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Multiple positive doubly periodic solutions for a singular semipositone telegraph equation with a parameter

Fanglei Wang[1*] and Yukun An[2]

*Correspondence:
wang-fanglei@hotmail.com
[1]College of Science, Hohai University, Nanjing, 210098, P.R. China
Full list of author information is available at the end of the article

Abstract
In this paper, we study the multiplicity of positive doubly periodic solutions for a singular semipositone telegraph equation. The proof is based on a well-known fixed point theorem in a cone.
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Keywords: semipositone telegraph equation; doubly periodic solution; singular; cone; fixed point theorem

1 Introduction

Recently, the existence and multiplicity of positive periodic solutions for a scalar singular equation or singular systems have been studied by using some fixed point theorems; see [1–9]. In [10], the authors show that the method of lower and upper solutions is also one of common techniques to study the singular problem. In addition, the authors [11] use the continuation type existence principle to investigate the following singular periodic problem:

$$\left(\left|u'\right|^{p-2}u'\right)' + h(u)u' = g(u) + c(t).$$

More recently, using a weak force condition, Wang [12] has built some existence results for the following periodic boundary value problem:

$$\begin{cases} u_{tt} - u_{xx} + c_1 u_t + a_{11}(t,x)u + a_{12}(t,x)v = f_1(t,x,u,v) + \chi_1(t,x), \\ v_{tt} - v_{xx} + c_2 v_t + a_{21}(t,x)u + a_{22}(t,x)v = f_2(t,x,u,v) + \chi_2(t,x). \end{cases}$$

The proof is based on Schauder's fixed point theorem. For other results concerning the existence and multiplicity of positive doubly periodic solutions for a single regular telegraph equation or regular telegraph system, see, for example, the papers [13–17] and the references therein. In these references, the nonlinearities are nonnegative.

On the other hand, the authors [18] study the semipositone telegraph system

$$\begin{cases} u_{tt} - u_{xx} + c_1 u_t + a_1(t,x)u = b_1(t,x)f(t,x,u,v), \\ v_{tt} - v_{xx} + c_2 v_t + a_2(t,x)v = b_2(t,x)g(t,x,u,v), \end{cases}$$

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where the nonlinearities f, g may change sign. In addition, there are many authors who have studied the semipositone equations; see [19, 20].

Inspired by the above references, we are concerned with the multiplicity of positive doubly periodic solutions for a general singular semipositone telegraph equation

$$\begin{cases} u_{tt} - u_{xx} + cu_t + a(t,x)u = \lambda f(t,x,u), \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x), \end{cases} \tag{1}$$

where $c > 0$ is a constant, $\lambda > 0$ is a positive parameter, $a(t,x) \in C(R \times R, R)$, $f(t,x,u)$ may change sign and is singular at $u = 0$, namely,

$$\lim_{u \to 0^+} f(t,x,u) = +\infty.$$

The main method used here is the following fixed-point theorem of a cone mapping.

Lemma 1.1 [21] *Let E be a Banach space, and $K \subset E$ be a cone in E. Assume Ω_1, Ω_2 are open subsets of E with $0 \in \Omega_1$, $\overline{\Omega}_1 \subset \Omega_2$, and let $T : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$ be a completely continuous operator such that either*

(i) $\|Tu\| \le \|u\|$, $u \in K \cap \partial\Omega_1$ *and* $\|Tu\| \ge \|u\|$, $u \in K \cap \partial\Omega_2$; *or*

(ii) $\|Tu\| \ge \|u\|$, $u \in K \cap \partial\Omega_1$ *and* $\|Tu\| \le \|u\|$, $u \in K \cap \partial\Omega_2$.

Then T has a fixed point in $K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

The paper is organized as follows. In Section 2, some preliminaries are given. In Section 3, we give the main result.

2 Preliminaries

Let $\top^2$ be the torus defined as

$$\top^2 = (R/2\pi Z) \times (R/2\pi Z).$$

Doubly 2π-periodic functions will be identified to be functions defined on $\top^2$. We use the notations

$$L^p(\top^2), \qquad C(\top^2), \qquad C^\alpha(\top^2), \qquad D(\top^2) = C^\infty(\top^2), \ldots$$

to denote the spaces of doubly periodic functions with the indicated degree of regularity. The space $D'(\top^2)$ denotes the space of distributions on $\top^2$.

By a doubly periodic solution of Eq. (1) we mean that a $u \in L^1(\top^2)$ satisfies Eq. (1) in the distribution sense, *i.e.*,

$$\int_{\top_2} u(\varphi_{tt} - \varphi_{xx} - c\varphi_t + a(t,x)\varphi)\,dt\,dx = \lambda \int_{\top^2} f(t,x,u)\varphi\,dt\,dx.$$

First, we consider the linear equation

$$u_{tt} - u_{xx} + cu_t - \xi u = h(t,x), \quad \text{in } D'(\top^2), \tag{2}$$

where $c > 0$, $\mu \in R$, and $h(t,x) \in L^1(\top^2)$.

Let $\pounds_\xi$ be the differential operator

$$\pounds_\xi u = u_{tt} - u_{xx} + cu_t - \xi u,$$

acting on functions on $\top^2$. Following the discussion in [14], we know that if $\xi < 0$, $\pounds_\xi$ has the resolvent R_ξ,

$$R_\xi : L^1(\top^2) \to C(\top^2), \qquad h_i(t,x) \mapsto u_i(t,x),$$

where $u(t,x)$ is the unique solution of Eq. (2), and the restriction of R_ξ on $L^p(\top^2)$ $(1 < p < \infty)$ or $C(\top^2)$ is compact. In particular, $R_\xi : C(\top^2) \to C(\top^2)$ is a completely continuous operator.

For $\xi = -c^2/4$, the Green function $G(t,x)$ of the differential operator $\pounds_\xi$ is explicitly expressed; see Lemma 5.2 in [14]. From the definition of $G(t,x)$, we have

$$\underline{G} := \operatorname{ess\,inf} G(t,x) = e^{-3c\pi/2}/\left(1 - e^{-c\pi}\right)^2,$$
$$\overline{G} := \operatorname{ess\,sup} G(t,x) = \left(1 + e^{-c\pi}\right)/2\left(1 - e^{-c\pi}\right)^2.$$

For convenience, we assume the following condition holds throughout this paper:

(H1) $a(t,x) \in C(\top^2, R)$, $0 \le a(t,x) \le \frac{c^2}{4}$ on $\top^2$, and $\int_{\top^2} a(t,x)\,dt\,dx > 0$.

Finally, if $-\xi$ is replaced by $a(t,x)$ in Eq. (2), the author [13] has proved the following unique existence and positive estimate result.

Lemma 2.1 *Let $h(t,x) \in L^1(\top^2)$. Then Eq.* (2) *has a unique solution $u(t,x) = P[h(t,x)]$, $P : L^1(\top^2) \to C(\top^2)$ is a linear bounded operator with the following properties*:

(i) *$P : C(\top^2) \to C(\top^2)$ is a completely continuous operator*;

(ii) *If $h(t,x) > 0$, a.e $(t,x) \in \top^2$, $P[h(t,x)]$ has the positive estimate*

$$\underline{G}\|h\|_{L^1} \le P\left[h(t,x)\right] \le \frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}. \tag{3}$$

3 Main result

Theorem 3.1 *Assume* (H1) *holds. In addition, if $f(t,x,u)$ satisfies*

(H2) $\lim_{u\to 0^+} f(t,x,u) = +\infty$, *uniformly* $(t,x) \in \top^2$,

(H3) $f : \top^2 \times (0,+\infty) \to (-\infty,+\infty)$ *is continuous*,

(H4) *there exists a nonnegative function $h(t,x) \in C(\top^2)$ such that*

$$f(t,x,u) + h(t,x) \ge 0, \quad (t,x) \in \top^2, u > 0,$$

(H5) $\int_{\top^2} F_\infty(t,x)\,dt\,dx = +\infty$, *where the limit function* $F_\infty(t,x) = \liminf_{u\to+\infty} \frac{f(t,x,u)}{u}$, *then Eq.* (1) *has at least two positive doubly periodic solutions for sufficiently small* λ.

$C(\top^2)$ is a Banach space with the norm $\|u\| = \max_{(t,x)\in\top^2} |u(t,x)|$. Define a cone $K \subset C(\top^2)$ by

$$K = \left\{u \in C\left(\top^2\right) : u \ge 0, u(t,x) \ge \delta\|u\|\right\},$$

where $\delta = \frac{\underline{G}^2\|a\|_{L^1}}{\overline{G}} \in (0,1)$. Let $\partial K_r = \{u \in K : \|u\| = r\}$, $[u]^+ = \max\{u, 0\}$. By Lemma 2.1, it is easy to obtain the following lemmas.

Lemma 3.2 *If $h(t,x) \in C(\top^2)$ is a nonnegative function, the linear boundary value problem*

$$\begin{cases} u_{tt} - u_{xx} + cu_t + a(t,x)u = \lambda h(t,x), \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x) \end{cases}$$

has a unique solution $\omega(t,x)$. The function $\omega(t,x)$ satisfies the estimates

$$\lambda\underline{G}\|h\|_{L^1} \le \omega(t,x) = \lambda P\big(h(t,x)\big) \le \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}.$$

Lemma 3.3 *If the boundary value problem*

$$\begin{cases} u_{tt} - u_{xx} + cu_t + a(t,x)u = \lambda[f(t,x,[u(t,x) - \omega(t,x)]^+) + h(t,x)], \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x) \end{cases}$$

has a solution $\widetilde{u}(t,x)$ with $\|\widetilde{u}\| > \lambda\frac{\overline{G}^2}{\underline{G}^3\|a\|^2_{L^1}}\|h\|_{L^1}$, then $u^*(t,x) = \widetilde{u}(t,x) - \omega(t,x)$ is a positive doubly periodic solution of Eq. (1).

Proof of Theorem 3.1 Step 1. Define the operator T as follows:

$$(Tu)(t,x) = \lambda P\big[f\big(t,x,\big[u(t,x) - \omega(t,x)\big]^+\big) + h(t,x)\big].$$

We obtain the conclusion that $T(K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\}) \subseteq K$, and $T : K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\} \to K$ is completely continuous.

For any $u \in K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\}$, then $[u(t,x) - \omega(t,x)]^+ > 0$, and T is defined. On the other hand, for $u \in K\backslash\{u \in K : [u(t,x) - \omega(t,x)]^+ = 0\}$, the complete continuity is obvious by Lemma 2.1. And we can have

$$\begin{aligned} (Tu)(t,x) &= \lambda P\big[f\big(t,x,\big[u(t,x) - \omega(t,x)\big]^+\big) + h(t,x)\big] \\ &\ge \lambda\underline{G}\big\|f\big(t,x,\big[u(t,x) - \omega(t,x)\big]^+\big) + h(t,x)\big\|_{L^1} \\ &\ge \underline{G}\frac{\underline{G}\|a\|_{L^1}}{\overline{G}}\big\|T(u)\big\| \\ &\ge \delta\|Tu\|. \end{aligned}$$

Thus, $T(K\backslash\{u \in K : u(t,x) \le \omega(t,x)\}) \subseteq K$.

Now we prove that the operator T has one fixed point $\widetilde{u} \in K$ and $\|\widetilde{u}\| > \lambda\frac{\overline{G}^2}{\underline{G}^3\|a\|^2_{L^1}}\|h\|_{L^1}$ for all sufficiently small λ.

Since $\int_{\top^2} F_\infty(t,x)\,dt\,dx = +\infty$, there exists $r_1 \ge 2$ such that

$$\int_{\top^2}\frac{f(t,x,u)}{u}\,dt\,dx \ge \frac{1}{\delta}, \quad u \ge \delta r_1.$$

Furthermore, we have $\int_{\top^2} f(t,x,\delta r_1)\,dt\,dx \geq r_1 \geq 2$. It follows that

$$\begin{aligned}&\int_{\top^2}\left[\max\left\{f(t,x,u):\frac{\delta}{2}r_1 \leq u \leq r_1\right\} + h(t,x)\right]dt\,dx\\&\quad\geq \int_{\top^2} f(t,x,\delta r_1)\,dt\,dx \geq r_1 \geq 2.\end{aligned}$$

Let $\Phi(t,x) = \max\{f(t,x,u) : \frac{\delta}{2}r_1 \leq u \leq r_1\} + h(t,x)$. Then $\Phi \in L^1(\top^2)$ and $\int_{\top^2} \Phi(t,x)\,dt\,dx > 0$. Set

$$\lambda^* = \min\left\{\frac{\delta^2}{2\underline{G}\|h\|_{L^1}}, \frac{2\underline{G}\|a\|_{L^1}}{\overline{G}\|\Phi\|_{L^1}}\right\}.$$

For any $u \in \partial K_{r_1}$ and $0 < \lambda < \lambda^*$, we can verify that

$$\begin{aligned}u(t,x) - \omega(t,x) &\geq \delta\|u\| - \omega(t,x)\\&= \delta r_1 - \omega(t,x)\\&\geq \delta r_1 - \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}\\&\geq \delta r_1 - \frac{\delta r_1}{2}\\&= \frac{\delta r_1}{2}.\end{aligned}$$

Then we have

$$\begin{aligned}\|Tu\| &= \lambda\left\|P\left[f\left(t,x,\left[u(t,x)-\omega(t,x)\right]^+\right) + h(t,x)\right]\right\|\\&\leq \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\left\|f\left(t,x,\left[u(t,x)-\omega(t,x)\right]^+\right) + h(t,x)\right\|_{L^1}\\&\leq \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\left\|\Phi(t,x)\right\|_{L^1}\\&< 2 \leq r_1 = \|u\|.\end{aligned}$$

On the other hand,

$$\liminf_{u\to+\infty}\frac{f(t,x,u-\omega(t,x))}{u} = \liminf_{u\to+\infty}\frac{f(t,x,u)}{u} = F_\infty(t,x).$$

By the Fatou lemma, one has

$$\begin{aligned}&\liminf_{u\to+\infty}\int_{\top^2}\frac{f(t,x,u-\omega(t,x)) + h(t,x)}{u}\,dt\,dx\\&\quad\geq \int_{\top^2}\liminf_{u\to+\infty}\frac{f(t,x,u)+h(t,x)}{u}\,dt\,dx\\&\quad= \int_{\top^2}F_\infty(t,x)\,dt\,dx = +\infty.\end{aligned}$$

Hence, there exists a positive number $r_2 > \delta r_2 > r_1$ such that

$$\int_{\top^2} \frac{f(t,x,u-\omega(t,x)) + h(t,x)}{u}\,dt\,dx \geq \lambda^{-1}\delta^{-1}\underline{G}^{-1}\left(4\pi^2\right)^{-1}, \quad u \geq \delta r_2.$$

Hence, we have

$$\int_{\top^2} f\big(t,x,u-\omega(t,x)\big) + h(t,x)\,dt\,dx \geq \lambda^{-1}\underline{G}^{-1}\left(4\pi^2\right)^{-1} r_2, \quad u \geq \delta r_2.$$

For any $u \in \partial K_{r_2}$, we have $\delta r_2 = \delta\|u\| \leq u(t,x) \leq \|u\| = r_2$. On the other hand, since $0 < \lambda < \lambda^*$, we can get

$$\begin{aligned} u(t,x) - \omega(t,x) &\geq \delta r_2 - \omega(t,x) \\ &\geq \delta\frac{r_2}{\delta} - \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}} \\ &\geq \delta r_2 - \delta \\ &> 0. \end{aligned}$$

From above, we can have

$$\begin{aligned} \|Tu\| &\geq \lambda P\big[f\big(t,x,\big[u(t,x)-\omega(t,x)\big]^+\big) + h(t,x)\big] \\ &\geq \lambda\underline{G}\big\|f\big(t,x,\big[u(t,x)-\omega(t,x)\big]^+\big) + h(t,x)\big\|_{L^1} \\ &\geq \lambda\underline{G}4\pi^2\lambda^{-1}\underline{G}^{-1}\left(4\pi^2\right)^{-1} r_2 \\ &= r_2. \end{aligned}$$

Therefore, by Lemma 1.1, the operator T has a fixed point $\widetilde{u}(t,x) \in K$ and

$$\begin{aligned} &r_2 \geq \|\widetilde{u}\| \geq r_1, \\ &\widetilde{u}(t,x) - \omega(t,x) \geq \delta r_1 - \lambda\frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1} \geq \delta r_1 - \frac{\overline{G}}{\underline{G}\|a\|_{L^1}}\|h\|_{L^1}\frac{\delta^2}{\underline{G}\|h\|_{L^1}} \geq \delta. \end{aligned}$$

So, Eq. (1) has a positive solution $\widehat{u}(t,x) = \widetilde{u}(t,x) - \omega(t,x) \geq \delta$.

Step 2. By conditions (H2) and (H3), it is clear to obtain that

$$u_0 = \inf\left\{u \in K : f(t,x,u) \leq 0, (t,x) \in \top^2\right\} > 0.$$

Let $r_4 = \min\{\frac{\delta}{2}, \frac{\delta\|u_0\|}{2}\}$. For any $u \in (0, r_4]$, we have $f(t,x,u) > 0$. Then define the operator A as follows:

$$(Au)(t,x) = \lambda\widehat{P}\big[f\big(t,x,u(t,x)\big)\big].$$

It is easy to prove that $A(K \cap \{u \in C(\top^2) : 0 < \|u\| < r_4\}) \subseteq K$, and $A : K \cap \{u \in C(\top^2) : 0 < \|u\| < r_4\} \to K$ is completely continuous.

And for any $\rho > 0$, define

$$M(\rho) = \max\left\{f(t,x,u) : u \in R^+, \delta\rho \leq u \leq \rho, (t,x) \in \top^2\right\} > 0.$$

Furthermore, for any $u \in \partial K_{r_4}$, we have

$$\begin{aligned}\|Au\| &= \lambda \|\widehat{P}[f(t,x,u(t,x))]\| \\ &\le \lambda \frac{\overline{G}}{\underline{G}\|a\|_{L^1}} \|f(t,x,u(t,x))\|_{L^1} \\ &\le \lambda \frac{\overline{G}}{\underline{G}\|a\|_{L^1}} M(r_4)4\pi^2.\end{aligned}$$

Thus, from the above inequality, there exists $\overline{\lambda}$ such that

$$\|Au\| < \|u\|, \quad \text{for } u \in \partial K_{r_4}, 0 < \lambda < \overline{\lambda}.$$

Since $\lim_{u\to 0^+} f(t,x,u) = \infty$, then there is $0 < r_3 < \frac{r_4}{2}$ such that

$$f(t,x,u) \ge \mu u, \quad \text{for } u \in R^+ \text{ with } 0 < u \le r_3,$$

where μ satisfies $\lambda \underline{G}\mu\delta > 1$. For any $u \in \partial K_{r_3}$, then we have

$$f(t,x,u) \ge \mu u(t,x), \quad \text{for } (t,x) \in \top^2.$$

By Lemma 2.1, it is clear to obtain that

$$\begin{aligned}\|Au\| &= \lambda \|\widehat{P}[f(t,x,u(t,x))]\| \\ &\ge \lambda \underline{G}\|f(t,x,u(t,x))\|_{L^1} \\ &\ge \lambda \underline{G}\mu\delta r_3 \\ &> r_3 = \|u\|.\end{aligned}$$

Therefore, by Lemma 1.1, A has a fixed point in $\overline{u}(t,x) \in K$ and $\|\overline{u}\| \le r_4 \le \frac{\delta}{2}$, which is another positive periodic solution of Eq. (1).

Finally, from Step 1 and Step 2, Eq. (1) has two positive doubly periodic solutions $\widehat{u}(t,x)$ and $\overline{u}(t,x)$ for sufficiently small λ. □

Example Consider the following problem:

$$\begin{cases} u_{tt} - u_{xx} + 2u_t + \sin^2(t+x)u = \lambda[\frac{1}{u} + \min\{u^2, \frac{u}{|1-\frac{t}{\pi}||1-\frac{x}{\pi}|}\} - 10], \\ u(t+2\pi, x) = u(t, x+2\pi) = u(t,x). \end{cases}$$

It is clear that $f(t,x,u)$ satisfies the conditions (H1)-(H5).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
This paper is concerned with a singular semipositone telegraph equation with a parameter and represents a somewhat interesting contribution in the investigation of the existence and multiplicity of doubly periodic solutions of the telegraph equation. All authors typed, read and approved the final manuscript.

Author details
[1]College of Science, Hohai University, Nanjing, 210098, P.R. China. [2]Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, P.R. China.

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