Xin and Cheng *Boundary Value Problems* (2016) 2016:139
DOI 10.1186/s13661-016-0648-x

Boundary Value Problems
a SpringerOpen Journal

RESEARCH Open Access

Positive periodic solution for ϕ-Laplacian Rayleigh equation with strong singularity

Yun Xin[1*] and Zhibo Cheng[2,3]

*Correspondence: xy_1982@126.com
[1]College of Computer Science and Technology, Henan Polytechnic University, Jiaozuo, 454000, China
Full list of author information is available at the end of the article

Abstract

In this paper, we discuss a kind of ϕ-Laplacian Rayleigh equation with strong singularity

$$(\phi(u'(t)))' + f(t, u'(t)) + g(u(t-\tau)) = e(t).$$

By application of the Manásevich-Mawhin continuation theorem, we obtain the existence of a positive periodic solution for this equation.

MSC: 34K13; 34C25

Keywords: positive periodic solution; ϕ-Laplacian; strong singularity; Rayleigh equation

1 Introduction

In this paper, we investigate the following ϕ-Laplacian Rayleigh equation:

$$\left(\phi\left(u'(t)\right)\right)' + f\left(t, u'(t)\right) + g\left(u(t-\tau)\right) = e(t), \tag{1.1}$$

where $f : \mathbb{R} \to \mathbb{R}$ is a L^2-Carathéodory function, *i.e.*, it is measurable in the first variable and continuous in the second variable, and for every $0 < r < s$ there exists $h_{r,s} \in L^2[0, \omega]$ such that $|g(t, x(t))| \leq h_{r,s}$ for all $x \in [r, s]$ and a.e. $t \in [0, \omega]$; f is a T-periodic function about t and $f(t, 0) = 0$; $g : (0, +\infty) \to \mathbb{R}$ is a continuous function that has a strong singularity at the origin;

$$\lim_{u \to 0^+} \int_u^1 g(s)\, ds = +\infty. \tag{1.2}$$

$e \in L^p(\mathbb{R})$ is a T-periodic function and $1 \leq p \leq \infty$, τ is a constant, and $0 \leq \tau < T$.

Moreover, let $\phi : \mathbb{R} \to \mathbb{R}$ be a continuous function, with $\phi(0) = 0$, which satisfies

- (A_1) $(\phi(u_1) - \phi(u_2))(u_1 - u_2) > 0$ for $\forall u_1 \neq u_2$, $u_1, u_2 \in \mathbb{R}$;
- (A_2) there exists a function $d : [0, +\infty] \to [0, +\infty]$, and $d(u) \to +\infty$ as $u \to +\infty$, such that $\phi(u) \cdot u \geq d(|u|)|u|$ for $\forall u \in \mathbb{R}$.

It is easy to see that ϕ represents a large class of nonlinear operators, including $\phi_p : \mathbb{R} \to \mathbb{R}$ is a p-Laplacian, *i.e.*, $\phi_p(u) = |u|^{p-2}u$ for $u \in \mathbb{R}$.

As is well known, the Rayleigh equation can be derived from many fields, such as the physics, mechanics, and engineering technique fields, and an important question is whether this equation can support periodic solutions. In 1977, Gaines and Mawhin [1] introduced some continuation theorems and applied this theorem to a discussion of the existence of solutions for the Rayleigh equation [1], p.99,

$$u''(t) + f\big(u'(t)\big) + g\big(t, u(t)\big) = 0. \tag{1.3}$$

Gaines and Mawhin's work has attracted the attention of many scholars in differential equations. More recently, the existence of periodic solutions for the Rayleigh equation was extensively studied (see [2–11] and the references therein). In 2001, by using the method of upper and lower solutions, Habets and Torres [3] investigated the existence 2π-periodic solutions of (1.3) by assuming that $g = g(t, u, u')$ is bounded (or bounded from below). Afterwards, by application of the time map continuation theorem, Wang [11] discussed the existence of periodic solutions of a kind of Rayleigh equation

$$u''(t) + f\big(u'(t)\big) + g\big(u(t)\big) = p(t).$$

In this direction, the researchers in [12–20] discussed the p-Laplacian Rayleigh equation. In 2006, by employing Mawhin's continuous theorem, Cheung and Ren [12] studied how the existence of the p-Laplacian Rayleigh equation

$$\big(\varphi_p\big(u'(t)\big)\big)' + f\big(u'(t)\big) + \beta g\big(u\big(t - \tau(t)\big)\big) = e(t)$$

under various assumptions is obtained. Recently, Xin and Cheng [21] discussed a kind of ϕ-Laplacian Rayleigh equation,

$$\big(\phi\big(u'(t)\big)\big)' + f\big(t, u'(t)\big) + g\big(t, u(t)\big) = e(t). \tag{1.4}$$

By using the Manásevich-Mawhin continuation theorem and some analysis techniques, the authors established a sufficient condition for the existence and uniqueness of positive periodic solutions for (1.4).

In the above papers, the authors investigated several kinds of Rayleigh and p-Laplacian Rayleigh equations. However, as far as we know, the study of periodic solutions for the ϕ-Laplacian differential equation with strong singularity is relatively rare. In this paper, we try to fill this gap and establish the existence of positive periodic solutions of (1.1) by employing the Manásevich-Mawhin continuation theorem. Finally, a numerical example demonstrates the validity of the method.

2 Positive periodic solution for (1.1)

In this section, we will consider the existence of positive periodic solution for (1.1) with strong singularity. First of all, we embed equation (1.1) into the following equation family with a parameter $\lambda \in (0, 1]$:

$$\big(\phi\big(u'(t)\big)\big)' + \lambda f\big(t, u'(t)\big) + \lambda g\big(u(t - \tau)\big) = \lambda e(t). \tag{2.1}$$

The following lemma is a consequence of Theorem 3.1 of [22].

Lemma 2.1 *Assume that there exist positive constants E_1, E_2, E_3, and $E_1 < E_2$ such that the following conditions hold*:

(1) *Each possible periodic solution u to equation* (2.1) *such that $E_1 < u(t) < E_2$, for all $t \in [0,T]$ and $\|u'\| < E_3$, here $\|u'\| := \max_{t\in[0,T]} |u'(t)|$.*

(2) *Each possible solution C to the equation*

$$g(C) - \frac{1}{T}\int_0^T e(t)\,dt = 0$$

satisfies $E_1 < C < E_2$.

(3) *We have*

$$\left(g(E_1) - \frac{1}{T}\int_0^T e(t)\,dt\right)\left(g(E_2) - \frac{1}{T}\int_0^T e(t)\,dt\right) < 0.$$

Then (1.1) *has at least one T-periodic solution.*

For the sake of convenience, we list the following assumptions which will be used repeatedly in the sequel:

(H_1) There exist constants $0 < d_1 < d_2$ such that $g(u) - e(t) > 0$ for $u \in (0, d_1)$ and $g(u) - e(t) < 0$ for $u \in (d_2, +\infty)$.

(H_2) There exist positive constants a, b such that

$$g(u) \le au + b, \quad \text{for all } u > 0. \tag{2.2}$$

(H_3) There exist constants α and $m > 1$ such that

$$f(t,u)u \ge \alpha |u|^m, \quad \text{for } (t,u) \in [0,T] \times \mathbb{R}.$$

(H_4) There exist positive constants β and γ such that

$$\left|f(t,u)\right| \le \beta |u|^{m-1} + \gamma, \quad \text{for } (t,u) \in [0,T] \times \mathbb{R}.$$

Lemma 2.2 *Assume that* (A_1) *and* (H_1) *hold. Then there exists a point $t_1 \in [0,T]$ such that*

$$d_1 < u(t_1) \le d_2. \tag{2.3}$$

Proof Let $\underline{t}$, $\bar{t}$, respectively, be the global minimum point and the global maximum point $u(t)$ on $[0,T]$; then $u'(\underline{t}) = 0$ and $u'(\bar{t}) = 0$, and we claim that

$$\bigl(\phi\bigl(u'(\underline{t})\bigr)\bigr)' \ge 0. \tag{2.4}$$

In fact, if (2.4) does not hold, then $(\phi(u'(\underline{t})))' < 0$ and there exists $\varepsilon > 0$ such that $(\phi(u'(t)))' < 0$ for $t \in (\underline{t}-\varepsilon, \underline{t}+\varepsilon)$. Therefore, $\phi(u'(t))$ is strictly decreasing for $t \in (\underline{t}-\varepsilon, \underline{t}+\varepsilon)$. From ($A_1$), we know that $u'(t)$ is strictly decreasing for $t \in (\underline{t}-\varepsilon, \underline{t}+\varepsilon)$. This contradicts the definition of $\underline{t}$. Thus, (2.4) is true. From $f(t,0) = 0$, (2.1) and (2.4), we have

$$g\bigl(u(\underline{t}-\tau)\bigr) - e(\underline{t}) \le 0. \tag{2.5}$$

Similarly, we can get

$$g\big(u(\bar{t}-\tau)\big)-e(\bar{t})\geq 0. \tag{2.6}$$

From (H_1), (2.5), and (2.6), we have

$$u(\underline{t}-\tau)\geq d_1 \quad\text{and}\quad u(\bar{t}-\tau)\leq d_2.$$

In view of u being a continuous function, there exists a point $t_1\in[0,T]$, such that

$$d_1\leq u(t_1)\leq d_2. \qquad\square$$

Lemma 2.3 *Assume that* (A_1) *and* (H_1), (H_3) *hold. Then there exists a positive constant* M_1 *such that*

$$u(t)<M_1. \tag{2.7}$$

Proof Multiplying both sides of (2.1) by $u'(t)$ and integrating over the interval $[0,T]$, we have

$$\begin{aligned}&\int_0^T\big(\phi\big(u'(t)\big)\big)'u'(t)\,dt+\lambda\int_0^T f\big(t,u'(t)\big)u'(t)\,dt+\lambda\int_0^T g\big(u(t-\tau)\big)u'(t)\,dt\\&\quad=\lambda\int_0^T e(t)u'(t)\,dt.\end{aligned} \tag{2.8}$$

Moreover, we have

$$\begin{aligned}\int_0^T\big(\phi\big(u'(t)\big)\big)'u'(t)\,dt&=\int_0^T u'(t)d\big(\phi\big(u'(t)\big)\big)\\&=\big[\phi\big(u'(t)\big)u'(t)\big]_0^T-\int_0^T\phi\big(u'(t)\big)\,du'(t)=0\end{aligned} \tag{2.9}$$

and

$$\begin{aligned}\int_0^T g\big(u(t-\tau)\big)u'(t)\,dt&=\int_0^T g\big(u(t-\tau)\big)\,du(t)\\&=\int_0^T g\big(u(t-\tau)\big)\,du(t-\tau)=0,\end{aligned} \tag{2.10}$$

since $du(t)=\frac{du(t-\tau)}{d(t-\tau)}\,dt=du(t-\tau)$.

Substituting (2.9) and (2.10) into (2.8), we have

$$\int_0^T f\big(t,u'(t)\big)u'(t)\,dt=\int_0^T e(t)u'(t)\,dt. \tag{2.11}$$

Thus, we have

$$\left|\int_0^T f\big(t,u'(t)\big)u'(t)\,dt\right|=\left|\int_0^T e(t)u'(t)\,dt\right|.$$

From (H_3), we can get

$$\left|\int_0^T f(t,u'(t))u'(t)\,dt\right| \geq \alpha\int_0^T \left|u'(t)\right|^m dt.$$

Therefore, we can get

$$\begin{aligned}\alpha\int_0^T \left|u'(t)\right|^m dt &\leq \int_0^T \left|e(t)\right|\left|u'(t)\right| dt\\ &\leq \left(\int_0^T \left|e(t)\right|^{\frac{m}{m-1}} dt\right)^{\frac{m-1}{m}}\left(\int_0^T \left|u'(t)\right|^m dt\right)^{\frac{1}{m}}\\ &= \|e\|_{\frac{m}{m-1}}\left(\int_0^T \left|u'(t)\right|^m dt\right)^{\frac{1}{m}},\end{aligned}$$

where $\|e\|_{\frac{m}{m-1}} = (\int_0^T |e(t)|^{\frac{m}{m-1}} dt)^{\frac{m-1}{m}}$. It is easy to see that there exists a positive constant M_1' (independent of λ) such that

$$\int_0^T \left|u'(t)\right|^m dt \leq M_1'. \tag{2.12}$$

From Lemma 2.2 and the Hölder inequality, we have

$$\begin{aligned}u(t) &= \left|\int_{t_1}^t u'(t)\,dt\right| \leq u(t_1) + \int_0^T \left|u'(t)\right| dt\\ &\leq d_2 + \int_0^T \left|u'(t)\right| dt\\ &\leq d_2 + T^{\frac{m-1}{m}}\left(\int_0^T \left|u'(t)\right|^m dt\right)^{\frac{1}{m}}\\ &\leq d_2 + T^{\frac{m-1}{m}}\left(M_1'\right)^{\frac{1}{m}} := M_1.\end{aligned}$$

□

Lemma 2.4 *Assume that* (A_1), (A_2), *and* (H_1)-(H_4) *hold. Then there exists a positive constant* M_2 *such that*

$$\left\|u'\right\| < M_2. \tag{2.13}$$

Proof Integrating both sides of (2.1) over $[0,T]$, we have

$$\int_0^T \left[f\left(t,u'(t)\right) + g\left(u(t-\tau)\right) - e(t)\right] dt = 0. \tag{2.14}$$

Therefore, we get from (2.7), (2.14), (H_2), and (H_4)

$$\begin{aligned}\int_0^T \left|g\left(u(t-\tau)\right)\right| dt &= \int_{g(u(t-\tau))\geq 0} g\left(u(t-\tau)\right) dt - \int_{g(u(t-\tau))\leq 0} g\left(u(t-\tau)\right) dt\\ &= 2\int_{g(u(t-\tau))\geq 0} g\left(u(t-\tau)\right) dt + \int_0^T f\left(t,u'(t)\right) dt - \int_0^T e(t)\,dt\end{aligned}$$

$$\begin{aligned}
&\leq 2\int_{g(u(t-\tau))\geq 0}\bigl(ax(t-\tau)+b\bigr)\,dt+\int_0^T\bigl|f\bigl(t,u'(t)\bigr)\bigr|\,dt+\int_0^T\bigl|e(t)\bigr|\,dt\\
&\leq 2a\int_0^T\bigl|x(t-\tau)\bigr|\,dt+2bT+\beta\int_0^T\bigl|u'(t)\bigr|^{m-1}\,dt+\gamma T+\|e\|_2T^{\frac{1}{2}}\\
&\leq 2aM_1T+2bT+\beta T^{\frac{1}{m}}\left(\int_0^T\bigl|u'(t)\bigr|^m\,dt\right)^{\frac{m-1}{m}}+\gamma T+\|e\|_2T^{\frac{1}{2}}\\
&\leq 2aM_1T+2bT+\beta T^{\frac{1}{m}}M_1'^{\frac{m-1}{m}}+\gamma T+\|e\|_2T^{\frac{1}{2}}.
\end{aligned}\tag{2.15}$$

As $u(0)=u(T)$, there exists $t_2\in[0,T]$ such that $u'(t_2)=0$, while $\phi(0)=0$, and we have

$$\begin{aligned}
\bigl|\phi\bigl(u'(t)\bigr)\bigr| &= \left|\int_{t_2}^t\bigl(\phi\bigl(u'(s)\bigr)\bigr)'\,ds\right|\\
&\leq \lambda\int_0^T\bigl|f\bigl(t,u'(t)\bigr)\bigr|\,dt+\lambda\int_0^T\bigl|g\bigl(u(t-\tau)\bigr)\bigr|\,dt+\lambda\int_0^T\bigl|e(t)\bigr|\,dt,
\end{aligned}\tag{2.16}$$

where $t\in[t_2,t_2+T]$. In view of (2.7), (2.12), (2.15), (2.16), and (H_4), we have

$$\begin{aligned}
\bigl\|\phi\bigl(u'\bigr)\bigr\| &= \max_{t\in[0,T]}\bigl\{\bigl|\phi\bigl(u'(t)\bigr)\bigr|\bigr\}\\
&= \max_{t\in[t_2,t_2+T]}\left\{\left|\int_{t_0}^t\bigl(\phi\bigl(u'(s)\bigr)\bigr)'\,ds\right|\right\}\\
&\leq \int_0^T\bigl|f\bigl(t,u'(t)\bigr)\bigr|\,dt+\int_0^T\bigl|g\bigl(u(t-\tau)\bigr)\bigr|\,dt+\int_0^T\bigl|e(t)\bigr|\,dt\\
&\leq \beta\int_0^T\bigl|u'(t)\bigr|^{m-1}\,dt+\gamma T+\int_0^T\bigl|g\bigl(u(t-\tau)\bigr)\bigr|\,dt+\int_0^T\bigl|e(t)\bigr|\,dt\\
&\leq 2\bigl(aM_1T+bT+\beta T^{\frac{1}{m}}M_1'^{\frac{m-1}{m}}+\gamma T+\|e\|_2T^{\frac{1}{2}}\bigr):=M_2'.
\end{aligned}\tag{2.17}$$

We claim that there exists a positive constant $M_2>M_2'+1$ such that, for all $t\in\mathbb{R}$,

$$\bigl\|u'\bigr\|\leq M_2.\tag{2.18}$$

In fact, if u' is not bounded, then from the definition of d, there exists a positive constant M_2'' such that $d(|u'|)>M_2''$ for some $u'\in\mathbb{R}$. However, from (A_2), we have

$$d\bigl(\bigl|u'\bigr|\bigr)\bigl|u'\bigr|\leq\phi\bigl(u'\bigr)u'\leq\bigl|\phi\bigl(u'\bigr)\bigr|\bigl|u'\bigr|\leq M_2'\bigl|u'\bigr|.$$

Then we can get

$$d\bigl(\bigl|u'\bigr|\bigr)\leq M_2',\quad\text{for all } u\in\mathbb{R},$$

which is a contradiction. So, (2.18) holds. □

Lemma 2.5 *Assume that* (A_1), (A_2), *and* (H_1)-(H_4) *hold. Then there exists a positive constant M_3 such that*

$$u(t)\geq M_3.\tag{2.19}$$

Proof From (2.1), we have

$$\big(\phi\big(u'(t+\tau)\big)\big)' + \lambda f\big(t+\tau, u(t+\tau)\big) + \lambda g\big(u(t)\big) = \lambda e(t+\tau). \tag{2.20}$$

Multiplying both sides of (2.20) by $u'(t)$ and integrating on $[\xi, t]$, here $\xi \in [0, T]$, we get

$$\begin{aligned}
\lambda \int_{u(\xi)}^{u(t)} g(u)\,du = \lambda \int_{\xi}^{t} g\big(u(s)\big)u'(s)\,ds \\
&= -\int_{\xi}^{t} \big(\phi\big(u'(s+\tau)\big)\big)'u'(s)\,ds - \lambda \int_{\xi}^{t} f\big(s+\tau, u(s+\tau)\big)u'(s)\,ds \\
&\quad + \lambda \int_{\xi}^{t} e(s+\tau)u'(s)\,ds.
\end{aligned} \tag{2.21}$$

By (2.13) and (2.17), we can get

$$\begin{aligned}
&\left|\int_{\xi}^{t} \big(\phi\big(u'(t+\tau)\big)\big)'u'(s)\,ds\right| \\
&\quad \le \int_{\xi}^{t} \big|\big(\phi\big(u'(s+\tau)\big)\big)'\big|\big|u'(s)\big|\,ds \\
&\quad \le \|u'\| \int_{0}^{T} \big|\big(\phi\big(u'(t+\tau)\big)\big)'\big|\,dt \\
&\quad \le \lambda\|u'\|\left(\int_{0}^{T} \big|f\big(t, u'(t)\big)\big|\,dt + \int_{0}^{T} \big|g\big(u(t-\tau)\big)\big|\,dt + \int_{0}^{T} \big|e(t)\big|\,dt\right) \\
&\quad \le 2\lambda M_2\big(aM_1T + bT + \beta T^{\frac{1}{m}} M_1'^{\frac{m-1}{m}} + \gamma T + T^{\frac{1}{2}}\|e\|_2\big).
\end{aligned}$$

Moreover, from (H_3) and (2.18), we have

$$\begin{aligned}
&\left|\int_{\xi}^{t} f\big(s+\tau, u(s+\tau)\big)u'(s)\,ds\right| \le \int_{0}^{T} \big|f\big(s+\tau, u(s+\tau)\big)\big|\big|u'(s)\big|\,ds \le M_2\big(\beta M_2^{m-1}T + \gamma T\big), \\
&\left|\int_{\xi}^{t} e(t+\tau)u'(t)\,dt\right| \le M_2\sqrt{T}\|e\|_2.
\end{aligned}$$

Form (2.21), we have

$$\begin{aligned}
&\left|\int_{u(\xi)}^{u(t)} g_0(u)\,du\right| \\
&\quad \le M_2\big(aM_1T + bT + \beta T^{\frac{1}{m}} M_1'^{\frac{m-1}{m}} + \gamma T + T^{\frac{1}{2}}\|e\|_2 + \beta M_2^{m-1}T + \gamma T + \sqrt{T}\|e\|_2\big) \\
&\quad := M_3'.
\end{aligned} \tag{2.22}$$

From the strong force condition (1.2), we know that there exists a constant $M_3 > 0$ such that

$$u(t) \ge M_3, \quad \forall t \in [\xi, T]. \tag{2.23}$$

Similarly, we can consider $t \in [0, \xi]$. □

By Lemmas 2.1-2.5, we obtain the following main result.

Theorem 2.1 *Assume that conditions* (A_1), (A_2), *and* (H_1)-(H_4) *hold. Then* (1.1) *has a positive T-periodic solution.*

Proof Let $E_1 < \min\{d_1, M_3\}$, $E_2 > \max\{d_2, M_1\}$, $E_3 > M_2$ are constants, from Lemmas 2.2-2.5, we see that the periodic solution u to (2.1) satisfies

$$E_1 < u(t) < E_2, \qquad \|u'\| < E_3. \tag{2.24}$$

Then condition (1) of Lemma 2.1 is satisfied. For a possible solution D to the equation

$$g(D) - \frac{1}{T}\int_0^T e(t)\,dt = 0,$$

$E_1 < D < E_2$ is satisfied. Therefore, condition (2) of Lemma 2.1 holds. Finally, we consider condition (3) of Lemma 2.1 also to be satisfied. In fact, from (H_1), we have

$$g(E_1) - \frac{1}{T}\int_0^T e(t)\,dt > 0$$

and

$$g(E_2) - \frac{1}{T}\int_0^T e(t)\,dt < 0.$$

So condition (3) is also satisfied. By application of Lemma 2.1, we see that (1.1) has at least one positive periodic solution. □

We illustrate our results with one example.

Example 2.1 Consider the following second-order ϕ-Laplacian Rayleigh equation:

$$\bigl(\phi\bigl(u'(t)\bigr)\bigr)' + \bigl(20 - 16\sin^2 t\bigr)\bigl(u'(t)\bigr)^{m-1} + \frac{1}{u^\kappa(t)} - 10u(t) = e^{\cos^2 t}, \tag{2.25}$$

where $\phi(u) = ue^{|u|^2}$, $m > 1$, and $\kappa \geq 1$.

Comparing (2.25) to (1.1), it is easy to see that $g(t,u) = \frac{1}{u^\kappa(t)} - 10u(t)$, $f(t,w) = (20 - 16\sin^2 t)w^{m-1}$, $e(t) = e^{\cos^2 t}$, $T = \pi$. Obviously, we get

$$\bigl(ue^{|u|^2} - ve^{|v|^2}\bigr)(u-v) \geq \bigl(|u|e^{|u|^2} - |v|e^{|v|^2}\bigr)\bigl(|u| - |v|\bigr) \geq 0$$

and

$$\phi(u)\cdot u = |u|^2 e^{|u|^2}.$$

So, conditions (A_1) and (A_2) hold. Moreover, it is easily seen that there exists a constant $d_2 = 1$ such that (H_1) holds. As $a = 10$ and $b = 1$, condition (H_2) holds. Consider $f(t,w)w = (20 - 16\cos^2 t)w^m \geq 4w^m$, here $\alpha = 4$, and $|f(t,w)| = |(20 - 16\cos^2 t)w^{m-1}| \leq 20|w|^{m-1} + 1$, here $\beta = 20$, $\gamma = 1$; thus, we see that conditions (H_3) and (H_4) hold. Therefore, by Theorem 2.1, we know that (2.25) has one positive periodic solution.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
YX and ZC worked together in the derivation of the mathematical results. Both authors read and approved the final manuscript.

Author details
[1]College of Computer Science and Technology, Henan Polytechnic University, Jiaozuo, 454000, China. [2]School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, 454000, China. [3]Department of Mathematics, Sichuan University, Chengdu, 610064, China.

Acknowledgements
YX and ZC would like to thank the referee for invaluable comments and insightful suggestions. This work was supported by National Natural Science Foundation of China (No. 11501170), China Postdoctoral Science Foundation funded project, Fundamental Research Funds for the Universities of Henan Province (NSFRF140142), Education Department of Henan Province project (No. 16B110006), Henan Polytechnic University Outstanding Youth Fund (J2015-02) and Henan Polytechnic University Doctor Fund (B2013-055).

Received: 2 June 2016 Accepted: 22 July 2016 Published online: 02 August 2016

References
1. Gaines, R, Mawhin, J: Coincidence Degree, and Nonlinear Differential Equations. Lecture Notes in Mathematics, vol. 568. Springer, Berlin (1977)
2. Gao, H, Liu, B: Existence and uniqueness of periodic solutions for forced Rayleigh-type equations. Appl. Math. Comput. **211**, 148-154 (2009)
3. Habets, P, Torres, P: Some multiplicity results for periodic solutions of a Rayleigh differential equation. Dyn. Contin. Discrete Impuls. Syst., Ser. A Math. Anal. **8**, 335-351 (2001)
4. Ma, T: Periodic solutions of Rayleigh equations via time-maps. Nonlinear Anal. **75**, 4137-4144 (2012)
5. Nakao, M: Global solutions to the initial-boundary value problem for the quasilinear viscoelastic equation with a derivative nonlinearity. Opusc. Math. **34**, 569-590 (2014)
6. Radulescu, V: Qualitative Analysis of Nonlinear Elliptic Partial Differential Equations: Monotonicity, Analytic, and Variational Methods. Contemporary Mathematics and Its Applications, vol. 6. Hindawi Publishing Corporation, New York (2008)
7. Radulescu, V, Repovš, D: Partial Differential Equations with Variable Exponents: Variational Methods and Qualitative Analysis. Monographs and Research Notes in Mathematics. CRC Press, Boca Raton (2015)
8. Wang, L, Shao, J: New results of periodic solutions for a kind of forced Rayleigh-type equations. Nonlinear Anal., Real World Appl. **11**, 99-105 (2010)
9. Wang, Y: New results of periodic solutions for a Rayleigh equation with a deviating argument. Z. Angew. Math. Phys. **60**, 583-593 (2009)
10. Wang, Y, Dai, X: Existence and stability of periodic solutions of a Rayleigh type equation. Bull. Aust. Math. Soc. **79**, 377-390 (2009)
11. Wang, Z: On the existence of periodic solutions of Rayleigh equations. Z. Angew. Math. Phys. **56**, 592-608 (2005)
12. Cheung, W, Ren, J: Periodic solutions for *p*-Laplacian Rayleigh equations. Nonlinear Anal. **65**, 2003-2012 (2006)
13. Beirão da Veiga, H: On nonlinear potential theory, and regular boundary points, for the *p*-Laplacian in *N* space variables. Adv. Nonlinear Anal. **3**, 45-67 (2014)
14. Dosla, D, Marini, M: Positive decaying solutions for differential equations with ϕ-Laplacian. Bound. Value Probl. **2015**, 95 (2015)
15. Du, B, Lu, S: On the existence of periodic solutions to a *p*-Laplacian Rayleigh equation. Indian J. Pure Appl. Math. **40**, 253-266 (2009)
16. Gao, F, Lu, S, Zhang, W: Periodic solutions for a Rayleigh type *p*-Laplacian equation with sign-variable coefficient of nonlinear term. Appl. Math. Comput. **216**, 2010-2015 (2010)
17. He, Z, Wang, W, Yi, X: Existence and uniqueness of periodic solutions for Rayleigh type *p*-Laplacian equation. J. Comput. Appl. Math. **232**, 558-564 (2009)
18. Herlea, D: Positive solutions for second-order boundary-value problems with ϕ-Laplacian. Electron. J. Differ. Equ. **2016**, 51 (2016)
19. Li, C, Agarwal, R, Tang, C: Infinitely many periodic solutions for ordinary *p*-Laplacian systems. Adv. Nonlinear Anal. **4**, 251-261 (2015)
20. Peng, S, Zhu, S: Periodic solutions for a Rayleigh system of *p*-Laplacian type. Nonlinear Anal. **70**, 274-279 (2009)
21. Xin, Y, Cheng, Z: Existence and uniqueness of a positive periodic solution for Rayleigh type ϕ-Laplacian equation. Adv. Differ. Equ. **2014**, 225 (2014)
22. Manásevich, R, Mawhin, J: Periodic solutions for nonlinear systems with *p*-Laplacian-like operator. J. Differ. Equ. **145**, 367-393 (1998)