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A regularity criterion for the generalized Hall-MHD system

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Abstract
This paper proves a regularity criterion for the 3D generalized Hall-MHD system.

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1 Introduction

In this paper, we consider the following 3D generalized Hall-MHD system:

$$\partial_t u + u \cdot \nabla u + \nabla \left(\pi + \frac{1}{2}|b|^2 \right) + (-\Delta)^\alpha u = b \cdot \nabla b, \tag{1.1}$$

$$\partial_t b + u \cdot \nabla b - b \cdot \nabla u + (-\Delta)^\beta b + \operatorname{rot}(\operatorname{rot} b \times b) = 0, \tag{1.2}$$

$$\operatorname{div} u = \operatorname{div} b = 0, \tag{1.3}$$

$$(u, b)(\cdot, 0) = (u_0, b_0) \quad \text{in } \mathbb{R}^3. \tag{1.4}$$

Here u, π, and b denote the velocity, pressure, and magnetic field of the fluid, respectively. $0 < \alpha, \beta$ are two constants. The fractional Laplace operator $(-\Delta)^\alpha$ is defined through the Fourier transform, namely, $\widehat{(-\Delta)^\alpha f}(\xi) = |\xi|^{2\alpha} \hat{f}(\xi)$.

The applications of the Hall-MHD system cover a very wide range of physical objects, for example, magnetic reconnection in space plasmas, star formation, neutron stars, and the geo-dynamo.

When the Hall effect term $\operatorname{rot}(\operatorname{rot} b \times b)$ is neglected, the system (1.1)-(1.4) reduces to the well-known generalized MHD system, which has received attention in many studies [1–6].

When $\alpha = \beta = 1$, the system (1.1)-(1.4) reduces to the well-known Hall-MHD system, which has received many studies [7–11]. Reference [7] gave a derivation of the isentropic Hall-MHD system from a two-fluid Euler-Maxwell system. Chae-Degond-Liu [8] proved the local existence of smooth solutions. Chae and Schonbek [9] showed the time-decay and some regularity criteria were proved in [10–12]. Some other relevant results about Hall-MHD equations can be found in [13–17].

The local well-posedness is established in Wan and Zhou [18] when $0 < \alpha \leq 1$ and $\frac{1}{2} < \beta \leq 1$.

When $\frac{3}{4} \leq \alpha < \frac{5}{4}$ and $1 \leq \beta < \frac{7}{4}$, Jiang and Zhu [19] prove the following regularity criteria:

$$\nabla b \in L^t(0,T;L^s) \quad \text{with } \frac{2\beta}{t} + \frac{3}{s} \leq 2\beta - 1, \frac{3}{2\beta-1} < s \leq \infty,$$

and one of the following two conditions:

$$u \in L^p(0,T;L^q) \quad \text{with } \frac{2\alpha}{p} + \frac{3}{q} \leq 2\alpha - 1, \frac{3}{2\alpha-1} < q \leq \frac{6\alpha}{2\alpha-1},$$

or

$$\Lambda^\alpha u \in L^p(0,T;L^q) \quad \text{with } \frac{2\alpha}{p} + \frac{3}{q} \leq 3\alpha - 1, \frac{3}{3\alpha-1} < q \leq \frac{6\alpha}{3\alpha-1}.$$

When $1 \leq \alpha < \frac{5}{4}$ and $1 \leq \beta < \frac{7}{4}$, Ye [20] showed the following regularity criterion:

$$u \in L^p(0,T;L^q) \quad \text{and} \quad \nabla b \in L^\ell(0,T;L^k),$$

where p, q, ℓ, and k satisfy the relation

$$\frac{3}{q} + \frac{2\alpha}{p} \leq 2\alpha - 1, \qquad \frac{3}{q} + \frac{2\beta}{p} \leq 2\beta - 1, \qquad \frac{3}{k} + \frac{2\beta}{\ell} \leq 2\beta - 1,$$

and

$$\max\left(\frac{3}{2\alpha-1}, \frac{3}{2\beta-1}\right) < q \leq \infty, \qquad \frac{3}{2\beta-1} < k \leq \infty.$$

The aim of this paper is to refine the results in [19, 20] as follows.

Theorem 1.1 *Let* $\frac{3}{4} \leq \alpha < \frac{5}{4}$ *and* $\frac{3}{4} \leq \beta < \frac{7}{4}$ *and* $u_0, b_0 \in H^2$ *with* $\operatorname{div} u_0 = \operatorname{div} b_0 = 0$ *in* $\mathbb{R}^3$. *If* ∇u *and* ∇b *satisfy*

$$\begin{aligned} &\nabla u \in L^{\frac{2\alpha}{2\alpha-\gamma_1}}(0,T;\dot{B}^{-\gamma_1}_{\infty,\infty}), \\ &\nabla b \in L^{\frac{2\beta}{2\beta-1-\gamma_2}}(0,T;\dot{B}^{-\gamma_2}_{\infty,\infty}) \quad \text{with } 0 < \gamma_1 < 2\alpha \text{ and } 0 < \gamma_2 < 2\beta - 1, \end{aligned} \tag{1.5}$$

with $0 < T < \infty$, *then the solution* (u,b) *can be extended beyond* T.

In the following proof, we will use the following bilinear products and commutator estimates due to Kato-Ponce [21]:

$$\left\|\Lambda^s(fg)\right\|_{L^p} \leq C\left(\left\|\Lambda^s f\right\|_{L^{p_1}}\|g\|_{L^{q_1}} + \|f\|_{L^{p_2}}\left\|\Lambda^s g\right\|_{L^{q_2}}\right), \tag{1.6}$$

$$\left\|\Lambda^s(fg) - f\Lambda^s g\right\|_{L^p} \leq C\left(\|\nabla f\|_{L^{p_1}}\left\|\Lambda^{s-1} g\right\|_{L^{q_1}} + \left\|\Lambda^s f\right\|_{L^{p_2}}\|g\|_{L^{q_2}}\right), \tag{1.7}$$

with $s > 0$, $\Lambda := (-\Delta)^{\frac{1}{2}}$, and $\frac{1}{p} = \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{p_2} + \frac{1}{q_2}$.

We will also use the improved Gagliardo-Nirenberg inequalities [22–24]:

$$\|\nabla u\|_{L^3}^3 \leq C\|\nabla u\|_{\dot{B}^{-\gamma_1}_{\infty,\infty}}\|\nabla u\|^2_{\dot{H}^{\frac{\gamma_1}{2}}}, \tag{1.8}$$

$$\|\nabla b\|_{L^3}^3 \leq C\|\nabla b\|_{\dot{B}^{-\gamma_2}_{\infty,\infty}}\|\nabla b\|^2_{\dot{H}^{\frac{\gamma_2}{2}}}, \tag{1.9}$$

$$\|\nabla b\|_{L^{p_3}} \le C\|\nabla b\|_{\dot{B}^{-\gamma_2}_{\infty,\infty}}^{1-\theta_1}\|\nabla b\|_{\dot{H}^s}^{\theta_1}, \tag{1.10}$$

$$\big\|\Lambda^{2-\beta} b\big\|_{L^{q_3}} \le C\|\nabla b\|_{\dot{B}^{-\gamma_2}_{\infty,\infty}}^{\theta_1}\|\nabla b\|_{\dot{H}^s}^{\theta_1}, \tag{1.11}$$

with $\theta_1 := \frac{\gamma_2}{s+\gamma_2}$, $s := 1 + \gamma_2 - \beta$, $p_3 := \frac{2}{\theta_1}$ and $q_3 := \frac{2}{1-\theta_1}$

$$\|\nabla b\|_{L^{p_4}} \le C\|\nabla b\|_{\dot{B}^{-\gamma_2}_{\infty,\infty}}^{1-\theta_2}\|\Delta b\|_{\dot{H}^s}^{\theta_2}, \tag{1.12}$$

$$\big\|\Lambda^{3-\beta} b\big\|_{L^{q_4}} \le C\|\nabla b\|_{\dot{B}^{-\gamma_2}_{\infty,\infty}}^{\theta_2}\|\Delta b\|_{\dot{H}^s}^{1-\theta_2}, \tag{1.13}$$

with $\theta_2 := \frac{\gamma_2}{1+s+\gamma_2}$, $p_4 := \frac{2}{\theta_2}$, and $q_4 := \frac{2}{1-\theta_2}$. We have

$$\big\|\Lambda^{2-\beta} b\big\|_{L^{p_5}} \le C\|\nabla b\|_{\dot{B}^{-\gamma_2}_{\infty,\infty}}^{1-\theta_3}\|\Delta b\|_{\dot{H}^s}^{\theta_3}, \tag{1.14}$$

$$\|\Delta b\|_{L^{q_5}} \le C\|\nabla b\|_{\dot{B}^{-\gamma_2}_{\infty,\infty}}^{\theta_3}\|\Delta b\|_{\dot{H}^s}^{1-\theta_3}, \tag{1.15}$$

with $\theta_3 := \frac{s}{1+s+\gamma_2}$, $p_5 := \frac{2}{\theta_3}$, and $q_5 := \frac{2}{1-\theta_3}$.

2 Proof of Theorem 1.1

This section is devoted to the proof of Theorem 1.1, we only need to prove *a priori* estimates.

First, testing (1.1) by u and using (1.3), we see that

$$\frac{1}{2}\frac{d}{dt}\int |u|^2\,dx + \int \big|\Lambda^\alpha u\big|^2\,dx = \int (b\cdot\nabla)b\cdot u\,dx.$$

Testing (1.2) by b and using (1.3), we find that

$$\frac{1}{2}\frac{d}{dt}\int |b|^2\,dx + \int \big|\Lambda^\beta b\big|^2\,dx = \int (b\cdot\nabla)u\cdot b\,dx.$$

Summing up the above two equations, we get the well-known energy inequality

$$\frac{1}{2}\int \big(|u|^2+|b|^2\big)\,dx + \int_0^T\int \big(\big|\Lambda^\alpha u\big|^2 + \big|\Lambda^\beta b\big|^2\big)\,dx\,dt \le \frac{1}{2}\int \big(|u_0|^2+|b_0|^2\big)\,dx. \tag{2.1}$$

Testing (1.1) by $-\Delta u$ and using (1.3), we infer that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int |\nabla u|^2\,dx + \int \big|\Lambda^{1+\alpha} u\big|^2\,dx \\
&\quad= \int u\cdot\nabla u\cdot\Delta u\,dx - \int b\cdot\nabla b\cdot\Delta u\,dx \\
&\quad= -\sum_{i,j}\int \partial_j u_i\,\partial_i u\,\partial_j u\,dx + \sum_{i,j}\int \partial_j b_i\,\partial_i b\,\partial_j u\,dx + \sum_{i,j}\int b_i\partial_i\partial_j b\,\partial_j u\,dx \\
&\quad\le C\|\nabla u\|_{L^3}^3 + C\|\nabla b\|_{L^3}^3 + \sum_{i,j}\int b_i\partial_i\partial_j b\,\partial_j u\,dx =: I_1 + I_2 + I_3.
\end{aligned} \tag{2.2}$$

Testing (1.2) by $-\Delta b$ and using (1.3), we deduce that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int|\nabla b|^2\,dx+\int\left|\Lambda^{1+\beta}b\right|^2dx\\
&\quad=\int u\cdot\nabla b\cdot\Delta b\,dx-\int b\cdot\nabla u\cdot\Delta b\,dx+\int(\operatorname{rot}b\times b)\Delta\operatorname{rot}b\,dx\\
&\quad=-\sum_{i,j}\int\partial_j u_i\,\partial_i b\,\partial_j b\,dx+\sum_{i,j}\int\partial_j b_i\,\partial_i u\,\partial_j b\,dx\\
&\qquad+\sum_{i,j}\int b_i\partial_i\partial_j u\,\partial_j b\,dx-\sum_i\int(\operatorname{rot}b\times\partial_i b)\,\partial_i\operatorname{rot}b\,dx\\
&\quad\le C\|\nabla u\|_{L^3}^3+C\|\nabla b\|_{L^3}^3+\sum_{i,j}\int b_i\partial_i\partial_j u\,\partial_j b\,dx-\sum_i\int(\operatorname{rot}b\times\partial_i b)\,\partial_i\operatorname{rot}b\,dx\\
&\quad=:I_1+I_2+I_4+I_5.
\end{aligned}\tag{2.3}$$

Summing up (2.2) and (2.3), using (1.6), (1.8), (1.9), (1.10), (1.11), and $I_3+I_4=0$, we derive

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int\left(|\nabla u|^2+|\nabla b|^2\right)dx+\int\left(\left|\Lambda^{1+\alpha}u\right|^2+\left|\Lambda^{1+\beta}b\right|^2\right)dx\\
&\quad\le C\|\nabla u\|_{L^3}^3+C\|\nabla b\|_{L^3}^3-\sum_i\int\Lambda^{1-\beta}(\operatorname{rot}b\times\partial_i b)\cdot\Lambda^{\beta-1}\partial_i\operatorname{rot}b\,dx\\
&\quad\le C\|\nabla u\|_{L^3}^3+C\|\nabla b\|_{L^3}^3+C\|\nabla b\|_{L^{p_3}}\left\|\Lambda^{2-\beta}b\right\|_{L^{q_3}}\left\|\Lambda^{1+\beta}b\right\|_{L^2}\\
&\quad\le C\|\nabla u\|_{\dot B^{-\gamma_1}_{\infty,\infty}}\|\nabla u\|^2_{\dot H^{\frac{\gamma_1}{2}}}+C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}\|\nabla b\|^2_{\dot H^{\frac{\gamma_2}{2}}}+C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}\|\nabla b\|_{\dot H^s}\left\|\Lambda^{1+\beta}b\right\|_{L^2}\\
&\quad\le C\|\nabla u\|_{\dot B^{-\gamma_1}_{\infty,\infty}}\|\nabla u\|_{L^2}^{2(1-\frac{\gamma_1}{2\alpha})}\left\|\Lambda^{1+\alpha}u\right\|_{L^2}^{2\cdot\frac{\gamma_1}{2\alpha}}+C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}\|\nabla b\|_{L^2}^{2(1-\frac{\gamma_2}{2\beta})}\left\|\Lambda^{1+\beta}b\right\|_{L^2}^{2\cdot\frac{\gamma_2}{2\beta}}\\
&\qquad+C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}\|\nabla b\|_{L^2}^{1-\frac{s}{\beta}}\left\|\Lambda^{1+\beta}b\right\|_{L^2}^{1+\frac{s}{\beta}}\\
&\quad\le\frac{1}{2}\left\|\Lambda^{1+\alpha}u\right\|_{L^2}^2+\frac{1}{2}\left\|\Lambda^{1+\beta}b\right\|_{L^2}^2+C\|\nabla u\|_{\dot B^{-\gamma_1}_{\infty,\infty}}^{\frac{2\alpha}{2\alpha-\gamma_1}}\|\nabla u\|_{L^2}^2\\
&\qquad+C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}^{\frac{2\beta}{2\beta-\gamma_2}}\|\nabla b\|_{L^2}^2+C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}^{\frac{2\beta}{2\beta-1-\gamma_2}}\|\nabla b\|_{L^2}^2,
\end{aligned}$$

which gives

$$\left\|(u,b)\right\|_{L^\infty(0,T;H^1)}+\|u\|_{L^2(0,T;H^{1+\alpha})}+\|b\|_{L^2(0,T;H^{1+\beta})}\le C.\tag{2.4}$$

In the following proofs, we will use the following Sobolev embedding theorem:

$$\begin{aligned}
&\|\nabla u\|_{L^4}\le C\|u\|_{H^{1+\alpha}},\qquad\|\Delta u\|_{L^4}\le C\|u\|_{H^{2+\alpha}}\le C\left\|\Lambda^{2+\alpha}u\right\|_{L^2}+C,\\
&\|\nabla b\|_{L^4}\le C\|b\|_{H^{1+\beta}},\qquad\|\Delta b\|_{L^4}\le C\left\|\Lambda^{2+\beta}b\right\|_{L^2}+C.
\end{aligned}\tag{2.5}$$

Taking Δ to (1.1), testing by Δu and using (1.3), we have

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int|\Delta u|^2\,dx+\int\left|\Lambda^{2+\alpha}u\right|^2dx\\
&\quad=-\int\left(\Delta(u\cdot\nabla u)-u\cdot\nabla\Delta u\right)\Delta u\,dx+\int\left(\Delta(b\cdot\nabla b)-b\cdot\nabla\Delta b\right)\Delta u\,dx
\end{aligned}$$

$$\begin{aligned}&+\int b\cdot\nabla\Delta b\cdot\Delta u\,dx\\&=:I_6+I_7+I_8.\end{aligned}\tag{2.6}$$

Applying Δ to (1.2), testing by Δb and using (1.3), we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int|\Delta b|^2\,dx+\int\left|\Lambda^{2+\beta}b\right|^2dx\\&=-\int\big(\Delta(u\cdot\nabla b)-u\cdot\nabla\Delta b\big)\Delta b\,dx+\int\big(\Delta(b\cdot\nabla u)-b\cdot\nabla\Delta u\big)\Delta b\,dx\\&\quad+\int b\cdot\nabla\Delta u\cdot\Delta b\,dx-\int\Delta(\operatorname{rot}b\times b)\cdot\Delta\operatorname{rot}b\,dx\\&=:I_9+I_{10}+I_{11}+I_{12}.\end{aligned}\tag{2.7}$$

Note that $I_8+I_{11}=0$.

Using (1.7), (2.4), and (2.5), we bound $I_6+I_7+I_9+I_{10}$ as follows:

$$\begin{aligned}&I_6+I_7+I_9+I_{10}\\&\le C\|\nabla u\|_{L^4}\|\Delta u\|_{L^4}\|\Delta u\|_{L^2}+C\|\nabla b\|_{L^4}\|\Delta b\|_{L^4}\|\Delta u\|_{L^2}\\&\quad+C\big(\|\nabla b\|_{L^4}\|\Delta u\|_{L^4}+\|\nabla u\|_{L^4}\|\Delta b\|_{L^4}\big)\|\Delta b\|_{L^2}\\&\le C\|u\|_{H^{1+\alpha}}\|u\|_{H^{2+\alpha}}\|\Delta u\|_{L^2}+C\|b\|_{H^{1+\beta}}\|b\|_{H^{2+\beta}}\|\Delta u\|_{L^2}\\&\quad+C\big(\|b\|_{H^{1+\beta}}\|u\|_{H^{2+\alpha}}+\|u\|_{H^{1+\alpha}}\|b\|_{H^{2+\beta}}\big)\|\Delta b\|_{L^2}\\&\le\frac{1}{8}\|u\|_{H^{2+\alpha}}^2+\frac{1}{8}\|b\|_{H^{2+\beta}}^2+C\big(\|u\|_{H^{1+\alpha}}^2+\|b\|_{H^{1+\beta}}^2\big)\big(\|\Delta u\|_{L^2}^2+\|\Delta b\|_{L^2}^2\big).\end{aligned}$$

Using (1.6), (1.12), (1.13), (1.14), and (1.15), we bound I_{12} as follows:

$$\begin{aligned}I_{12}&=-\int(\operatorname{rot}b\times\Delta b)\cdot\Delta\operatorname{rot}b\,dx-2\sum_i\int(\partial_i\operatorname{rot}b\times\partial_i b)\Delta\operatorname{rot}b\,dx\\&=-\int\Lambda^{1-\beta}(\operatorname{rot}b\times\Delta b)\cdot\Lambda^{\beta-1}\Delta\operatorname{rot}b\,dx\\&\quad-2\sum_i\int\Lambda^{1-\beta}(\partial_i\operatorname{rot}b\times\partial_i b)\cdot\Lambda^{\beta-1}\Delta\operatorname{rot}b\,dx\\&\le C\big(\|\nabla b\|_{L^{p_4}}\left\|\Lambda^{3-\beta}b\right\|_{L^{q_4}}+\left\|\Lambda^{2-\beta}b\right\|_{L^{p_5}}\|\Delta b\|_{L^{q_5}}\big)\left\|\Lambda^{2+\beta}b\right\|_{L^2}\\&\le C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}\|\Delta b\|_{\dot H^s}\left\|\Lambda^{2+\beta}b\right\|_{L^2}\\&\le C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}\|\Delta b\|_{L^2}^{1-\frac{s}{\beta}}\left\|\Lambda^{2+\beta}b\right\|_{L^2}^{1+\frac{s}{\beta}}\\&\le\frac{1}{8}\left\|\Lambda^{2+\beta}b\right\|_{L^2}^2+C\|\nabla b\|_{\dot B^{-\gamma_2}_{\infty,\infty}}^{\frac{2\beta}{2\beta-1-\gamma_2}}\|\Delta b\|_{L^2}^2.\end{aligned}$$

Inserting the above estimates into (2.6) and (2.7), and summing up the results and using the Gronwall inequality, we arrive at

$$\left\|(u,b)\right\|_{L^\infty(0,T;H^2)}+\|u\|_{L^2(0,T;H^{2+\alpha})}+\|b\|_{L^2(0,T;H^{2+\beta})}\le C.$$

This completes the proof.

3 Conclusions

The applications of Hall-MHD system cover a very wide range of physical objects, such as magnetic reconnection in space plasmas, star formation, neutron stars, and the geodynamo. In this paper, we obtained a new regularity criterion that improves and extends some known regularity criteria of the 3D generalized Hall-MHD system.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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