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Existence of solutions for nonlinear mixed type integro-differential functional evolution equations with nonlocal conditions

Shengli Xie*

*Correspondence: slxie@aiai.edu.cn
Department of Mathematics and Physics, Anhui University of Architecture, Jinzhai Road, Hefei, Anhui 230022, P.R. China

Abstract

Using the Mönch fixed point theorem, this article proves the existence of mild solutions for nonlinear mixed type integro-differential functional evolution equations with nonlocal conditions in Banach spaces. Some restricted conditions on *a priori* estimation and measure of noncompactness estimation have been deleted, and compactness conditions of evolution operators or compactness conditions on a nonlinear term $f(t, X_r, X_r, X_r)$ have been weakened. Our results extend and improve many known results.

MSC: 34G20; 34K30

Keywords: integro-differential functional evolution equation; mild solution; nonlocal conditions; fixed point; Banach spaces

1 Introduction

Let $(X, \|\cdot\|)$ be a Banach space, $C[J,X] = \{x : J = [0,a] \to X, x(t) \text{ is continuous in } J\}$ with the norm $\|x\|_C = \sup_{t\in J} \|x(t)\|$. It is easy to verify that $C[J,X]$ is a Banach space. The space of X-valued Bochner integrable functions on J with the norm $\|x\|_1 = \int_0^a \|x(s)\|\,ds$ is denoted by $L[J,X]$. Consider the following nonlinear mixed type integro-differential functional evolution equations with nonlocal conditions in a Banach space $X(IVP)$,

$$x'(t) = A\left[x(t) + \int_0^t F(t-s)x(s)\,ds\right] + f\big(t, x(\sigma_1(t)), (Kx_{\sigma_2})(t), (Hx_{\sigma_3})(t)\big), \quad t \in J, \tag{1.1}$$

$$x(0) = g(x) + x_0, \tag{1.2}$$

where

$$(Kx_{\sigma_2})(t) = \int_0^t k\big(t,s,x(\sigma_2(s))\big)\,ds, \qquad (Hx_{\sigma_3})(t) = \int_0^a h\big(t,s,x(\sigma_3(s))\big)\,ds, \tag{1.3}$$

A is the generator of a strongly continuous semigroup in the Banach space X, and $F(t)$ is a bounded operator for $t \in J$, $x_0 \in X$, $f \in C[J \times X^3, X]$, $g : C[J,X] \to X$, $k \in C[\triangle \times X, X]$, $\triangle = \{(t,s) \in J \times J : s \le t\}$, $h \in C[J \times J \times X, X]$, $\sigma_i \in C[J,J]$ and $\sigma_i(t) \le t$ $(i = 1,2,3)$.

For the existence of mild solutions of integro-differential functional evolution equations in abstract spaces, there are many research results, see [1–16], and references therein. In order to obtain the existence and controllability of mild solutions in these study papers, usually, some restricted conditions on a priori estimation and compactness conditions of an evolution operator or compactness conditions on $f(t, X_r, X_r, X_r)$ are used.

Recently, using a fixed point theorem, Haribhau Laxman Tidkey and Machindra Baburao Dhakne [1] have studied the existence of mild solutions of *IVP* (1.1)-(1.2) when $\sigma_i(t) = t$ ($i = 1, 2, 3$), the compactness of the resolvent operator and the restricted condition

$$M_1\left[\|x_0\| + G_1 + Lrb + LKrb^2 + LK_1b^2 + LHrb^2 + LH_1b^2 + L_1b\right] \le r$$

with $M_1[Lb + LKb^2 + LHb^2] < 1$ is used. Malar [17] and Shi [18] studied the existence of mild solutions of semilinear mixed type integrodifferential evolution equations with the equicontinuous semigroup

$$\begin{cases} x'(t) = Ax(t) + f(t, x(t), \int_0^t a(t,s)k(s,x(s))\,ds, \\ \qquad\qquad \int_0^a b(t,s)h(s,x(s))\,ds), \quad t \in [0,a], \\ x(0) = x_0 + g(x). \end{cases} \tag{1.4}$$

Solvability of the scalar equation

$$m(t) = K_1 + K_2 \int_0^t h\big(s, m(s), n(s), q(s)\big)\,ds, \quad t \in J$$

and the restricted condition on measure of noncompactness estimation

$$\int_0^t \big[\eta_1(s) + k_1\eta_2(s) + k_2\eta_3(s)\big]\,ds \le K$$

are used in [17]. But estimations (3.15) and (3.21) in [18] seem to be incorrect, as they have no meaning.

In this paper, using the Mönch fixed point theorem, we investigate the existence of mild solutions of *IVP* (1.1)-(1.2). Some restricted conditions on *a priori* estimation and measure of noncompactness estimation have been deleted, and compactness conditions of a resolvent operator or compactness conditions on a nonlinear term $f(t, X_r, X_r, X_r)$ have been weakened. Our results extend and improve some corresponding results in papers [1–4, 6–21].

2 Preliminaries

We will make the following assumptions:

(H_1) A generates a strongly continuous semigroup in the Banach space X.

(H_2) $F(t) \in B(X)$, $0 \le t \le a$. $F(t): Y \to Y$ and for $x(\cdot)$ continuous in Y, $AF(\cdot)x(\cdot) \in L^1[J, X]$. For $x \in X$, $F'(t)x$ is continuous in $t \in J$, where $B(X)$ is the space of all linear and bounded operators on X, and Y is the Banach space formed from $D(A)$, the domain of A, endowed with the graph norm.

Definition 2.1 [5] $R(t)$ is a resolvent operator of (1.1) with $f \equiv 0$ if $R(t) \in B(X)$ for $0 \le t \le a$ and satisfies the following conditions:

(1) $R(0) = I$, the identity operator on X,

(2) for all $x \in X$, $R(t)x$ is continuous for $0 \le t \le a$,
(3) $R(t) \in B(Y)$, $0 \le t \le a$; for $y \in Y$, $R(\cdot)y \in C^1[J,X] \cap C[J,Y]$ and

$$\begin{aligned}\frac{d}{dt}R(t)y &= A\left[R(t)y + \int_0^t F(t-s)R(s)y\,ds\right] \\ &= R(t)Ay + \int_0^t R(t-s)AF(s)y\,ds, \quad 0 \le t \le a.\end{aligned} \tag{2.1}$$

The resolvent operator $R(t)$ is said to be equicontinuous if $\{t \to R(t)x : x \in B\}$ is equicontinuous for the entire bounded set $B \subset X$ and $t > 0$. If $x \in C[J,X]$ satisfies the following integral equation:

$$x(t) = R(t)\big(x_0 + g(x)\big) + \int_0^t R(t-s)f\big(s, x\big(\sigma_1(s)\big), (Kx_{\sigma_2})(s), (Hx_{\sigma_3})(s)\big)\,ds, \quad t \in J,$$

then x is said to be a mild solution *IVP* (1.1)-(1.2).

Lemma 2.2 [14] *Let the conditions* (H_1), (H_2) *be satisfied. Then (1.1) with* $f \equiv 0$ *has a unique resolvent operator.*

The following lemma is obvious.

Lemma 2.3 *Let the resolvent operator* $R(t)$ *be equicontinuous. If there is* $\rho \in L[J,\mathbb{R}^+]$ *such that* $\|x(t)\| \le \rho(t)$ *for a.e.* $t \in J$, *then the set* $\{\int_0^t R(t-s)x(s)\,ds\}$ *is equicontinuous.*

Lemma 2.4 [22] *Let* $V \in C[J,E]$ *be an equicontinuous bounded subset. Then* $\alpha(V(t)) \in C[J,\mathbb{R}^+]$ ($\mathbb{R}^+ = [0,\infty)$), $\alpha(V) = \max_{t\in J}\alpha(V(t))$.

Lemma 2.5 [23] *Let* $V = \{x_n\} \subset L[J,E]$ *and there exists* $\sigma \in L[J,\mathbb{R}^+]$ *such that* $\|x_n(t)\| \le \sigma(t)$ *for any* $x \in V$ *and a.e.* $t \in J$. *Then* $\alpha(V(t)) \in L[J,\mathbb{R}^+]$ *and*

$$\alpha\left(\left\{\int_0^t x_n(s)\,ds : n \in \mathbb{N}\right\}\right) \le 2\int_0^t \alpha\big(V(s)\big)\,ds, \quad t \in J.$$

Lemma 2.6 [24] (Mönch) *Let* E *be a Banach space,* Ω *a closed convex subset in* E *and* $y_0 \in \Omega$. *Suppose that the continuous operator* $F : \Omega \to \Omega$ *has the following property:*

$$V \subset \Omega \text{ countable}, V \subset \overline{co}\big(\{y_0\} \cup F(V)\big) \Rightarrow V \text{ is relatively compact.}$$

Then F *has a fixed point in* Ω.

For $V \subset C[J,X]$, let $V(t) = \{x(t) : x \in V\}$, $V_{\sigma_i}(t) = \{x(\sigma_i(t)) : x \in V\}$ $(i = 1,2,3)$, $(KV)(t) = \{(Kx)(t) : x \in V\}$, $(HV)(t) = \{(Hx)(t) : x \in V\}$ $(t \in J)$, $X_r = \{x \in X : \|x\| \le r\}$ and $S_r = \{x \in C[J,X] : \|x\|_C \le r\}$ for any $r > 0$. $\alpha(\cdot)$ and $\alpha_C(\cdot)$ denote the Kuratowski measure of noncompactness in X and $C[J,X]$ respectively. For details on the properties of noncompact measure, we refer the reader to [22].

3 Existence of a mild solution

We make the following assumptions for convenience.

(H_3) There exist constants $l_g > 0$, $M > 0$ and $4l_gM < 1$ such that

$$\|g(x) - g(y)\| \leq l_g \|x - y\|_C, \quad x, y \in C[J, X],$$

and $g(0) = 0$.

(H_3') $g : C[J, X] \to E$ is continuous, compact and there exists a constant $N \geq 0$ such that $\|g(x)\| \leq N$.

(H_4) There exists $q \in C[J, \mathbb{R}^+]$ such that

$$\|f(t, x, y, z)\| \leq q(t)\big(\|x\| + \|y\| + \|z\|\big), \quad t \in J, x, y, z \in X.$$

(H_5) There exist $k_0 \in C[\triangle, \mathbb{R}^+]$, $h_0 \in C[J \times J, \mathbb{R}^+]$ such that

$$\|k(t, s, x)\| \leq k_0(t, s)\|x\|, \quad (t, s) \in \triangle, x \in X,$$
$$\|h(t, s, x)\| \leq h_0(t, s)\|x\|, \quad t, s \in J, x \in X.$$

(H_6) For any $r > 0$ and a bounded set $V_i \subset X_r$, there exist constants $l_i > 0$ $(i = 1, 2, 3)$ such that

$$\alpha\big(f(t, V_1, V_2, V_3)\big) \leq l_1\alpha(V_1) + l_2\alpha(V_2) + l_3\alpha(V_3), \quad t \in J.$$

(H_7) For any $r > 0$ and a bounded set $V \subset X_r$,

$$\alpha\big(k(t, s, V)\big) \leq k_0(t, s)\alpha(V), \quad (t, s) \in \triangle,$$
$$\alpha\big(h(t, s, V)\big) \leq h_0(t, s)\alpha(V), \quad t, s \in J.$$

(H_8) The resolvent operator $R(t)$ is equicontinuous and $\|R(t)\| \leq Me^{-wt}$ for $t \in J$ and some positive number

$$w = \max\big\{2Mq_0(1 + K_0a + H_0a), 4M(l_1 + 2l_2aK_0 + 2l_3aH_0)\big\},$$

where $K_0 = \max_{(t,s)\in\triangle} k_0(t, s)$, $H_0 = \max_{t,s\in J} h_0(t, s)$, $q_0 = \max_{t\in J} q(t)$.

Without loss of generality, we always suppose that $x_0 = 0$.

Theorem 3.1 *Let conditions (H_1), (H_2), (H_3)-(H_8) be satisfied. Then IVP (1.1)-(1.2) has at least one mild solution.*

Proof Let

$$(Fx)(t) = R(t)g(x) + \int_0^t R(t - s)f\big(s, x(\sigma_1(s)), (Kx_{\sigma_2})(s), (Hx_{\sigma_3})(s)\big)\, ds, \quad t \in J. \tag{3.1}$$

We have by (H_3), (H_4) and (H_5),

$$\begin{aligned}
&\|(Fx)(t)\| \\
&\quad\leq \|R(t)g(x)\| + \int_0^t \|R(t-s)\| \|f(s, x(\sigma_1(s)), (Kx_{\sigma_2})(s), (Hx_{\sigma_3})(s))\| \, ds \\
&\quad\leq M\|g(x)\| + M\int_0^t e^{-w(t-s)} q(s)\left(\|x(\sigma_1(s))\| + \|(Kx)(\sigma_2(s))\| + \|(Hx)(\sigma_3(s))\|\right) ds \\
&\quad\leq l_g M\|x\|_C \\
&\qquad + Mq_0 \int_0^t e^{w(s-t)}\left(\|x(s)\| + \int_0^s k_0(s,r)\|x(r)\|\, dr + \int_0^a h_0(s,r)\|x(r)\|\, dr\right) ds \\
&\quad\leq l_g M\|x\|_C + Mq_0 \int_0^t e^{w(s-t)}(1 + K_0 a + H_0 a)\|x\|_C \, ds \\
&\quad\leq l_g M\|x\|_C + Mq_0(1 + K_0 a + H_0 a)w^{-1}\|x\|_C \leq \|x\|_C. \qquad (3.2)
\end{aligned}$$

Let

$$B_R = \{x \in C[J,X] : \|x\|_C \leq R\}.$$

Then B_R is a closed convex subset in $C[J,X]$, $0 \in B_R$ and $F : B_R \to B_R$. Similar to the proof of [6] and [9], it is easy to verify that F is a continuous operator from B_R into B_R. For $x \in B_R$, $s \in J$, (H_4) and (H_5) imply

$$\|f(s, x(\sigma_1(s)), (Kx_{\sigma_2})(s), (Hx_{\sigma_3})(s))\| \leq q(s)\left(1 + \int_0^s k_0(s,r)\, dr + \int_0^a h_0(s,r)\, dr\right) R. \quad (3.3)$$

We can show from (3.3), (H_8) and Lemma 2.3 that $F(B_R)$ is an equicontinuous subset in $C[J,X]$.

Let $V \subset B_R$ be a countable set and $V \subset \overline{co}(\{0\} \cup F(V))$, then

$$V(t) \subset \overline{co}(\{0\} \cup (FV)(t)). \quad (3.4)$$

From equicontinuity of $F(B_R)$ and (3.4), we know that V is an equicontinuous subset in $C[J,X]$. By the properties of noncompact measure, the conditions (H_3), (H_6), (H_7), (3.4) and Lemma 2.5, we have

$$\begin{aligned}
\alpha(V(t)) &\leq \alpha((FV)(t)) \\
&\leq \|R(t)\|\alpha(g(V)) + 2\int_0^t \|R(t-s)\|\alpha(f(s, V_{\sigma_1}(s), (KV_{\sigma_2})(s), (HV_{\sigma_3})(s))) \, ds \\
&\leq l_g M\alpha_C(V) + 2M\int_0^t e^{w(s-t)}\left[l_1\alpha(V(\sigma_1(s))) + 2l_2\int_0^s k_0(s,r)\alpha(V(\sigma_2(r)))\, dr\right. \\
&\qquad \left. + 2l_3\int_0^a h_0(s,r)\alpha(V(\sigma_3(r)))\, dr\right] ds \\
&\leq l_g M\alpha_C(V) \\
&\qquad + 2M\int_0^t e^{w(s-t)}\left[l_1\alpha(V(s))\, ds + 2\left(l_2\int_0^s K_0 + l_3\int_0^a H_0\right)\alpha(V(r))\, dr\right] ds
\end{aligned}$$

$$\leq l_g M\alpha_C(V) + 2M(l_1 + 2l_2 aK_0 + 2l_3 aH_0)\alpha_C(V)\int_0^t e^{w(s-t)}\,ds$$
$$\leq l_g M\alpha_C(V) + 2M(l_1 + 2l_2 aK_0 + 2l_3 aH_0)w^{-1}\alpha_C(V) \leq \frac{3}{4}\alpha_C(V), \quad t \in J. \quad (3.5)$$

(3.5) together with Lemma 2.4 imply that $\alpha_C(V) \leq \frac{3}{4}\alpha_C(V)$, and so $\alpha_C(V) = 0$. Hence V is relatively compact in $C[J,X]$. Lemma 2.6 implies that F has a fixed point in $C[J,X]$. Then *IVP* (1.1)-(1.2) has at least one mild solution. The proof is completed. □

Theorem 3.2 *Let the conditions* (H_1), (H_2) *and* (H_3')-(H_8) *be satisfied. Then IVP (1.1)-(1.2) has at least one mild solution.*

Proof Similar to (3.2) and (3.5), it is easy to verify

$$\big\|(Fx)(t)\big\| \leq MN + Mq_0(1 + K_0 a + H_0 a)w^{-1}\|x\|_C = MN + \eta\|x\|_C,$$

where $\eta = Mq_0(1+K_0a+H_0a)w^{-1} < 1$. Taking $R > MN(1-\eta)^{-1}$, let $B_R = \{x \in C[J,X] : \|x\|_C \leq R\}$. We have $F : B_R \to B_R$ and the inequality (3.5) is transformed into $\alpha(V(t)) \leq \frac{1}{2}\alpha_C(V)$, $t \in J$.

The other proof is similar to the proof of Theorem 3.1, we omit it. □

4 An example

Let $X = L^2[0,\pi]$. Consider the following partial functional integro-differential equation with a nonlocal condition,

$$\begin{cases} u_t(t,y) = u_y(t,y) + \int_0^t F(t-s)u_y(s,y)\,ds + \gamma_1 \sin u(t-r,y) \\ \qquad + \int_0^t \frac{\gamma_2 u(s-r,y)\,ds}{(1+t)} + \int_0^a \frac{\gamma_3 u(s-r,y)\,ds}{(1+t)(1+s)^2}, \quad 0 \leq t \leq a, \\ u(0,y) = u_0(y) + \gamma_4 u(y), \end{cases} \quad (4.1)$$

where $r, \gamma_i \in \mathbb{R}$ $(i = 1,2,3,4)$, $\sigma_1(t) = \sigma_2(t) = \sigma_3(t) = t - r$, $0 \leq r \leq t \leq a$, $F(t)$ satisfies the condition (H_2),

$$f\big(t, u(\sigma(t)), (Ku_\sigma)(t), (Su_\sigma)(t)\big)(y)$$
$$= \gamma_1 \sin u(t-r,y) + \int_0^t \frac{\gamma_2 u(s-r,y)\,ds}{(1+t)} + \int_0^a \frac{\gamma_3 u(s-r,y)\,ds}{(1+t)(1+s)^2}, \quad (4.2)$$

$$k\big(t,s,u(\sigma(s))\big)(y) = \frac{u(s-r,y)}{1+t}, \qquad h\big(t,s,u(\sigma(s))\big)(y) = \frac{u(s-r,y)}{(1+t)(1+s)^2}, \quad (4.3)$$

$$g(u)(y) = \gamma_4 u(y). \quad (4.4)$$

Let the operator A be defined by $Aw = w'$, $w \in D(A)$ with the domain

$$D(A) = \big\{w \in E : w' \in E, w' \text{ is almost everywhere bounded}\big\}.$$

Then A generates a translation semigroup $R(t)$ and $R(t)$ is equicontinuous. The problem (4.1) can be regarded as a form of *IVP* (1.1)-(1.2). We have by (4.2), (4.3) and (4.4),

$$\big\|f(t,u,v,z)\big\| \leq |\gamma|\big(\|u\| + \|v\| + \|z\|\big), \quad |\gamma| = \max\big\{|\gamma_1|, |\gamma_2|, |\gamma_3|\big\}, u, v, z \in X,$$

$$\|k(t,s,u)\| \le \|u\|, \qquad \|h(t,s,u)\| \le \|u\|, \quad u \in X,$$

and

$$\|g(u) - g(v)\| \le |\gamma_4| \|u - v\|_C, \quad g(0) = 0.$$

γ_4 and M can be chosen such that $4M|\gamma_4| < 1$. In addition, for any $r > 0$ and a bounded set $V_i \subset X_r$ ($i = 1, 2, 3$), we can show that by the diagonal method,

$$\alpha\big(f(t, V_1, V_2, V_3)\big) \le |\gamma| \big(\alpha(V_1) + \alpha(V_2) + \alpha(V_3)\big), \quad t \in J,$$
$$\alpha\big(k(t,s,V_1)\big) \le \alpha(V_1), \quad t,s \in \triangle,$$
$$\alpha\big(h(t,s,V_1)\big) \le \alpha(V_1), \quad t,s \in [0,a].$$

Hence all the conditions of Theorem 3.1 are satisfied, the problem (4.1) has at least one mild solution in $C[J,X]$.

Competing interests
The author declares that they have no competing interests.

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