ as $\alpha \to +\infty$, and

$$g(\alpha^*) = \frac{1}{2}\alpha^{*2} - \frac{B^{2(p+2)}}{2(p+2)}\alpha^{*2(p+2)} = E_1,$$

where α^* is given in (3.4). Now we establish (3.6) by contradiction. Suppose (3.6) does not hold, then it follows from the continuity of $(u(t), v(t))$ that there exists $t_0 \in (0, T)$ such that

$$(k_1||\nabla u(t_0)||_2^2 + k_2||\nabla v(t_0)||_2^2 + (g_1 \circ \nabla u)(t_0) + (g_2 \circ \nabla v)(t_0))^{1/2} = \alpha^*.$$

By (3.7), we observe that

$$E(t_0) \geq g\left((k_1||\nabla u(t_0)||_2^2 + k_2||\nabla v(t_0)||_2^2 + (g_1 \circ \nabla u)(t_0) + (g_2 \circ \nabla v)(t_0))^{1/2}\right) = g(\alpha^*) = E_1.$$

This is impossible since $E(t) \leq E(0) < E_1$ for all $t \in [0, T)$. Hence (3.6) is established. □

The following integral inequality plays an important role in our proof of the energy decay of the solutions to problem (1.1).

Lemma 3.4. [28]*Assume that the function* $\phi : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ *is a non-increasing function and that there exists a constant* $c > 0$ *such that*

$$\int_t^\infty \varphi(s)ds \leq c\varphi(t)$$

for every $t \in [0, \infty)$. *Then*

$$\varphi(t) \le \varphi(0) \exp(1 - t/c)$$

for every $t \ge c$.

Theorem 3.5. *Let (2.1) and (G1) hold. If the initial data* $(u_0, u_1) \in H_0^1(\Omega) \times L^2(\Omega)$, $(v_0, v_1) \in H_0^1(\Omega) \times L^2(\Omega)$*satisfy* $E(0) < E_1$ *and*

$$(k_1||\nabla u_0||_2^2 + k_2||\nabla v_0||_2^2)^{1/2} < \alpha^*, \tag{3.8}$$

where the constants α_*, E_1 *are defined in (3.4), then the corresponding solution to (1.1) globally exists, i.e.* $T = \infty$. *Moreover, if the initial energy* $E(0)$ *and k such that*

$$1 - \eta\left(\frac{2(p+2)}{(p+1)}E(0)\right)^{(p+1)} - \frac{5(1-k)(p+2)}{2k(p+1)} > 0,$$

where $k = \min\{k_1, k_2\}$, *then the energy decay is*

$$E(t) \le E(0) \exp(1 - aC^{-1}t)$$

for every $t \ge aC^{-1}$, *where C is some positive constant.*

Proof. In order to get $T = \infty$, by (2.2), it suffices to show that

$$||u_t(t)||_2^2 + ||v_t(t)||_2^2 + k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2$$

is bounded independently of t. Since $E(0) < E_1$ and

$$(k_1||\nabla u_0||_2^2 + k_2||\nabla v_0||_2^2)^{1/2} < \alpha^*,$$

it follows from Lemma 3.3 that

$$k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 \le k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 + (g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t) < \alpha^{*2},$$

which implies that

$$\begin{aligned} I(t) &\ge k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 + [(g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)] - 2(p+2)\int_\Omega F(u,v)dx \\ &\ge k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 - 2(p+2)\int_\Omega F(u,v)dx \\ &= k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 - (||u+v||_{2(p+2)}^{2(p+2)} + 2||uv||_{p+2}^{p+2}) \\ &\ge k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 - \eta(k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2)^{p+2} \ge 0, \quad \text{for } t \in [0, T), \end{aligned}$$

where we have used (3.3). Furthermore, by (2.3) and (2.4), we get

$$\begin{aligned} J(t) &\ge \left(\frac{1}{2} - \frac{1}{2(p+2)}\right)\left[(1 - \int_0^t g_1(s)ds)||\nabla u(t)||_2^2 + \left(1 - \int_0^t g_2(s)ds\right)||\nabla v(t)||_2^2\right] \\ &+ \left(\frac{1}{2} - \frac{1}{2(p+2)}\right)\left[(g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)\right] + \frac{1}{2(p+2)}I(t) \\ &\ge \frac{p+1}{2(p+2)}\left[k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 + (g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)\right] + \frac{1}{2(p+2)}I(t) \ge 0, \end{aligned}$$

from which, the definition of $E(t)$ and $E(t) \le E(0)$, we deduce that

$$\left[k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2\right] \le \frac{2(p+2)}{(p+1)}J(t) \le \frac{2(p+2)}{(p+1)}E(t) \le \frac{2(p+2)}{(p+1)}E(0), \tag{3.9}$$

for $t \in [0, T)$. So it follows from (16) and Lemma 3.1 that

$$\frac{p+1}{2(p+2)}\left[k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2\right] + \frac{1}{2}(||u_t(t)||_2^2 + ||v_t(t)||_2^2) \le J(t) + \frac{1}{2}(||u_t(t)||_2^2 + ||v_t(t)||_2^2)$$
$$= E(t) \le E(0) < E_1, \ \forall t \in [0, T),$$

which implies

$$||u_t(t)||_2^2 + ||v_t(t)||_2^2 + k_1||\nabla u(t)||_2^2 + k_2||\nabla v(t)||_2^2 < CE_1,$$

where C is a positive constant depending only on p.

Next we want to derive the decay rate of energy function for problem (1.1). By multiplying the first equation of system (1.1) by u and the second equation of system (1.1) by v, integrating over $\Omega \times [t_1, t_2]$ $(0 \le t_1 \le t_2)$, using integration by parts and summing up, we have

$$\begin{aligned}
&\int_\Omega u_t(t)u(t)dx|_{t_1}^{t_2} - \int_{t_1}^{t_2} ||u_t(t)||_2^2 dt + \int_\Omega v_t(t)v(t)dx|_{t_1}^{t_2} - \int_{t_1}^{t_2} ||v_t(t)||_2^2 dt \\
= &-\int_{t_1}^{t_2} (\nabla u(t), \nabla u_t(t))dt - \int_{t_1}^{t_2} (\nabla v(t), \nabla v_t(t))dt - \int_{t_1}^{t_2} ||\nabla u(t)||_2^2 dt - \int_{t_1}^{t_2} ||\nabla v(t)||_2^2 dt \\
&- \int_{t_1}^{t_2} \int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dxdt - \int_{t_1}^{t_2} \int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dxdt \\
&+2(p+2)\int_{t_1}^{t_2} \int_\Omega F(u,v)dxdt,
\end{aligned}$$

which implies

$$\begin{aligned}
&2\int_{t_1}^{t_2} E(t)dt - 2(p+1)\int_{t_1}^{t_2}\int_\Omega F(u,v)dxdt \\
= &-\int_\Omega u_t(t)u(t)dx|_{t_1}^{t_2} - \int_\Omega v_t(t)v(t)dx|_{t_1}^{t_2} + 2\int_{t_1}^{t_2} ||u_t(t)||_2^2 dt + 2\int_{t_1}^{t_2} ||v_t(t)||_2^2 dt \\
&+ \int_{t_1}^{t_2} (g_1 \circ \nabla u)(t)dt + \int_{t_1}^{t_2} (g_2 \circ \nabla v)(t)dt - \int_{t_1}^{t_2} \int_0^t g_1(\tau)d\tau ||\nabla u(t)||_2^2 dt \\
&- \int_{t_1}^{t_2} \int_0^t g_2(\tau)d\tau ||\nabla v(t)||_2^2 dt - \int_{t_1}^{t_2} (\nabla u(t), \nabla u_t(t))dt - \int_{t_1}^{t_2} (\nabla v(t), \nabla v_t(t))dt \\
&- \int_{t_1}^{t_2} \int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dxdt - \int_{t_1}^{t_2} \int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dxdt.
\end{aligned} \tag{3.10}$$

For the 11th term on the right-hand side of (3.10), one has

$$\begin{aligned}
&-2\int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dx = 2\int_\Omega \int_0^t g_1(t-\tau)\nabla u(\tau)\nabla u(t)d\tau dx \\
= &\int_0^t g_1(t-\tau)(||\nabla u(t)||_2^2 + ||\nabla u(\tau)||_2^2)d\tau - \int_0^t g_1(t-\tau)(||\nabla u(t) - \nabla u(\tau)||_2^2)d\tau.
\end{aligned} \tag{3.11}$$

Similarly,

$$\begin{aligned}
&-2\int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dx \\
= &\int_0^t g_2(t-\tau)(||\nabla v(t)||_2^2 + ||\nabla v(\tau)||_2^2)d\tau - \int_0^t g_2(t-\tau)(||\nabla v(t) - \nabla v(\tau)||_2^2)d\tau.
\end{aligned} \tag{3.12}$$

Combining (3.10), (3.11) with (3.12), we have

$$\begin{aligned}
&2\int_{t_1}^{t_2} E(t)dt - 2(p+1)\int_{t_1}^{t_2}\int_\Omega F(u,v)dxdt \\
&= -\int_\Omega u_t(t)u(t)dx|_{t_1}^{t_2} - \int_\Omega v_t(t)v(t)dx|_{t_1}^{t_2} + 2\int_{t_1}^{t_2} ||u_t(t)||_2^2 dt + 2\int_{t_1}^{t_2} ||v_t(t)||_2^2 dt \\
&+\frac{1}{2}\int_{t_1}^{t_2}(g_1\circ\nabla u)(t)dt + \frac{1}{2}\int_{t_1}^{t_2}(g_2\circ\nabla v)(t)dt - \frac{1}{2}\int_{t_1}^{t_2}\int_0^t g_1(\tau)d\tau||\nabla u(t)||_2^2 dt \\
&-\frac{1}{2}\int_{t_1}^{t_2}\int_0^t g_2(\tau)d\tau||\nabla v(t)||_2^2 dt - \int_{t_1}^{t_2}(\nabla u(t),\nabla u_t(t))dt - \int_{t_1}^{t_2}(\nabla v(t),\nabla v_t(t))dt \\
&+\frac{1}{2}\int_{t_1}^{t_2}\int_0^t g_1(t-\tau)||\nabla u(\tau)||_2^2 d\tau dt + \frac{1}{2}\int_{t_1}^{t_2}\int_0^t g_2(t-\tau)||\nabla v(\tau)||_2^2 d\tau dt \qquad (3.13)\\
&\le -\int_\Omega u_t(t)u(t)dx|_{t_1}^{t_2} - \int_\Omega v_t(t)v(t)dx|_{t_1}^{t_2} + 2\int_{t_1}^{t_2} ||u_t(t)||_2^2 dt + 2\int_{t_1}^{t_2} ||v_t(t)||_2^2 dt \\
&+\frac{1}{2}\int_{t_1}^{t_2}(g_1\circ\nabla u)(t)dt + \frac{1}{2}\int_{t_1}^{t_2}(g_2\circ\nabla v)(t)dt - \int_{t_1}^{t_2}(\nabla u(t),\nabla u_t(t))dt \\
&-\int_{t_1}^{t_2}(\nabla v(t),\nabla v_t(t))dt + \frac{1}{2}\int_{t_1}^{t_2}\int_0^t g_1(t-\tau)||\nabla u(\tau)||_2^2 d\tau dt \\
&+\frac{1}{2}\int_{t_1}^{t_2}\int_0^t g_2(t-\tau)||\nabla v(\tau)||_2^2 d\tau dt.
\end{aligned}$$

Now we estimate every term of the right-hand side of the (3.13). First, by Hölder's inequality and Poincaré's inequality

$$\begin{aligned}
\int_\Omega |u(t)u_t(t)|dx + \int_\Omega |v(t)v_t(t)|dx &\le \frac{1}{2}||u(t)||_2^2 + \frac{1}{2}||u_t(t)||_2^2 + \frac{1}{2}||v(t)||_2^2 + \frac{1}{2}||v_t(t)||_2^2 \\
&\le \frac{\lambda}{2}||\nabla u(t)||_2^2 + \frac{1}{2}||u_t(t)||_2^2 + \frac{\lambda}{2}||\nabla v(t)||_2^2 + \frac{1}{2}||v_t(t)||_2^2,
\end{aligned}$$

where λ being the first eigenvalue of the operator - Δ under homogeneous Dirichlet boundary conditions. Then, by (3.9), we see that

$$\int_\Omega |u(t)u_t(t)|dx + \int_\Omega |v(t)v_t(t)|dx \le c_1 E(t),$$

where c_1 is a constant independent on u and v, from which follows that

$$\int_\Omega |u(t)u_t(t)|dx|_{t_1}^{t_2} + \int_\Omega |v(t)v_t(t)|dx|_{t_1}^{t_2} \le 2c_1 E(t_1). \qquad (3.14)$$

Since $0 \le J(t) \le E(t)$, from (3.2) we deduce that

$$\int_{t_1}^{t_2}(||\nabla u_t(t)||_2^2 + ||\nabla v_t(t)||_2^2)dt \le E(t_1).$$

Hence, by Poincaré inequality we get

$$2\int_{t_1}^{t_2} ||u_t(t)||_2^2 dt + 2\int_{t_1}^{t_2} ||v_t(t)||_2^2 dt \le 2c_2 E(t_1), \qquad (3.15)$$

where c_2 is a constant independent on u and v. In addition, using Young's inequality for convolution $||f * g||_q \le ||f||_r\, ||g||_s$ with $1/q = 1/r + 1/s - 1$ and $1 \le q, r, s \le \infty$, noting that if $q = 1$, then $r = 1$ and $s = 1$, we have

$$\begin{aligned}\int_{t_1}^{t_2}\int_0^t g_1(t-\tau)||\nabla u(\tau)||_2^2 d\tau dt &= ||g_1 * ||\nabla u||_2^2||_1 \le ||g_1||_1|| \; ||\nabla u||_2^2||_1 \\ &= \int_{t_1}^{t_2} g_1(t)dt \int_{t_1}^{t_2} ||\nabla u(t)||_2^2 dt \\ &\le (1-k_1)\int_{t_1}^{t_2} ||\nabla u(t)||_2^2 dt,\end{aligned} \tag{3.16}$$

and

$$\begin{aligned}\int_{t_1}^{t_2}\int_0^t g_2(t-\tau)||\nabla v(\tau)||_2^2 d\tau dt &= ||g_2 * ||\nabla v||_2^2||_1 \;\le\; ||g_2||_1||||\nabla v||_2^2||_1 \\ &= \int_{t_1}^{t_2} g_2(t)dt \int_{t_1}^{t_2} ||\nabla v(t)||_2^2 dt \\ &\le (1-k_2)\int_{t_1}^{t_2} ||\nabla v(t)||_2^2 dt.\end{aligned} \tag{3.17}$$

Hence, combining (3.9), (3.16) with (3.17) we then have

$$\begin{aligned}&\int_{t_1}^{t_2}\int_0^t g_1(t-\tau)||\nabla u(\tau)||_2^2 d\tau dt + \int_{t_1}^{t_2}\int_0^t g_2(t-\tau)||\nabla v(\tau)||_2^2 d\tau dt \\ &\le (1-k_1)\int_{t_1}^{t_2} ||\nabla u(t)||_2^2 dt + (1-k_2)\int_{t_1}^{t_2} ||\nabla v(t)||_2^2 dt \\ &\le (1-k)\int_{t_1}^{t_2} (||\nabla u(t)||_2^2 + ||\nabla v(t)||_2^2)dt \le \frac{2(1-k)(p+2)}{k(p+1)}\int_{t_1}^{t_2} E(t)dt.\end{aligned} \tag{3.18}$$

From (3.9), we also have

$$\begin{aligned}&\int_{t_1}^{t_2}\int_0^t g_1(t-\tau)||\nabla u(t)||_2^2 d\tau dt + \int_{t_1}^{t_2}\int_0^t g_2(t-\tau)||\nabla v(t)||_2^2 d\tau dt \\ &\le (1-k_1)\int_{t_1}^{t_2} ||\nabla u(t)||_2^2 dt + (1-k_2)\int_{t_1}^{t_2} ||\nabla v(t)||_2^2 dt \\ &\le (1-k)\int_{t_1}^{t_2} (||\nabla u(t)||_2^2 + ||\nabla v(t)||_2^2)dt \le \frac{2(1-k)(p+2)}{k(p+1)}\int_{t_1}^{t_2} E(t)dt.\end{aligned} \tag{3.19}$$

Combining (3.18) with (3.19), we deduce that

$$\begin{aligned}&\frac{1}{2}\int_{t_1}^{t_2} (g_1 \circ \nabla u)(t)dt + \frac{1}{2}\int_{t_1}^{t_2} (g_2 \circ \nabla v)(t)dt \\ &\le \int_{t_1}^{t_2}\int_0^t g_1(t-\tau)(||\nabla u(\tau)||_2^2 + ||\nabla u(t)||_2^2)d\tau dt + \int_{t_1}^{t_2}\int_0^t g_2(t-\tau) \\ &(||\nabla v(\tau)||_2^2 + ||\nabla v(t)||_2^2)d\tau dt \le \frac{4(1-k)(p+2)}{k(p+1)}\int_{t_1}^{t_2} E(t)dt.\end{aligned} \tag{3.20}$$

Finally, we also have the following estimate

$$\begin{aligned}
&\int_{t_1}^{t_2} (\nabla u(t), \nabla u_t(t))dt + \int_{t_1}^{t_2} (\nabla v(t), \nabla v_t(t))dt \\
&= \frac{1}{2}\int_{t_1}^{t_2} \frac{d}{dt}||\nabla u(t)||_2^2 dt + \frac{1}{2}\int_{t_1}^{t_2} \frac{d}{dt}||\nabla v(t)||_2^2 dt \\
&= \frac{1}{2}(||\nabla u(t_2)||_2^2 - ||\nabla u(t_1)||_2^2) + \frac{1}{2}(||\nabla v(t_2)||_2^2 - ||\nabla v(t_1)||_2^2) \\
&\le \frac{2(p+2)}{k(p+1)}E(t_1) \le c_3 E(t_1).
\end{aligned} \tag{3.21}$$

where c_3 is a constant independent on u and v. Combining (3.13)-(3.21), we obtain

$$2\int_{t_1}^{t_2} E(t)dt - 2(p+1)\int_{t_1}^{t_2}\int_\Omega F(u,v)dxdt \le CE(t_1) + \frac{5(1-k)(p+2)}{k(p+1)}\int_{t_1}^{t_2} E(t)dt. \tag{3.22}$$

where C is a constant independent on u.

On the other hand, from (3.3) and (3.9), we have

$$\begin{aligned}
2(p+1)\int_\Omega F(u,v)dx &= \frac{p+1}{p+2}\left(||u+v||_{2(p+2)}^{2(p+2)} + 2||uv||_{(p+2)}^{(p+2)}\right) \\
&\le \frac{p+1}{p+2}\eta(k_1||\nabla u||_2^2 + k_2||\nabla v||_2^2)^{(p+2)} \\
&\le 2\eta\left(\frac{2(p+2)}{(p+1)}E(0)\right)^{(p+1)} E(t),
\end{aligned}$$

which implies

$$2\int_{t_1}^{t_2} E(t)dt - 2(p+1)\int_{t_1}^{t_2}\int_\Omega F(u,v)dxdt \ge 2\left(1-\eta(\frac{2(p+2)}{(p+1)}E(0))^{(p+1)}\right)\int_{t_1}^{t_2} E(t)dt. \tag{3.23}$$

Note that $E(0) < E_1$, we see that

$$1-\eta(\frac{2(p+2)}{(p+1)}E(0))^{(p+1)} > 0.$$

Thus, combining (3.22) with (3.23), we have

$$2\left(1-\eta(\frac{2(p+2)}{(p+1)}E(0))^{(p+1)}\right)\int_{t_1}^{t_2} E(t)dt \le CE(t_1) + \frac{5(1-k)(p+2)}{k(p+1)}\int_{t_1}^{t_2} E(t)dt,$$

that is

$$2\left(1-\eta(\frac{2(p+2)}{(p+1)}E(0))^{(p+1)} - \frac{5(1-k)(p+2)}{2k(p+1)}\right)\int_{t_1}^{t_2} E(t)dt \le CE(t_1). \tag{3.24}$$

Denote

$$a = 2\left(1-\eta(\frac{2(p+2)}{(p+1)}E(0))^{(p+1)} - \frac{5(1-k)(p+2)}{2k(p+1)}\right).$$

We rewrite (3.24)

$$a \int_t^\infty E(\tau)d\tau \le CE(t)$$

for every $t \in [0, \infty)$.

Since $a > 0$ from the assumption conditions, by Lemma 3.4, we obtain the following energy decay for problem (1.1) as

$$E(t) < E(0)\exp(1 - aC^{-1}t)$$

for every $t \ge Ca^{-1}$. □

4. Blow-up of solution

In this section, we deal with the blow-up solutions of the system (1.1). Set

$$\theta_i = k_i - \frac{1}{4(p+2)(p+1)} \int_0^\infty g_i(s)ds, \quad i = 1, 2. \tag{4.1}$$

From the assumption (*G*2), we have $\theta_i > 0$ (i = 1, 2). Similarly Lemma 3.2, we have

Lemma 4.1. *Assume (2.1) holds. Then there exists* $\eta_1 > 0$ *such that for any* $(u, v) \in H_0^1(\Omega) \times H_0^1(\Omega)$, *we have*

$$||u+v||_{2(p+2)}^{2(p+2)} + 2||uv||_{p+2}^{p+2} \le \eta_1(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2)^{p+2}, \tag{4.2}$$

where the constants θ_i (i = 1, 2) *are defined in (4.1).*

To prove our result and for the sake of simplicity, we take $a = b = 1$ and introduce the following:

$$B_1 = \eta_1^{\frac{1}{2(p+2)}}, \quad \alpha_* = B_1^{-\frac{p+2}{p+1}}, \quad E_2 = \left(\frac{1}{2} - \frac{1}{2(p+2)}\right)\alpha_*^2. \tag{4.3}$$

Then we have

Lemma 4.2. *Let* (*G*1), *G*(2) *and (2.1) hold. Let* (*u, v*) *be the solution of the system (1.1). Assume further that* $E(0) < E_2$ *and*

$$(\theta_1||\nabla u_0||_2^2 + \theta_2||\nabla v_0||_2^2)^{1/2} > \alpha_*, \tag{4.4}$$

where the constants θ_i (i = 1, 2) *are defined in (4.1). Then there exists a constant* $\alpha_2 > \alpha_*$ *such that*

$$(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2)^{1/2} \ge \alpha_2, \quad \text{for } t \in (0, T). \tag{4.5}$$

Proof. We first note that, by (2.5), (4.2) and the definition of B_1, we have

$$\begin{aligned} E(t) &\ge \frac{1}{2}(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2) - \frac{1}{2(p+2)}(||u+v||_{2(p+2)}^{2(p+2)} + 2||uv||_{p+2}^{p+2}) \\ &\ge \frac{1}{2}(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2) - \frac{B_1^{2(p+2)}}{2(p+2)}(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2)^{p+2} \\ &= \frac{1}{2}\alpha^2 - \frac{B_1^{2(p+2)}}{2(p+2)}\alpha^{2(p+2)}, \end{aligned} \tag{4.6}$$

where $\alpha = (\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2)^{1/2}$. It is not hard to verify that g is increasing for $0 < \alpha < \alpha_*$, decreasing for $\alpha > \alpha_*$, $g(\alpha) \to -\infty$ as $\alpha \to +\infty$, and

$$g(\alpha_*) = \frac{1}{2}\alpha_*^2 - \frac{B_1^{2(p+2)}}{2(p+2)}\alpha_*^{2(p+2)} = E_2,$$

where α_* is given in (4.3). Since $E(0) < E_2$, there exists $\alpha_2 > \alpha_*$ such that $g(\alpha_2) = E(0)$.

Set $\alpha_0 = (\theta_1||\nabla u_0||_2^2 + \theta_2||\nabla v_0||_2^2)^{1/2}$, then by (4.6) we get $g(\alpha_0) \le E(0) = g(\alpha_2)$, which implies that $\alpha_0 \ge \alpha_2$. Now, to establish (4.5), we suppose by contradiction that

$$(\theta_1||\nabla u(t_0)||_2^2 + \theta_2||\nabla v(t_0)||_2^2)^{1/2} < \alpha_2,$$

for some $t_0 > 0$. By the continuity of $\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2$ we can choose t_0 such that

$$(\theta_1||\nabla u(t_0)||_2^2 + \theta_2||\nabla v(t_0)||_2^2)^{1/2} > \alpha_*.$$

Again, the use of (4.6) leads to

$$E(t_0) \ge g((\theta_1||\nabla u(t_0)||_2^2 + \theta_2||\nabla v(t_0)||_2^2)^{1/2}) > g(\alpha_2) = E(0).$$

This is impossible since $E(t) \le E(0)$ for all $t \in [0, T)$. Hence (4.5) is established. □

Theorem 4.3. *Assume* $(G1)$, $(G2)$ *and (2.1) hold. Then any solution of problem (1.1) with initial data satisfying*

$$(\theta_1||\nabla u_0||_2^2 + \theta_2||\nabla v_0||_2^2)^{1/2} > \alpha_* \quad \text{and} \quad E(0) < E_2$$

blows up in finite time, where the constants θ_i $(i = 1, 2)$ *are defined in (4.1) and* α_*, E_2 *are defined in (4.3).*

Proof. Assume by contradiction that the solution (u, v) is global. Then, for any $T > 0$ we consider $H(t) : [0, T] \to \mathbb{R}_+$ defined by

$$H(t) = ||u(t)||_2^2 + ||v(t)||_2^2 + \int_0^t ||\nabla u(\tau)||_2^2 d\tau + \int_0^t ||\nabla v(\tau)||_2^2 d\tau + (T-t)(||\nabla u_0||_2^2 + ||\nabla v_0||_2^2) + \beta(t + s_0)^2,$$

where β and s_0 are positive constants to be determined later. A direct computation yields

$$\begin{aligned} H'(t) &= 2\int_\Omega u(t)u_t(t)dx + 2\int_\Omega v(t)v_t(t)dx + ||\nabla u(t)||_2^2 + ||\nabla v(t)||_2^2 \\ &\quad - ||\nabla u_0||_2^2 - ||\nabla v_0||_2^2 + 2\beta(t + s_0) \\ &= 2\int_\Omega u(t)u_t(t)dx + 2\int_\Omega v(t)v_t(t)dx + 2\int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau \\ &\quad + 2\int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau + 2\beta(t + s_0) \end{aligned}$$

and

$$\begin{aligned} H''(t) &= 2\int_\Omega u(t)u_{tt}(t)dx + 2\int_\Omega v(t)v_{tt}(t)dx + 2||u_t(t)||_2^2 + 2||v_t(t)||_2^2 \\ &\quad + 2(\nabla u(t), \nabla u_t(t)) + 2(\nabla v(t), \nabla v_t(t)) + 2\beta \text{ for a.e. } t \in [0, T]. \end{aligned}$$

Multiplying the first equation of system (1.1) by u and the second equation of system (1.1) by v, integrating over Ω, using integration by parts and summing up, we have

$$
\begin{aligned}
&(u_{tt}, u(t)) + (v_{tt}, v(t)) + (\nabla u(t), \nabla u_t(t)) + (\nabla v(t), \nabla v_t(t)) \\
&= -||\nabla u(t)||_2^2 - ||\nabla v(t)||_2^2 - \int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau) d\tau u(t) dx \\
&\quad - \int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau) d\tau v(t) dx + 2(p+2)\int_\Omega F(u,v)dx,
\end{aligned}
$$

which implies

$$
\begin{aligned}
H''(t) &= 2||u_t(t)||_2^2 + 2||v_t(t)||_2^2 - 2||\nabla u(t)||_2^2 - 2||\nabla v(t)||_2^2 + 4(p+2)\int_\Omega F(u,v)dx \\
&\quad - 2\int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dx - 2\int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dx + 2\beta.
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
&H(t)H''(t) - \frac{p+3}{2}H'(t)^2 \\
&= 2H(t)\left(||u_t(t)||_2^2 + ||v_t(t)||_2^2 - ||\nabla u(t)||_2^2 - ||\nabla v(t)||_2^2 + 2(p+2)\int_\Omega F(u,v)dx + 2\beta\right) \\
&\quad -2H(t)\left(\int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dx + \int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dx\right) \\
&\quad -2(p+3)\Big(\int_\Omega u(t)u_t(t)dx + \int_\Omega v(t)v_t(t)dx + \int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau \\
&\quad + \int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau + \beta(t+s_0)\Big)^2 \\
&= 2H(t)\left(||u_t(t)||_2^2 + ||v_t(t)||_2^2 - ||\nabla u(t)||_2^2 - ||\nabla v(t)||_2^2 + 2(p+2)\int_\Omega F(u,v)dx + 2\beta\right) \\
&\quad -2H(t)\left(\int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dx + \int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dx\right) \\
&\quad +2(p+3)\left(G(t) - (H(t) - (T-t)(||\nabla u_0||_2^2 + ||\nabla v_0||_2^2))\Psi(t)\right),
\end{aligned}
\tag{4.7}
$$

where $\Psi(t)$, $G(t)$: $[0, T] \to \mathbb{R}_+$ are the functions defined by

$$
\Psi(t) = ||u_t(t)||_2^2 + ||v_t(t)||_2^2 + \int_0^t ||\nabla u_t(\tau)||_2^2 d\tau + \int_0^t ||\nabla v_t(\tau)||_2^2 d\tau + \beta
$$

and

$$
\begin{aligned}
G(t) &= \left(||u(t)||_2^2 + ||v(t)||_2^2 + \int_0^t ||\nabla u(\tau)||_2^2 d\tau + \int_0^t ||\nabla v(\tau)||_2^2 d\tau + \beta(t+s_0)^2\right)\Psi(t) \\
&\quad - \Big(\int_\Omega u(t)u_t(t)dx + \int_\Omega v(t)v_t(t)dx + \int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau \\
&\quad + \int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau + \beta(t+s_0)\Big)^2.
\end{aligned}
$$

Using the Schwarz inequality, we have

$$\left(\int_\Omega u(t)u_t(t)dx\right)^2 \le ||u(t)||_2^2||u_t(t)||_2^2, \quad \left(\int_\Omega v(t)v_t(t)dx\right)^2 \le ||v(t)||_2^2||v_t(t)||_2^2,$$

$$\left(\int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau\right)^2 \le \int_0^t ||\nabla u(\tau)||_2^2 d\tau \int_0^t ||\nabla u_t(\tau)||_2^2 d\tau,$$

$$\left(\int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau\right)^2 \le \int_0^t ||\nabla v(\tau)||_2^2 d\tau \int_0^t ||\nabla v_t(\tau)||_2^2 d\tau,$$

$$\begin{aligned}\int_\Omega u(t)u_t(t)dx \int_\Omega v(t)v_t(t)dx &\le ||u(t)||_2||v_t(t)||_2||u_t(t)||_2||v(t)||_2 \\ &\le \frac{1}{2}||u(t)||_2^2||v_t(t)||_2^2 + \frac{1}{2}||u_t(t)||_2^2||v(t)||_2^2,\end{aligned}$$

$$\begin{aligned}\beta(t+s_0)\int_\Omega u(t)u_t(t)dx &\le \sqrt{\beta}\sqrt{\beta}(t+s_0)||u(t)||_2||u_t(t)||_2 \\ &\le \frac{1}{2}\beta||u(t)||_2^2 + \frac{1}{2}\beta(t+s_0)^2||u_t(t)||_2^2,\end{aligned}$$

and

$$\beta(t+s_0)\int_\Omega v(t)v_t(t)dx \le \frac{1}{2}\beta||v(t)||_2^2 + \frac{1}{2}\beta(t+s_0)^2||v_t(t)||_2^2.$$

Similarly, we have

$$\int_\Omega u(t)u_t(t)dx \int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau \le \frac{1}{2}||u(t)||_2^2 \int_0^t ||\nabla u_t(\tau)||_2^2 d\tau + \frac{1}{2}||u_t(t)||_2^2 \int_0^t ||\nabla u(\tau)||_2^2 d\tau,$$
$$\int_\Omega u(t)u_t(t)dx \int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau \le \frac{1}{2}||u(t)||_2^2 \int_0^t ||\nabla v_t(\tau)||_2^2 d\tau + \frac{1}{2}||u_t(t)||_2^2 \int_0^t ||\nabla v(\tau)||_2^2 d\tau,$$
$$\int_\Omega v(t)v_t(t)dx \int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau \le \frac{1}{2}||v(t)||_2^2 \int_0^t ||\nabla u_t(\tau)||_2^2 d\tau + \frac{1}{2}||v_t(t)||_2^2 \int_0^t ||\nabla u(\tau)||_2^2 d\tau,$$
$$\int_\Omega v(t)v_t(t)dx \int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau \le \frac{1}{2}||v(t)||_2^2 \int_0^t ||\nabla v_t(\tau)||_2^2 d\tau + \frac{1}{2}||v_t(t)||_2^2 \int_0^t ||\nabla v(\tau)||_2^2 d\tau,$$

and

$$\begin{aligned}&\int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau \int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau \\ &\le \frac{1}{2}\int_0^t ||\nabla u(\tau)||_2^2 d\tau \int_0^t ||\nabla v_t(\tau)||_2^2 d\tau + \frac{1}{2}\int_0^t ||\nabla u_t(\tau)||_2^2 d\tau \int_0^t ||\nabla v(\tau)||_2^2 d\tau,\end{aligned}$$

$$\beta(t+s_0)\int_0^t (\nabla u(\tau), \nabla u_t(\tau))d\tau \le \frac{1}{2}\beta\int_0^t ||\nabla u(\tau)||_2^2 d\tau + \frac{1}{2}\beta(t+s_0)^2\int_0^t ||\nabla u_t(\tau)||_2^2 d\tau,$$

$$\beta(t+s_0)\int_0^t (\nabla v(\tau), \nabla v_t(\tau))d\tau \le \frac{1}{2}\beta\int_0^t ||\nabla v(\tau)||_2^2 d\tau + \frac{1}{2}\beta(t+s_0)^2\int_0^t ||\nabla v_t(\tau)||_2^2 d\tau.$$

The previous inequalities entail $G(t) \ge 0$ for every $[0, T]$. Using (4.7), we get

$$H(t)H''(t) - \frac{p+3}{2}H'(t)^2 \ge H(t)L(t) \ \text{ for a.e. } t \in [0, T], \tag{4.8}$$

where

$$\begin{aligned}L(t) = &-2(p+2)(||u_t(t)||_2^2 + ||v_t(t)||_2^2) - 2||\nabla u(t)||_2^2 - 2||\nabla v(t)||_2^2 + 4(p+2)\int_\Omega F(u,v)dx \\ &- 2\left(\int_\Omega\int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dx + \int_\Omega\int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dx\right) \\ &- 2(p+3)\left(\int_0^t ||\nabla u_t(\tau)||_2^2 d\tau + \int_0^t ||\nabla v_t(\tau)||_2^2 d\tau\right) - 2(p+1)\beta.\end{aligned} \tag{4.9}$$

For the fifth term on the right-hand side of (4.9), we have

$$\begin{aligned}
&-\int_\Omega \int_0^t g_1(t-\tau)\Delta u(\tau)d\tau u(t)dx = \int_0^t g_1(t-\tau)\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau \\
&= \int_0^t g_1(t-\tau)\int_\Omega \nabla u(t)\nabla(u(\tau)-u(t))dxd\tau + \int_0^t g_1(t-\tau)||\nabla u(t)||_2^2 d\tau \\
&= \int_0^t g_1(t-\tau)\int_\Omega \nabla u(t)\nabla(u(\tau)-u(t))dxd\tau + \int_0^t g_1(\tau)||\nabla u(t)||_2^2 d\tau.
\end{aligned} \tag{4.10}$$

Similarly,

$$\begin{aligned}
&-\int_\Omega \int_0^t g_2(t-\tau)\Delta v(\tau)d\tau v(t)dx \\
&= \int_0^t g_2(t-\tau)\int_\Omega \nabla v(t)\nabla(v(\tau)-v(t))dxd\tau + \int_0^t g_2(\tau)||\nabla v(t)||_2^2 d\tau.
\end{aligned} \tag{4.11}$$

Combining (4.9), (4.10) with (4.11), we get

$$\begin{aligned}
L(t) = &-2(p+2)(||u_t(t)||_2^2 + ||v_t(t)||_2^2) - 2(1-\int_0^t g_1(\tau)d\tau)||\nabla u(t)||_2^2 \\
&- 2(1-\int_0^t g_2(\tau)d\tau)||\nabla v(t)||_2^2 + 4(p+2)\int_\Omega F(u,v)dx - 2(p+1)\beta \\
&+ 2\int_0^t g_1(t-\tau)\int_\Omega \nabla u(t)\nabla(u(\tau)-u(t))dxd\tau - 2(p+3)\int_0^t ||\nabla u_t(\tau)||_2^2 d\tau \\
&+ 2\int_0^t g_2(t-\tau)\int_\Omega \nabla v(t)\nabla(v(\tau)-v(t))dxd\tau - 2(p+3)\int_0^t ||\nabla v_t(\tau)||_2^2 d\tau.
\end{aligned} \tag{4.12}$$

Since

$$\begin{aligned}
&2\int_0^t g_1(t-\tau)\int_\Omega \nabla u(t)\nabla(u(\tau)-u(t))dxd\tau \\
&\geq -2\left((p+2)\int_0^t g_1(t-\tau)||\nabla u(\tau)-\nabla u(t)||_2^2 d\tau + \frac{1}{4(p+2)}\int_0^t g_1(\tau)||\nabla u(t)||_2^2 d\tau\right) \\
&= -2(p+2)(g_1\circ\nabla u)(t) - \frac{1}{2(p+2)}\int_0^t g_1(\tau)||\nabla u(t)||_2^2 d\tau,
\end{aligned} \tag{4.13}$$

and

$$\begin{aligned}
&2\int_0^t g_2(t-\tau)\int_\Omega \nabla v(t)\nabla(v(\tau)-v(t))dxd\tau \\
&\geq -2(p+2)(g_2\circ\nabla v)(t) - \frac{1}{2(p+2)}\int_0^t g_2(\tau)||\nabla v(t)||_2^2 d\tau,
\end{aligned} \tag{4.14}$$

inserting (4.13) and (4.14) into (4.12), we have

$$\begin{aligned}
L(t) \geq & -2(p+2)(||u_t(t)||_2^2 + ||v_t(t)||_2^2 + (g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)) + 4(p+2)\int_\Omega F(u,v)dx \\
& -2\left(1 - \int_0^t g_1(\tau)d\tau + \frac{1}{4(p+2)}\int_0^t g_1(\tau)d\tau\right)||\nabla u(t)||_2^2 - 2\left(1 - \int_0^t g_2(\tau)d\tau\right)||\nabla v(t)||_2^2 \\
& -\frac{1}{2(p+2)}\int_0^t g_2(\tau)||\nabla v(t)||_2^2 d\tau - 2(p+3)\left(\int_0^t ||\nabla v_t(\tau)||_2^2 d\tau\right. \\
& \left. + \int_0^t ||\nabla u_t(\tau)||_2^2 d\tau\right) - 2(p+1)\beta \\
\geq & -4(p+2)E(t) + 2(p+1)\left(1 - \int_0^t g_1(\tau)d\tau\right)||\nabla u(t)||_2^2 - \frac{1}{2(p+2)}\int_0^t g_1(\tau)||\nabla u(t)||_2^2 d\tau \\
& + 2(p+1)\left(1 - \int_0^t g_2(\tau)d\tau\right)||\nabla v(t)||_2^2 - \frac{1}{2(p+2)}\int_0^t g_2(\tau)||\nabla v(t)||_2^2 d\tau \\
& - 2(p+3)\left(\int_0^t ||\nabla v_t(\tau)||_2^2 d\tau + \int_0^t ||\nabla u_t(\tau)||_2^2 d\tau\right) - 2(p+1)\beta.
\end{aligned}$$

Using (3.2) for $s = 0$, we have

$$\begin{aligned}
L(t) \geq & -4(p+2)E(0) + 2(p+1)\left(1 - \int_0^t g_1(\tau)d\tau\right)||\nabla u(t)||_2^2 - \frac{1}{2(p+2)}\int_0^t g_1(\tau)||\nabla u(t)||_2^2 d\tau \\
& + 2(p+1)\left(1 - \int_0^t g_2(\tau)d\tau\right)||\nabla v(t)||_2^2 - \frac{1}{2(p+2)}\int_0^t g_2(\tau)||\nabla v(t)||_2^2 d\tau \\
& + 2(p+1)\left(\int_0^t ||\nabla v_t(\tau)||_2^2 d\tau + \int_0^t ||\nabla u_t(\tau)||_2^2 d\tau\right) - 2(p+1)\beta \\
\geq & \, 4(p+2)\left(\frac{p+1}{2(p+2)}\left((1 - \int_0^t g_1(\tau)d\tau) - \frac{1}{4(p+2)(p+1)}\int_0^t g_1(\tau)d\tau\right)\right)||\nabla u(t)||_2^2 \\
& + 4(p+2)\left(\frac{p+1}{2(p+2)}\left((1 - \int_0^t g_2(\tau)d\tau) - \frac{1}{4(p+2)(p+1)}\int_0^t g_2(\tau)d\tau\right)\right)||\nabla v(t)||_2^2 \\
& - 4(p+2)E(0) - 2(p+1)\beta \\
\geq & \, 4(p+2)\left(\frac{p+1}{2(p+2)}\left(k_1 - \frac{1}{4(p+2)(p+1)}\int_0^t g_1(\tau)d\tau\right)\right)||\nabla u(t)||_2^2 - 2(p+1)\beta \\
& + 4(p+2)\left(\frac{p+1}{2(p+2)}\left(k_2 - \frac{1}{4(p+2)(p+1)}\int_0^t g_2(\tau)d\tau\right)\right)||\nabla v(t)||_2^2 - 4(p+2)E(0) \\
\geq & \, 4(p+2)\left(\frac{p+1}{2(p+2)}(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2) - E(0) - \frac{p+1}{2(p+2)}\beta\right).
\end{aligned}$$

Since

$$(\theta_1||\nabla u_0||_2^2 + \theta_2||\nabla v_0||_2^2)^{1/2} > \alpha_* \quad \text{and} \quad E(0) < E_2,$$

by Lemma 4.2, there exists a constant $\alpha > \alpha_*$ such that

$$(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2)^{1/2} \geq \alpha_2, \tag{4.15}$$

which implies

$$\frac{p+1}{2(p+2)}(\theta_1||\nabla u(t)||_2^2 + \theta_2||\nabla v(t)||_2^2) \geq \frac{p+1}{2(p+2)}\alpha_2^2 > E_2 > E(0).$$

Thus, we can let β satisfy

$$(p+1)\beta < 2(p+2)(E_2 - E(0)),$$

which implies that there exists $\delta > 0$ (independent of T) such that

$$L(t) \geq \delta \quad \text{for } t \in [0, T]. \tag{4.16}$$

From (4.15) and the definition of $H(t)$, there also exists $\rho > 0$ (independent of T) such that

$$H(t) \geq \rho \quad \text{for } t \in [0, T]. \tag{4.17}$$

By (4.8), (4.16) and (4.17) it follows that

$$H(t)H''(t) - \frac{p+3}{2}H'(t)^2 \geq \delta\rho \quad \text{for a.e. } t \in [0, T].$$

Moreover, we let s_0 satisfy that

$$\beta s_0 + \int_\Omega u_0 u_1 dx + \int_\Omega v_0 v_1 dx > 0,$$

which means $H'(0) > 0$. Thus by $H''(t) > 0$ we see that $H(t)$ and $H'(t)$ is strictly increasing on $[0, T]$.

Setting $y(t) = H(t)^{-(p+1)/2}$, then we have

$$y'(t) = -\frac{p+1}{2}H(t)^{-(p+3)/2}H'(t) < 0,$$

and

$$y''(t) \leq -\frac{p+1}{2}\delta\rho y(t)^{\frac{p+5}{p+1}}$$

for all $t \in [0, T]$, which implies that $y(t)$ reaches 0 in finite time, say as $t \to T^*$. Since T^* is independent of the initial choice of T, we may assume that $T^* < T$. This tells us that

$$\lim_{t \to T*} H(t) = \infty.$$

In turn, this implies that

$$\lim_{t \to T*} (||\nabla u(t)||_2^2 + ||\nabla v(t)||_2^2) = \infty. \tag{4.18}$$

Indeed, if

$$\lim_{t \to T*} (||u(t)||_2^2 + ||v(t)||_2^2) = \infty,$$

then (4.18) immediately follows. On the contrary, if $||u(t)||_2^2 + ||v(t)||_2^2$ remains bounded on $[0, T^*)$, then

$$\lim_{t \to T*} \left(\int_0^t ||\nabla u(\tau)||_2^2 \tau + \int_0^t ||\nabla v(\tau)||_2^2 \tau \right) = \infty$$

so that again (4.18) is satisfied. This implies a contradiction, i.e. $T < \infty$. □

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Authors' contributions

FL and HGAO carried out all studies in this article.

Competing interests

The authors declare that they have no competing interests.

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