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Long-time dynamics of N-dimensional structure equations with thermal memory

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Abstract

This paper is concerned with the long-time behavior for a class of N-dimensional thermoelastic coupled structure equations with structural damping and past history thermal memory

$$u_{tt} + \Delta^2 u + \nu\Delta\theta + \Delta^2 u_t - \left[\sigma\left(\int_\Omega (\nabla u)^2\,dx\right) + \phi\left(\int_\Omega \nabla u \nabla u_t\,dx\right)\right]\Delta u + f_1(u) = q_1(x), \quad \text{in } \Omega \times R^+,$$

$$\theta_t - \iota\Delta\theta - (1-\iota)\int_0^\infty k(s)\Delta\theta(t-s)\,ds - \nu\Delta u_t + f_2(\theta) = q_2(x), \quad \text{with } 0 \le \iota < 1.$$

This system arises from a model of the nonlinear thermoelastic coupled vibration structure with the clamped ends for simultaneously considering the medium damping, the viscous effect and the nonlinear constitutive relation and thermoelasticity based on a theory of non-Fourier heat flux laws. By considering the case where the internal (structural) damping is present, for $0 \le \iota < 1$, we show that the thermal source term $f_2(\theta)$ is crucial to stabilizing the system and guarantees the existence of a global attractor for the above mentioned system in the present method.

Keywords: thermoelastic coupled structure; internal (structural) damping; thermal memory; asymptotically smooth; global attractor

1 Introduction

In this paper, we study the N-dimensional nonlinear thermoelastic coupled structure equations with structural damping and past history thermal memory

$$u_{tt} + \Delta^2 u + \nu\Delta\theta + \Delta^2 u_t - \left[\sigma\left(\int_\Omega (\nabla u)^2\,dx\right) + \phi\left(\int_\Omega \nabla u \nabla u_t\,dx\right)\right]\Delta u + f_1(u) = q_1(x), \quad \text{in } \Omega \times R^+ \tag{1}$$

$$\theta_t - \iota\Delta\theta - (1-\iota)\int_0^\infty k(s)\Delta\theta(t-s)\,ds - \nu\Delta u_t + f_2(\theta) = q_2(x), \quad \text{with } 0 \le \iota < 1 \tag{2}$$

which arise from a model of the nonlinear thermoelastic coupled vibration structure with the clamped ends for simultaneously considering the medium damping, the viscous effect and the nonlinear constitutive relation and thermoelasticity based on a theory of non-Fourier heat flux. The system is supplemented with the boundary conditions

$$u|_{\partial\Omega} = \frac{\partial u}{\partial \nu}\bigg|_{\partial\Omega} = 0, \qquad \theta|_{\partial\Omega} = 0 \tag{3}$$

for every $t > 0$, and the initial conditions

$$u(x,0) = u^0(x), \qquad u_t(x,0) = u^1(x), \qquad \theta(x,0) = \theta^0(x) \tag{4}$$

for every $x \in \Omega$, where $u^0(x)$, $u^1(x)$ and θ_0 are assigned initial value functions.

Here the unknown variables $u(x,t)$ and $\theta(x,t)$ represent the vertical deflection of the structure and the vertical component of the temperature gradient, respectively. The subscript t denotes derivative with respect to t. Ω is a bounded domain of R^N with a smooth boundary $\partial\Omega$, $\sigma(\cdot)$ and $\phi(\cdot)$ are the nonlinearity of the material and both continuous nonnegative nonlinear real functions, $f_1(u)$ and $f_2(\theta)$ are the source terms, $k(s)$ is memory kernel and $q_1(x)$ is the lateral load distribution, $q_2(x)$ is the external heat supply, ν is a positive constant. What is more, the source terms $f_1(u)$ and $f_2(\theta)$ are essentially $|u|^\rho u$ and $|\theta|^\varrho \theta$, respectively, with $0 < \rho, \varrho \leq \frac{2}{N-2}$ if $N \geq 3$ and $\rho, \varrho > 0$ if $N = 1, 2$, and the memory kernel $k : R^+ \mapsto R$ is assumed to be a positive bounded convex function vanishing at infinity and the assumptions on nonlinear functions $\sigma(\cdot)$, $\phi(\cdot)$, $f_1(\cdot)$, $f_2(\cdot)$ and the external force function $q_1(x)$, $q_2(x)$ will be specified later.

Without considering the thermal effect, this problem of the infinite dimensional dynamical systems determined by the elastic structure is based on the one-dimensional uncoupled beam equation

$$u_{tt} + \alpha u_{xxxx} - \left(\beta + k \int_0^L u_x^2 \, dx\right) u_{xx} + \gamma u_{xxxxt} - \sigma \int_0^L u_x u_{xt} \, dx u_{xx} + \delta u_t = 0, \tag{5}$$

which was proposed as a model by introducing terms to account for effects of internal (structural) and external linear damping, and the stability theory under the clamped boundary conditions and the hinged boundary conditions was proved by Ball [1]. Ma and Narciso [2] proved the existence of global solutions and the existence of a global attractor for the Kirchhoff-type beam equation

$$u_{tt} + \triangle^2 u - M\left(\int_\Omega (\nabla u)^2 \, dx\right) \triangle u + f(u) + g(u_t) = h(x), \tag{6}$$

with nonlinear external damping but without structural damping, subjected to the conditions

$$u = \frac{\partial u}{\partial \nu} = 0 \quad \text{on } \partial\Omega \times R^+. \tag{7}$$

Without structural damping and thermal effects, this class of structure equations was studied by several authors, e.g., [3–7] and so on.

In the following we also make some comments about previous works for the long-time dynamics of thermoelastic coupled structure equations system with thermal effects.

As the case $\iota = 1$, Eq. (2) becomes the classical parabolic heat equation, thus thermoelastic coupled structure equations system without thermal memory term was considered by several authors. Giorgi et al. [8] studied a class of one-dimensional thermoelastic coupled beam equations with the classical parabolic heat equation but without structural damping

$$\begin{cases} u_{tt} + \triangle^2 u - (\beta + \|\nabla u\|^2_{L^2(0,l)})\triangle u - \triangle u_{tt} + f(u) + \triangle\theta = f, \\ \theta_t - \triangle\theta - \triangle u_t = g \end{cases} \tag{8}$$

subjected to the hinged conditions

$$u = \triangle u = 0, \qquad \theta = 0$$

and gave the existence and uniqueness of global weak solution and the existence of global attractor. Berti et al. [9] studied a class of one-dimensional thermoelastic coupled beam equations with strong external damping and the classical parabolic heat equations

$$\begin{cases} u_{tt} + u_{xxxx} - (\beta + \int_0^l u_x^2\,dx)u_{xx} + \theta_{xx} + u_{xxxxt} = 0, \\ \theta_t - \theta_{xx} - u_{xxt} = 0 \end{cases} \tag{9}$$

and proved the existence of solutions and the exponential decay property. In 2016, Fastovska [10] considered the existence of a compact global attractor for a nonlinear one-dimensional thermoelastic equation

$$\begin{cases} \alpha u_{tt} + k u_{xxxx} - f(u_x)_x - \theta_x = 0, \\ \gamma\theta_t - \beta\theta_{xx} - \theta u_{tx} = 0 \end{cases}$$

with thermally insulated and clamped boundary conditions

$$u(0,t) = u(l,t) = 0, \qquad u_x(0,t) = u_x(l,t) = 0, \qquad \theta_x(0,t) = \theta_x(l,t) = 0,$$

and this system arose in phase transitions in rods made of shape memory alloys whose free energy density had a potential of Ginzburg-Landau form. In addition, Sprekels et al. [11] studied the dynamics of a nonlinear one-dimensional differential equation with the strain

$$\begin{cases} u_{tt} - (f_1\theta + f_2)_x - \gamma\varepsilon_{xt} + \delta u_{xxxx} = 0, \\ C_v\theta_t - k\theta_{xx} - f_1\theta\varepsilon_t - \gamma\varepsilon_t^2 = 0, \\ \varepsilon = u_x \end{cases}$$

arising from the study of phase transitions in shape memory alloys.

As the case $0 \le \iota < 1$, the memory term in Eq. (2) indicates the heat flux depending on the temperature gradient and its past history. When $0 \le \iota < 1$, Barbose et al. [12] studied the long-time behavior for a class of two-dimensional thermoelastic coupled plate equations

$$\begin{cases} u_{tt} + \triangle^2 u - M(\int_\Omega |\nabla u|^2\,dx) \triangle u - \triangle u_{tt} + f(u) + \nu\triangle\theta = h(x), \\ \theta_t - \iota\triangle\theta - (1-\iota)\int_0^\infty k(t-s)\triangle\theta\,ds - \nu\triangle u_t = 0 \end{cases} \tag{10}$$

subjected to the hinged conditions

$$u = \triangle u = 0, \qquad \theta = 0, \qquad x \in \Gamma.$$

Potomkin [13] studied long-time behavior of thermoviscoelastic Berger two-dimensional plate equations

$$\begin{cases} u_{tt} + k_1(0) \triangle^2 u + \int_0^{+\infty} k_1'(s)\triangle^2 u(t-s)\,ds + (\Gamma - \int_\Omega |\nabla u|^2\,dx) \triangle u + \nu\triangle\theta = p(x), \\ \theta_t - \iota\triangle\theta - (1-\iota)\int_0^\infty k_2(s)\triangle\theta(t-s)\,ds - \nu\triangle u_t = 0 \end{cases}$$

with boundary conditions

$$u = k_1(0) \triangle u + \int_0^{+\infty} k_1'(s)\triangle^2 u(t-s)\,ds = 0, \qquad \nu = 0.$$

When the case $\iota = 0$, Wu [14] considered the following nonlinear plate equations with thermal memory effects due to non-Fourier heat flux laws:

$$\begin{cases} u_{tt} + \triangle(\triangle u + \theta) - \triangle u_t + f(u)a = 0, \\ \theta_t + \int_0^\infty k(s)[-\triangle\theta(t-s)]\,ds - \triangle u_t = 0 \end{cases} \tag{11}$$

subjected to the hinged conditions

$$u = \triangle u = 0, \qquad \theta = 0, \qquad x \in \Gamma$$

and gave the existence of a global attractor.

In their works [12] and [13], they did not show directly that the system had a bounded absorbing set, because they found some technical difficulty due to the 'extensibility' term and θ. Instead, they showed that the system was gradient. While directly proving the existence of a bounded absorbing set in the present work, we mainly use Nakao's lemma. The presence of $f_2(\cdot)$ is crucial and guarantees the existence of a global attractor for the above mentioned system in the present method.

In addition, we also refer the reader to [15, 16] and the references therein for thermoelastic coupled structure equations.

It is well known that the infinite dimensional dynamical systems determined by the elastic structure are different because of the difference of boundary conditions. However, the above mentioned systems are all subjected to the hinged boundary conditions. For long-time dynamics for the thermoelastic coupled structure equations with clamped boundary, we refer the reader to [17].

In this paper, our fundamental assumptions on $\sigma(\cdot)$, $\phi(\cdot)$, $f_1(\cdot)$, $f_2(\cdot)$, $g(\cdot)$, and $q(x)$ are given as follows.

Assumption 1 We assume that $\sigma(\cdot) \in C^1(R)$ satisfying

$$\sigma(z)z \geq \hat{\sigma}(z) \geq 0, \quad \forall z \geq 0, \tag{12}$$

where $\hat{\sigma}(z) = \int_0^z \sigma(s)\,ds$. This condition is promptly satisfied if $\sigma(\cdot)$ is nondecreasing with $\sigma(0) = 0$.

Assumption 2 We also assume that $\phi(\cdot) \in C^1(R)$ satisfying $\phi(0) = 0$ and $\phi(\cdot)$ is nondecreasing and

$$\phi(s)s \geq 0, \quad \forall s \in R^+. \tag{13}$$

Assumption 3 The function $f_1(\cdot) : R \longrightarrow R$ is of class $C^1(R)$ and satisfies $f(0) = 0$, and there exist constants k and $\rho > 0$ such that

$$\left|f_1(u) - f_1(v)\right| \leq k_1\left(1 + |u|^\rho + |v|^\rho\right)|u - v|, \quad \forall u, v \in R, \tag{14}$$

$$-a_0 \leq \hat{f}_1(u) \leq \frac{1}{2} f_1(u)u + a_1, \tag{15}$$

where $\hat{f}_1(z) = \int_0^z f_1(s)\,ds$.

Assumption 4 The function $f_2(\cdot) : R \longrightarrow R$ is of class $C^1(R)$ and satisfies $f_2(0) = 0$, and there exist constants k_2 and $\varrho > 0$ such that

$$\left(f_2(\theta) - f_2(\tilde{\theta})\right)(\theta - \tilde{\theta}) \geq k_2(\theta - \tilde{\theta})^{\varrho+2}, \quad \forall \theta, \tilde{\theta} \in R. \tag{16}$$

Assumption 5 The assumptions on $k(s)$ are as follows: k is vanishing at ∞; moreover,

$$-(1 - \iota)k'(s) = \mu(s), \tag{17}$$

where $\mu \in C^1(R^+) \cap L^1(R^+)$, and there exists a constant $\delta_1 > 0$, $\forall s \in R^+$, such that

$$\mu'(s) \leq -\delta_1 \mu(s), \quad s \geq 0. \tag{18}$$

Assumption 6 $q_1(x), q_2(x) \in L^2(\Omega)$.

Under the above assumptions, we prove the existence and uniqueness of global solutions and the existence of a global attractor for N-dimensional nonlinear thermoelastic coupled system (1)-(4) with structural damping and past history thermal memory.

2 Transformed system and basic spaces

Now, we observe that, because of the memory term with past history, problem (1)-(4) does not correspond to autonomous systems. Then we proceed as in Dafermos [18] and Giorgi [19] and Barbose et al. [12] and define a new variable $\chi = \chi^t(x,s)$ by

$$\chi = \chi^t(x,s) = \int_0^s \theta(x, t - \tau)\,d\tau, \quad (t,s) \in [0,\infty) \times R^+. \tag{19}$$

From the definition of χ, for all $t \geq 0$, we have $\chi^t(x,0) = 0$, $\Omega, t \in R^+$ and $\chi^0(s) = \chi_0(s)$ in $\Omega, s \in R^+$, where $\chi_0(s) = \int_0^s \theta_0(\tau)\,d\tau$, $s \in R^+$. Differentiate (19) with respect to t on both sides to get

$$\chi_t = -\left(\theta(x, t - s) - \theta(t)\right). \tag{20}$$

Thus

$$\triangle\chi_t = -\triangle\theta(x,t-s) + \triangle\theta(t) \tag{21}$$

and

$$\chi_t|_{t=0} = \theta_0 - \theta(x,-s) := o(s), \tag{22}$$

where o is the history of θ.

Differentiate (19) with respect to s on both sides to get

$$\chi_s = \theta(x,t-s). \tag{23}$$

Make the sum with (20) and (24) to get

$$\chi_t + \chi_s = \theta(x,t), \quad \Omega\times R^+\times R^+. \tag{24}$$

So

$$\triangle\chi_t + \triangle\chi_s = \triangle\theta(x,t), \quad \Omega\times R^+\times R^+. \tag{25}$$

From (21) and (25), we get

$$\triangle\chi_s = \triangle\theta(t-s). \tag{26}$$

Therefore thermal memory can be rewritten to be

$$-\int_0^\infty k(s)\triangle\theta(t-s)\,ds = -\int_0^\infty k(s)\,d\triangle\chi = -k(s)\triangle\chi|_0^\infty + \int_0^\infty k'(s)\triangle\chi\,ds. \tag{27}$$

Thus, from assumption (17) of kernel $k(s)$, problem (1)-(4) is transformed into the new system

$$u_{tt} + \triangle^2 u + \triangle^2 u_t + \nu\triangle\theta - \left[\sigma\left(\int_\Omega(\nabla u)^2\,dx\right) + \phi\left(\int_\Omega\nabla u\nabla u_t\,dx\right)\right]\triangle u + f_1(u)$$
$$= q_1(x), \tag{28}$$

$$\theta_t - \iota\triangle\theta - \int_0^\infty \mu(s)\triangle\chi^t(s)\,ds - \nu\triangle u_t + f_2(\theta) = q_2(x), \tag{29}$$

$$\chi_t^t(s) = \theta(t) - \chi_s^t(s), \quad \text{in } \Omega\times R^+\times R^+ \tag{30}$$

with the initial conditions

$$u(x,0) = u^0(x), \qquad u_t(x,0) = u^1(x), \qquad \theta(x,0) = \theta^0(x) \quad \text{in } \Omega, \tag{31}$$

$$\chi^0(x,s) = \chi_0(x,s) \quad \text{in } \Omega\times R^+ \tag{32}$$

and homogeneous boundary conditions

$$u|_{\partial\Omega} = \left.\frac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0, \qquad \theta|_{\partial\Omega} = 0, \qquad \chi|_{\partial\Omega} = 0. \tag{33}$$

Our analysis is based on the following Sobolev spaces. Let

$$L^2(\Omega), \qquad H_0^1(\Omega), \qquad W = H^4(\Omega), \qquad V = H_0^2(\Omega), \qquad U = \{\theta \in H^2(\Omega); \theta|_{\partial\Omega} = 0\},$$

and with respect to the new variable χ, we define the weighted space

$$L_\mu^2(R^+, H_0^1(\Omega)) = \left\{\chi : R^+ \to H_0^1(\Omega) \,\middle|\, \int_0^\infty \mu(s) \|\chi\|_{H_0^1(\Omega)}^2 \, ds\right\} \tag{34}$$

which is a Hilbert space with the inner product and the norm defined by

$$\langle \chi, \tilde{\chi} \rangle = \langle \chi, \tilde{\chi} \rangle_{L_\mu^2(R^+, H_0^1(\Omega))} = \int_0^\infty \mu(s) \int_\Omega \nabla\chi \nabla\tilde{\chi} \, dx \, ds,$$

$$\|\chi\|_\mu^2 = \int_0^\infty \mu(s) \|\chi\|_{H_0^1(\Omega)}^2 \, ds.$$

Then, for regular solutions, we consider the phase space

$$\mathbb{H}_1 = W \cap V \times W \cap V \times U \times L_\mu^2(R^+, U). \tag{35}$$

In the case of weak solutions, we consider the phase space

$$\mathbb{H}_0 = V \times L^2(\Omega) \times L^2(\Omega) \times L_\mu^2(R^+, H_0^1(\Omega)). \tag{36}$$

In $\mathbb{H}_0$ we adopt the norm defined by

$$\|(u, v, \theta, \chi)\|_{\mathbb{H}_0}^2 = \|u_{xx}\|^2 + \|v\|^2 + \|\theta\|^2 + \|\chi\|_\mu^2. \tag{37}$$

3 The existence of global solutions and global attractor

Firstly, using the classical Galerkin method, we can establish the existence and uniqueness of regular solution and weak solution to problem (28)-(33) as in the work of Cavalcanti et al. [20] We state it as follows.

Theorem 7 *Under Assumptions* 1-6, *for any initial data* $(u^0, u^1, \theta^0, \chi^0) \in \mathbb{H}_1$, *problem* (28)-(33) *has a unique regular solution* (u, θ, χ) *with*

$$\begin{aligned} &u \in L^\infty(R^+, W \cap V), \qquad u_t \in L^\infty(R^+, W \cap V), \qquad u_{tt} \in L^\infty(R^+, L^2(\Omega)), \\ &\theta \in L^\infty(R^+, U), \qquad \theta_t \in L^\infty(R^+, L^2(\Omega)), \qquad \chi \in L^\infty(R^+, L_\mu^2(R^+, U)). \end{aligned} \tag{38}$$

Theorem 8 *Under the assumptions of Theorem* 7, *if the initial data* $(u^0, u^1, \theta^0, \chi^0) \in \mathbb{H}_0$, *there exists a unique weak solution* (u, θ, χ) *of problem* (28)-(33) *such that*

$$(u, \theta, \chi) \in C(R^+, \mathbb{H}_0), \tag{39}$$

which depends continuously on the initial data with respect to the norm of $\mathbb{H}_0$.

Remark 9 In both cases

$$\|u_t\|^2 + \|\triangle u\|^2 + \|u\|_{\rho+2}^{\rho+2} + \|\theta\|^2 + \left\|\chi^t\right\|_\mu^2 \le C, \tag{40}$$

where C is a constant and C denotes a different constant in different expression of this paper.

Remark 10 Theorem 8 implies that problem (28)-(33) defines a nonlinear C_0-semigroup $S(t)$ on $\mathbb{H}_0$. Indeed, let us set $S(t)(u^0, u^1, \theta^0, \chi_0) = (u(t), u_t(t), \theta(t), \chi^t)$, where u is the unique solution corresponding to the initial data $(u^0, u^1, \theta^0, \chi_0) \in \mathbb{H}_0$. Moreover, the operator $S(t)$ defined in $\mathbb{H}_0$ meets the usual semigroup properties $S(t+\tau) = S(t)S(\tau)$, $S(0) = I$, $\forall t, \tau \in R$.

To prove the main result, we need the following Lemma 11 of Nakao and Lemma 12.

Lemma 11 ([21]) *Let $\varphi(t)$ be a nonnegative continuous function defined on $[0, T)$, $1 < T \le \infty$, which satisfies $\sup_{t\le s\le t+1} \varphi(s)^{1+\eta} \le M_0(\varphi(t) - \varphi(t+1)) + M_1$, $0 \le t \le T-1$, where M_0, M_1, η are positive constants. Then we have*

$$\varphi(t) \le \left(M_0^{-1}\eta(t-1)^+ + \left(\sup_{0\le s\le 1}\varphi(s)\right)^{-\eta}\right)^{-\frac{1}{\eta}} + M_1^{\frac{1}{\eta+1}}, \quad 0 \le t \le T.$$

Lemma 12 ([22]) *Assume that for any bounded positive invariant set $B \subset H$, and for any $\varepsilon > 0$, there exists $T = T(\varepsilon, B)$ such that $d(S(T)x, S(T)y) \le \varepsilon + \varpi_T(x,y)$, $\forall x, y \in B$, where $\varpi_T : H \times H \to R$ satisfies, for any sequence $\{z_n\} \subset B$, $\liminf_{m\to\infty} \liminf_{n\to\infty} \varpi_T(z_n, z_m) = 0$. Then $S(t)$ is asymptotically smooth.*

The main result of an absorbing set reads as follows.

Theorem 13 *Assume the hypotheses of Theorem* 8, *then the corresponding semigroup $S(t)$ of problem* (28)-(33) *has an absorbing set $\mathbb{B}$ in $\mathbb{H}_0$.*

Proof Now we show that the semigroup $S(t)$ has an absorbing set $\mathbb{B}$ in $\mathbb{H}_0$. Firstly, we can calculate the total energy functional

$$E(t) = \frac{1}{2}\left\{\|u_t\|^2 + \|\triangle u\|^2 + \hat{\sigma}\left(\|\nabla u\|^2\right) + \|\theta\|^2 + \|\chi\|_\mu^2\right\} + \int_\Omega \hat{f}_1(u)\,dx - \int_\Omega q_1 u\,dx. \tag{41}$$

Let us fix an arbitrary bounded set $B \subset \mathbb{H}_0$ and consider the solutions of problem (28)-(33) given by $(u(t), u_t(t), \theta(t), \chi) = S(t)(u^0, u^1, \theta^0, \chi^0)$ with $(u^0, u^1, \theta^0, \chi^0) \in B$. Our analysis is based on the modified energy function

$$\tilde{E}(t) = E(t) + a_0|\Omega| + \frac{1}{\lambda_1}\|q_1\|^2 \ge 0 \quad \left(\text{with } \hat{f}_1(u) \ge -a_0\right), \tag{42}$$

where $\lambda_1 > 0$ is the first eigenvalue of the operator $\triangle$ in $H_0^2(\Omega)$. It is easy to see that $\tilde{E}(t)$ dominates $\|(u(t), u_t(t), \theta(t))\|_{\mathbb{H}_0}^2$ and $\|\triangle u(s)\|^2 \le 4\tilde{E}(s)$.

By multiplying Eq. (28) by u and integrating over Ω

$$\begin{aligned}\|\triangle u\|^2 = &-\left[\sigma\left(\int_\Omega |\nabla u|^2\,dx\right) + \phi\left(\int_\Omega \nabla u \nabla u_t\right)dx\right]\|\nabla u\|^2 - \int_\Omega f_1(u)u\,dx + \|u_t\|^2 \\ &- \frac{d}{dt}(u_t, u) + \int_\Omega q_1 u\,dx - \int_\Omega \triangle u \triangle u_t\,dx - \nu \int_\Omega \theta \triangle u\,dx \qquad (43)\end{aligned}$$

and inserting it into $\tilde{E}(t)$, then integrating it from t_1 to t_2, and considering (12) and (15), we obtain that

$$\begin{aligned}\int_{t_1}^{t_2} \tilde{E}(t)\,ds \le &\int_{t_1}^{t_2} \|u_t\|^2\,ds - \frac{1}{2}\int_{t_1}^{t_2} \phi\left(\int_\Omega \nabla u \nabla u_t\,dx\right)\|\nabla u\|^2\,ds \\ &+ \frac{1}{2}\int_{t_1}^{t_2} \|\theta\|^2\,ds + \frac{1}{2}\int_{t_1}^{t_2} \|\chi\|_\mu^2\,ds - \frac{1}{2}\int_{t_1}^{t_2}\int_\Omega \triangle u \triangle u_t\,dx\,ds \\ &- \frac{1}{2}\left(\int_\Omega u_t(t_2)u(t_2)\,dx - \int_\Omega u_t(t_1)u(t_1)\,dx\right) - \frac{\nu}{2}\int_{t_1}^{t_2}\int_\Omega \theta \Delta u\,dx\,ds \\ &+ (a_0 + a_1)|\Omega| + \frac{1}{\lambda_1}\|q_1\|^2 - \frac{1}{2}\int_{t_1}^{t_2}\int_\Omega q_1 u\,dx\,ds, \qquad (44)\end{aligned}$$

where $t_1, t_2 \in [t, t+1]$.

Now let us begin to estimate the right-hand side of (44) to use the above Lemma 11 of Nakao.

First, multiplying Eq. (28) by u_t and integrating over Ω, and multiplying Eq. (29) by θ and integrating over Ω, and taking the inner product with (30) by χ in $L^2_\mu(R^+, H^1_0)$, then taking the sum and integrating from t to $t+1$, we get

$$\begin{aligned}&\int_t^{t+1} \|\triangle u_t\|^2\,d\tau + \frac{\iota}{2}\int_t^{t+1} \|\nabla\theta\|^2\,d\tau + \int_t^{t+1} \phi\left(\int_\Omega \nabla u \nabla u_t\,dx\right)\int_\Omega \nabla u \nabla u_t\,dx\,d\tau \\ &\quad+ \int_t^{t+1}\int_0^\infty \mu(s)\int_\Omega \nabla\chi_s \nabla\chi\,dx\,ds\,d\tau + \int_t^{t+1}\int_\Omega f_2(\theta)\theta\,dx\,d\tau \\ &= E(t) - E(t+1) + \int_t^{t+1}\int_\Omega q_2\theta\,dx\,d\tau. \qquad (45)\end{aligned}$$

Using Schwarz's inequality and Young's inequality and Holder's inequality (with $\frac{\varrho}{\varrho+2} + \frac{2}{\varrho+2} = 1$), we get

$$\begin{aligned}\int_t^{t+1}\int_\Omega q_2\theta\,dx\,d\tau &\le \int_t^{t+1} \|q_2\|\|\theta\|\,d\tau \\ &\le \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{4}\|q_2\|^{\frac{\varrho+2}{\varrho+1}} + \frac{\varepsilon}{|\Omega|^{\frac{\varrho}{2}}}\int_t^{t+1} \|\theta\|^{\varrho+2}\,d\tau \\ &\le \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{4}\|q_2\|^{\frac{\varrho+2}{\varrho+1}} + \varepsilon\int_t^{t+1} \|\theta\|_{\varrho+2}^{\varrho+2}\,d\tau.\end{aligned}$$

With

$$\frac{1}{2}\int_t^{t+1}\int_\Omega f_2(\theta)\theta\,dx\,d\tau \ge \frac{k_2}{2}\int_t^{t+1}\int_\Omega |\theta|^{\varrho+2}\,dx\,d\tau = \frac{k_2}{2}\int_t^{t+1} \|\theta\|_{\varrho+2}^{\varrho+2}\,d\tau,$$

as $\varepsilon \leq \frac{k_2}{2}$, from (45) we have

$$\begin{aligned}
&\int_t^{t+1} \|\triangle u_t\|^2 \,d\tau + \frac{\iota}{2}\int_t^{t+1} \|\nabla\theta\|^2 \,d\tau + \int_t^{t+1} \phi\left(\int_\Omega \nabla u \nabla u_t \,dx\right) \int_\Omega \nabla u \nabla u_t \,dx\,d\tau \\
&\quad + \int_t^{t+1}\int_0^\infty \mu(s) \int_\Omega \nabla\chi_s \nabla\chi \,dx\,ds\,d\tau + \frac{1}{2}\int_t^{t+1}\int_\Omega f_2(\theta)\theta \,dx\,d\tau \\
&\quad \leq E(t) - E(t+1) + \frac{\varepsilon^{-\frac{1}{\varrho+1}} |\Omega|^{\frac{\varrho}{2}}}{4} \|q_2\|^{\frac{\varrho+2}{\varrho+1}}. \qquad (46)
\end{aligned}$$

Taking into account assumptions (13) and (16) of $\phi(\cdot)$ and $f_2(\cdot)$, we have

$$E(t) + \frac{\varepsilon^{-\frac{1}{\varrho+1}} |\Omega|^{\frac{\varrho}{2}}}{4} \|q_2\|^{\frac{\varrho+2}{\varrho+1}} \geq E(t+1),$$

then we define an auxiliary function $I^2(t)$ by putting

$$I^2(t) = E(t) + \frac{\varepsilon^{-\frac{1}{\varrho+1}} |\Omega|^{\frac{\varrho}{2}}}{4} \|q_2\|^{\frac{\varrho+2}{\varrho+1}} - E(t+1) \geq 0.$$

Since $\frac{\iota}{2}\int_t^{t+1} \|\nabla\theta\|^2 \,d\tau$ is vanishing as $\iota = 0$, thus with $0 \leq \iota < 1$, we can only get

$$\begin{aligned}
&\int_t^{t+1} \|\triangle u_t\|^2 \leq I^2(t), \qquad \int_t^{t+1}\int_\Omega f_2(\theta)\theta \,dx\,d\tau \leq 2I^2(t), \\
&\int_t^{t+1}\int_0^\infty \mu(s)\int_\Omega \nabla\chi_s\nabla\chi \,dx\,ds\,d\tau \leq I^2(t).
\end{aligned} \qquad (47)$$

So from the first inequality of (47), we have

$$\int_t^{t+1} \|u_t\|^2 \,ds = \frac{1}{\lambda_1}\int_t^{t+1}\int_\Omega \triangle u_t^2 \,dx\,ds \leq \frac{1}{\lambda_1} I^2(t). \qquad (48)$$

Using twice Holder's inequality with $\frac{\varrho}{\varrho+2} + \frac{2}{\varrho+2} = 1$, combined with assumption (16) on $f_2(\theta)$ and the second inequality of (47), we have

$$\begin{aligned}
&\int_t^{t+1} \|\theta\|^2 \,ds \\
&\quad \leq \int_t^{t+1} \left(\int_\Omega 1^{\frac{\varrho+2}{\varrho}} \,dx\right)^{\frac{\varrho}{\varrho+2}} \left(\int_\Omega \theta^{\varrho+2} \,dx\right)^{\frac{\varrho+2}{2}} ds \\
&\quad \leq |\Omega|^{\frac{\varrho}{\varrho+2}} \left(\int_t^{t+1} 1^{\frac{\varrho+2}{\varrho}} \,ds\right)^{\frac{\varrho}{\varrho+2}} \left(\int_t^{t+1}\int_\Omega \theta^{\varrho+2} \,dx\,ds\right)^{\frac{2}{\varrho+2}} \\
&\quad \leq |\Omega|^{\frac{\varrho}{\varrho+2}} \left(\int_t^{t+1} \frac{1}{k_2}\int_\Omega f_2(\theta)\theta \,dx\,ds\right)^{\frac{2}{\varrho+2}} \\
&\quad \leq |\Omega|^{\frac{\varrho}{\varrho+2}} \left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}} I(t)^{\frac{4}{\varrho+2}}. \qquad (49)
\end{aligned}$$

In fact, as $0 < \iota < 1$, we have $\frac{\iota}{2}\int_t^{t+1}\|\nabla\theta\|^2\,ds \le I^2(t)$, thus we can directly get the estimate on $\int_t^{t+1}\|\theta\|^2\,ds$ without using assumption (16) on $f_2(\theta)$.

Using the mean value theorem with $\phi(0) = 0$ and considering the estimate of (40), then using Young's inequality combined with (47), we have

$$\begin{aligned}
&\frac{1}{2}\int_{t_1}^{t_2}\phi\biggl(\int_\Omega \nabla u\nabla u_t\,dx\biggr)\|\nabla u\|^2\,ds \\
&\quad= \frac{1}{2}\int_{t_1}^{t_2}\phi'(\xi_0)\int_\Omega \nabla u\nabla u_t\,dx\|\nabla u\|^2\,ds \\
&\quad\le \frac{1}{2}\int_{t_1}^{t_2} C\|\triangle u_t\|\|\triangle u\|\,ds \\
&\quad\le \frac{C}{4\eta}I^2(t) + \eta\sup_{t\le s\le t+1}\tilde{E}(s),
\end{aligned} \tag{50}$$

where ξ_0 is among 0 and $\int_\Omega \nabla u\nabla u_t\,dx$.

Considering assumption (18) on $\mu(s)$, we have

$$\begin{aligned}
&\int_t^{t+1}\int_0^\infty \mu(s)\int_\Omega \nabla\chi_s\nabla\chi\,dx\,ds\,d\tau \\
&\quad= -\frac{1}{2}\int_t^{t+1}\int_0^\infty \mu'(s)\|\nabla\chi\|^2\,ds\,d\tau \\
&\quad\ge \frac{\delta_1}{2}\int_t^{t+1}\|\chi\|_\mu^2\,d\tau.
\end{aligned} \tag{51}$$

Also, from the third inequality of (47), we have

$$\frac{1}{2}\int_t^{t+1}\|\chi\|_\mu^2\,d\tau \le \frac{1}{\delta_1}I^2(t). \tag{52}$$

Using Schwarz's inequality and Young's inequality, we get

$$\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega \triangle u\triangle u_t\,dx\,ds \le \frac{1}{4\eta}I(t)^2 + \eta\sup_{t\le s\le t+1}\tilde{E}(s). \tag{53}$$

Since (48), in view of the mean value theorem for integral, there exist number $t_1 \in [t, t+\frac{1}{4}]$ and number $t_2 \in [t+\frac{3}{4}, t+1]$ such that

$$\bigl\|u_t(t_1)\bigr\|^2 \le \frac{4}{\lambda_1}I(t)^2 \tag{54}$$

and

$$\bigl\|u_t(t_2)\bigr\|^2 \le \frac{4}{\lambda_1}I(t)^2. \tag{55}$$

Thus from Schwarz's inequality combined with (54)-(55), noting that $\|\triangle u(s)\|^2 \leq 4\tilde{E}(s)$, we have

$$\begin{aligned}
&\frac{1}{2}\left(\int_\Omega u_t(t_2)u(t_2)\,dx - \int_\Omega u_t(t_1)u(t_1)\,dx\right)\\
&\quad\leq \frac{1}{2}\left(\|u_t(t_2)\|\|u(t_2)\| + \|u_t(t_1)\|\|u(t_1)\|\right)\\
&\quad\leq \frac{1}{2\sqrt{\lambda_1}}\left(\|u_t(t_2)\|\|\triangle u(t_2)\| + \|u_t(t_1)\|\|\triangle u(t_1)\|\right)\\
&\quad\leq \frac{2}{\lambda_1} I(t) \sup_{t\leq s\leq t+1}\|\triangle u\|\\
&\quad\leq \frac{4}{\lambda_1^2\eta} I(t)^2 + \frac{\eta}{4}\sup_{t\leq s\leq t+1}\|\triangle u\|^2\\
&\quad\leq \frac{4}{\lambda_1^2\eta} I(t)^2 + \eta\sup_{t\leq s\leq t+1}\tilde{E}(s).
\end{aligned} \tag{56}$$

Also, by Young's inequality and considering (49), we have

$$\begin{aligned}
&-\frac{\nu}{2}\int_{t_1}^{t_2}\int_\Omega \theta\triangle u\,dx\,ds\\
&\quad\leq \frac{\nu}{2}\int_{t_1}^{t_2}\|\theta\|\|\triangle u\|\,ds\\
&\quad\leq \int_{t_1}^{t_2}\frac{\nu^2}{4\eta}\|\theta\|^2\,ds + \int_{t_1}^{t_2}\frac{\eta}{4}\|\triangle u\|^2\,ds\\
&\quad\leq \frac{\nu^2}{4\eta}\left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}}|\Omega|^{\frac{\varrho}{\varrho+2}} I(t)^{\frac{4}{\varrho+2}} + \eta\sup_{t\leq s\leq t+1}\tilde{E}(s).
\end{aligned} \tag{57}$$

Finally, using Young's inequality again, we get that

$$\frac{1}{2}\int_{t_1}^{t_2}\int_\Omega q_1 u(t)\,dx\,ds \leq \frac{1}{4\eta\lambda_1}\|q_1\|^2 + \eta\sup_{t\leq s\leq t+1}\tilde{E}(s). \tag{58}$$

Inserting (48)-(50), (52)-(53) and (56)-(58) into (44), we obtain

$$\begin{aligned}
\int_{t_1}^{t_2}\tilde{E}(s)\,ds \leq{}& \left[\frac{1}{2}|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}} + \frac{\nu^2}{4\eta}\left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}}|\Omega|^{\frac{\varrho}{\varrho+2}}\right] I(t)^{\frac{4}{\varrho+2}}\\
&+ \left(\frac{1}{\lambda_1} + \frac{C}{4\eta} + \frac{1}{\delta_1} + \frac{1}{4\eta} + \frac{4}{\lambda_1^2\eta} + \frac{\nu^2}{2\eta\lambda_3}\right) I(t)^2\\
&+ 5\eta\sup_{t\leq s\leq t+1}\tilde{E}(s) + (a_0 + a_1)|\Omega| + \left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q_1\|^2.
\end{aligned} \tag{59}$$

For the left-hand side of (59), we use the mean value theorem, then there exists number $\tau \in [t_1, t_2]$ such that

$$\int_{t_1}^{t_2}\tilde{E}(s)\,ds \geq \frac{1}{2}\tilde{E}(t+1) = \frac{1}{2}\left(\tilde{E}(t) - I(t)^2\right) + \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{8}\|q_2\|^{\frac{\varrho+2}{\varrho+1}}. \tag{60}$$

So we conclude that

$$\tilde{E}(t) \le I(t)^2 + 2\int_{t_1}^{t_2} \tilde{E}(s)\,ds - \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{8}\|q_2\|^{\frac{\varrho+2}{\varrho+1}}. \tag{61}$$

Inserting (59) into (61), we obtain that

$$\begin{aligned}\tilde{E}(t) \le{}& 2\left[\frac{1}{2}|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}} + \frac{\nu^2}{4\eta}\left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}}|\Omega|^{\frac{\varrho}{\varrho+2}}\right]I(t)^{\frac{4}{\varrho+2}} \\ &\times\left\{1 + 2\left(\frac{1}{\lambda_1} + \frac{C}{4\eta} + \frac{1}{\delta_1} + \frac{1}{4\eta} + \frac{4}{\lambda_1^2\eta} + \frac{\nu^2}{2\eta\lambda_3}\right)\right\}I(t)^2 \\ &+ 10\eta\sup_{t\le s\le t+1}\tilde{E}(s) + 2(a_0+a_1)|\Omega| \\ &+ 2\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q_1\|^2 - \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{8}\|q_2\|^{\frac{\varrho+2}{\varrho+1}}.\end{aligned} \tag{62}$$

Letting $0 < \eta < \frac{1}{10}$, considering the boundedness of $I(t)^{\frac{2\varrho}{\varrho+2}}$, then setting

$$\begin{aligned}C_1 ={}& 2\left[\frac{1}{2}|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}} + \frac{\nu^2}{4\eta}\left(\frac{2}{k_2}\right)^{\frac{2}{\varrho+2}}|\Omega|^{\frac{\varrho}{\varrho+2}}\right] \\ &+\left\{1 + 2\left(\frac{1}{\lambda_1} + \frac{C}{4\eta} + \frac{1}{\delta_1} + \frac{1}{4\eta} + \frac{4}{\lambda_1^2\eta} + \frac{\nu^2}{2\eta\lambda_3}\right)\right\}C,\end{aligned}$$

from (62) we get

$$\begin{aligned}\tilde{E}(t) \le{}& C_1 I(t)^{\frac{4}{\varrho+2}} + \frac{2}{1-10\eta}(a_0+a_1)|\Omega| \\ &+ \frac{1}{1-10\eta}\left[2\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q_1\|^2 + \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{8}\|q_2\|^{\frac{\varrho+2}{\varrho+1}}\right],\end{aligned} \tag{63}$$

then (63) can be rewritten as

$$\begin{aligned}\tilde{E}(t)^{1+\frac{\varrho}{2}} \le{}& C_1\big(\tilde{E}(t) - \tilde{E}(t+1)\big) + \left\{\frac{2}{1-10\eta}(a_0+a_1)|\Omega|\right. \\ &\left.+ \frac{1}{1-10\eta}\left[2\left(\frac{1}{\lambda_1} + \frac{1}{4\eta\lambda_1}\right)\|q_1\|^2 + \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{8}\|q_2\|^{\frac{\varrho+2}{\varrho+1}}\right]\right\}^{1+\frac{\varrho}{2}}.\end{aligned} \tag{64}$$

Using Nakao's Lemma 11, we conclude that

$$\begin{aligned}\tilde{E}(t) \le{}& \left(C_1^{-1}\frac{\varrho}{2}(t-1)^+ + \tilde{E}(0)^{-\frac{\varrho}{2}}\right)^{-\frac{2}{\varrho}} + \frac{2}{1-10\eta}(a_0+a_1)|\Omega| \\ &+ \frac{1}{1-10\eta}\left[\left(\frac{2}{\lambda_1} + \frac{2}{4\eta\lambda_1}\right)\|q_1\|^2 + \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{8}\|q_2\|^{\frac{\varrho+2}{\varrho+1}}\right].\end{aligned} \tag{65}$$

As $t \to \infty$, the first term on the right-hand side of (65) goes to zero, thus with $\tilde{E}(t) \geq \frac{1}{4}(\|\triangle u\|^2 + \|v\|^2 + \|\theta\|^2 + \|\chi\|_\mu^2)$, we conclude

$$\begin{aligned}\mathbb{B} = \bigg\{(u,v,\theta,\chi) \in \mathbb{H}_0 \bigg| \|\triangle u\|^2 + \|v\|^2 + \|\theta\|^2 + \|\chi\|_\mu^2 \leq \frac{8}{1-10\eta}(a_0 + a_1)|\Omega| \\ + \frac{4}{1-10\eta}\bigg[\bigg(\frac{2}{\lambda_1} + \frac{2}{4\eta\lambda_1}\bigg)\|q_1\|^2 + \frac{\varepsilon^{-\frac{1}{\varrho+1}}|\Omega|^{\frac{\varrho}{2}}}{8}\|q_2\|^{\frac{\varrho+2}{\varrho+1}}\bigg]\bigg\}\end{aligned} \tag{66}$$

is an absorbing set for $S(t)$ in $\mathbb{H}_0$. Theorem 13 is proved. □

The main result of a global attractor reads as follows.

Theorem 14 *Assume the hypotheses of Theorem* 8, *then the corresponding semigroup* $S(t)$ *of problem* (28)-(33) *is asymptotically compact in the space* $\mathbb{H}_0$.

Proof We are going to apply Lemmas 11 and 12 to prove the asymptotic smoothness. Given the initial data $(u^0, u^1, \theta^0, \chi^0)$ and $(v^0, v^1, \tilde{\theta}^0, \tilde{\chi}^0) \in B$ in a bounded invariant set $B \subset \mathbb{H}_0$, let (u,θ,χ), $(v,\tilde{\theta},\tilde{\chi})$ be the corresponding weak solutions of problem (28)-(33). Then the difference $w = u - v$, $\vartheta = \theta - \tilde{\theta}$, $\Pi = \chi - \tilde{\chi}$ is the weak solutions of

$$\begin{cases} w_{tt} + \triangle^2 w + \triangle^2 w_t + \nu\triangle\vartheta \\ \quad = \sigma(\|\nabla u\|^2)\triangle u - \sigma(\|\nabla v\|^2)\triangle v + J_1 - J_2, \\ \vartheta_t - \iota\triangle\vartheta - \int_0^\infty \mu(s)\triangle\Pi^t(s)\,ds - \nu\triangle w_t + J_3 = 0, \\ \Pi_t^t = -\Pi_s^t + \vartheta, \\ w(0) = u^0 - v^0, \qquad w_t(0) = u^1 - v^1, \qquad \vartheta(0) = \theta^0 - \tilde{\theta}_0, \qquad \Pi(0) = \chi^0 - \tilde{\chi}^0, \\ w|_{\partial\Omega} = \frac{\partial w}{\partial \nu}|_{\partial\Omega} = 0, \qquad \vartheta|_{\partial\Omega} = 0, \qquad \Pi|_{\partial\Omega} = 0, \end{cases} \tag{67}$$

where

$$\begin{aligned} J_1 &= \phi\bigg(\int_\Omega \nabla u\nabla u_t\,dx\bigg)\triangle u - \phi\bigg(\int_\Omega \nabla v\nabla v_t\,dx\bigg)\triangle v, \\ J_2 &= f_1(u) - f_1(v), \\ J_3 &= f_2(\theta) - f_2(\tilde{\theta}). \end{aligned} \tag{68}$$

Let us define

$$E_w(t) = \|w_t\|^2 + \|\triangle w\|^2 + \sigma\big(\|\nabla u\|^2\big)\|\nabla w\|^2 + \|\vartheta\|^2 + \|\Pi\|_\mu^2. \tag{69}$$

We can assume formally that w is sufficiently regular by density. Then, multiplying the first equation in (67) by w_t and integrating over Ω, and multiplying the second equation in (67) by ϑ and integrating over Ω, and taking the inner product with Π for the third equation in (67) in the space $L_\mu^2(R^+, H_0^1)$, then taking the sum, we get

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}E_w(t) + \|\triangle w_t\|^2 + \iota\|\nabla\vartheta\|^2 + \int_0^\infty \mu(s)\int_\Omega \nabla\Pi_s\nabla\Pi\,dx\,ds + \int_\Omega J_3\vartheta\,dx \\ &\quad = -\sigma'\big(\|\nabla u\|^2\big)\|\nabla w\|^2\int_\Omega \triangle u u_t\,dx + J_4\int_\Omega \triangle v w_t\,dx + \int_\Omega J_1 w_t\,dx - \int_\Omega J_2 w_t\,dx, \end{aligned} \tag{70}$$

where

$$J_4 = \sigma\left(\|\nabla u\|^2\right) - \sigma\left(\|\nabla v\|^2\right). \tag{71}$$

Let us estimate the right-hand side of (70).

Considering the continuity of $\sigma'(\cdot)$ and estimate (40), we have

$$-\sigma'\left(\|\nabla u\|^2\right)\|\nabla w\|^2 \le C\|\nabla w\|^2. \tag{72}$$

Applying the mean value theorem combined with estimate (40), by Young's inequality we get

$$J_4 \int_\Omega \triangle v w_t \, dx \le \frac{C^2}{\lambda_1}\|\nabla w\|^2 + \frac{1}{4}\|\triangle w_t\|^2. \tag{73}$$

Also use the mean value theorem combined with estimate (40) and Young's inequality to get

$$\begin{aligned}
&\int_\Omega J_1 w_t \, dx \\
&\quad= \int_\Omega \phi\left(\int_\Omega \nabla u \nabla u_t \, dx\right)\triangle w w_t \, dx \\
&\qquad- \int_\Omega \left[\phi\left(\int_\Omega \nabla u \nabla u_t \, dx\right) - \phi\left(\int_\Omega \nabla v \nabla v_t \, dx\right)\right]\triangle v w_t \, dx \\
&\quad= \int_\Omega \phi'(\xi_1)\int_\Omega \nabla u \nabla u_t \, dx \triangle w w_t \, dx - \int_\Omega \phi'(\xi_2)\int_\Omega \nabla w \nabla w_t \, dx \triangle v w_t \, dx \\
&\quad\le C\|\nabla w\|^2 + \frac{1}{4}\|\triangle w_t\|^2,
\end{aligned} \tag{74}$$

where ξ_1 is among 0 and $\int_\Omega \nabla u \nabla u_t \, dx$, and ξ_2 is among $\int_\Omega \nabla u \nabla u_t \, dx$ and $\int_\Omega \nabla v \nabla v_t \, dx$.

By Holder's inequality, Minkowski's inequality combined with the estimate of (40), then by $H_0^1(\Omega) \hookrightarrow\hookrightarrow L^{2(\rho+1)}(\Omega)$ (with $0 < \rho, \varrho \le \frac{2}{N-2}$ if $N \ge 3$ and $\rho, \varrho > 0$ if $N = 1, 2$) and Young's inequality, we obtain

$$\int_\Omega J_2 w_t \, dx \le C\|\nabla w\|\|w_t\| \le \frac{C^2}{\lambda_1}\|\nabla w\|^2 + \frac{1}{4}\|\triangle w_t\|^2. \tag{75}$$

On the other hand, combining with assumption (18) on $\mu(s)$, we obtain

$$\begin{aligned}
&\int_0^\infty \mu(s)\int_\Omega \nabla \Pi_s \nabla \Pi \, dx \, ds \\
&\quad= -\frac{1}{2}\int_0^\infty \mu'(s)\|\nabla \Pi\|^2 \, ds \\
&\quad\ge \frac{\delta_1}{2}\|\Pi\|_\mu^2.
\end{aligned} \tag{76}$$

Considering assumption (17) on $f_2(\cdot)$, we have

$$\int_\Omega J_3 \vartheta \, dx \ge k_2\|\vartheta\|_{\varrho+2}^{\varrho+2}. \tag{77}$$

Thus, by inserting (72)-(77) into (70), we get that

$$\frac{1}{2}\frac{d}{dt}E_w(t)+\frac{1}{4}\|\triangle w_t\|^2+\iota\|\nabla\vartheta\|^2+\frac{\delta_1}{2}\|\Pi\|_\mu^2+k_2\|\vartheta\|_{\varrho+2}^{\varrho+2}\le C\|\nabla w\|^2. \tag{78}$$

Then, integrating from t to $t+1$ and defining an auxiliary function $F^2(t)$, we get

$$\begin{aligned}&\frac{1}{4}\int_t^{t+1}\|\triangle w_t\|^2\,ds+\iota\int_t^{t+1}\|\nabla\vartheta\|^2\,ds+\int_t^{t+1}\frac{\delta_1}{2}\|\Pi\|_\mu^2\,ds+k_2\int_t^{t+1}\|\vartheta\|_{\varrho+2}^{\varrho+2}\,ds\\&\quad\le\frac{1}{2}\big(E_w(t)-E_w(t+1)\big)+C\int_t^{t+1}\|\nabla w\|^2\,ds\\&\quad=F(t)^2.\end{aligned} \tag{79}$$

As $\iota=0$, $\iota\int_t^{t+1}\|\nabla\vartheta\|^2\,d\tau$ is obsolescent, thus with $0\le\iota<1$ we can only get

$$\begin{aligned}&\frac{1}{4}\int_t^{t+1}\|\triangle w_t\|^2\,ds\le F(t)^2,\qquad\int_t^{t+1}\frac{\delta_1}{2}\|\Pi\|_\mu^2\,ds\le F(t)^2,\\&k_2\int_t^{t+1}\|\vartheta\|_{\varrho+2}^{\varrho+2}\,ds\le F(t)^2.\end{aligned} \tag{80}$$

Then, multiplying the first equation in (67) by w and integrating over Ω again, we obtain that

$$\begin{aligned}&\|\triangle w\|^2+\sigma\big(\|\nabla u\|^2\big)\|\nabla w\|^2\\&\quad=-\frac{d}{dt}\int_\Omega w_t w\,dx+\|w_t\|^2-\int_\Omega\triangle^2 w_t w\,dx+J_4\int_\Omega\triangle v w\,dx\\&\qquad-\int_\Omega J_2 w\,dx+\int_\Omega\left[\phi\left(\int_\Omega\nabla u\nabla u_t\,dx\right)-\phi\left(\int_\Omega\nabla v\nabla v_t\,dx\right)\right]\triangle v w\,dx\\&\qquad-\phi\left(\int_\Omega\nabla u\nabla u_t\,dx\right)\|\nabla w\|^2-\nu\int_\Omega\vartheta\triangle w\,dx.\end{aligned} \tag{81}$$

Integrating from t_1 to t_2, we get

$$\begin{aligned}&\int_{t_1}^{t_2}\big(\|\triangle w\|^2+\sigma\big(\|\nabla u\|^2\big)\|\nabla w\|^2\big)\,ds\\&\quad=\int_\Omega w_t(t_2)w(t_2)\,dx-\int_\Omega w_t(t_1)w(t_1)\,dx+\int_{t_1}^{t_2}\|w_t\|^2\,dt\\&\qquad-\int_{t_1}^{t_2}\int_\Omega\triangle w_t\triangle w\,dx\,ds+\int_{t_1}^{t_2}J_4\int_\Omega\triangle v w\,dx\,ds-\int_{t_1}^{t_2}\int_\Omega J_2 w\,dx\,ds\\&\qquad+\int_{t_1}^{t_2}\int_\Omega\left[\phi\left(\int_\Omega\nabla u\nabla u_t\,dx\right)-\phi\left(\int_\Omega\nabla v\nabla v_t\,dx\right)\right]\triangle v w\,dx\,ds\\&\qquad-\int_{t_1}^{t_2}\phi\left(\int_\Omega\nabla u\nabla u_t\,dx\right)\|\nabla w\|^2\,ds-\nu\int_{t_1}^{t_2}\int_\Omega\vartheta\triangle w\,dx\,ds.\end{aligned} \tag{82}$$

Now let us estimate the right-hand side of (82). Firstly, from the first inequality of (80), we infer that

$$\int_t^{t+1} \|w_t\|^2\,ds \le \int_t^{t+1} \frac{1}{\lambda_1}\|\triangle w_t\|^2\,ds \le \frac{4}{\lambda_1}F^2(t). \tag{83}$$

Thus there exist $t_1 \in [t, t+\frac{1}{4}]$ and $t_2 \in [t+\frac{3}{4}, t+1]$ such that

$$\|w_t(t_1)\|^2 \le \frac{16}{\lambda_1}F^2(t) \quad \text{and} \quad \|w_t(t_2)\|^2 \le \frac{16}{\lambda_1}F^2(t), \tag{84}$$

then we can deduce that

$$\begin{aligned}
&\int_\Omega w_t(t_2)w(t_2)\,dx - \int_\Omega w_t(t_1)w(t_1)\,dx \\
&\quad\le \frac{8}{\sqrt{\lambda_1}}F(t)\sup_{t\le\sigma\le t+1}\|\triangle w\| \\
&\quad\le \frac{64}{\lambda_1}F(t)^2 + \frac{1}{4}\sup_{t\le\sigma\le t+1}E_w(\sigma).
\end{aligned} \tag{85}$$

Use Schwarz's inequality and Young's inequality to obtain

$$\int_{t_1}^{t_2}\int_\Omega \triangle w_t \triangle w\,dx\,ds \le \frac{1}{4}\sup_{t\le\sigma\le t+1}E_w(\sigma) + 4F^2(t). \tag{86}$$

Apply the mean value theorem combined with estimate (40) to get

$$\int_{t_1}^{t_2} J_4 \int_\Omega \triangle v w\,dx\,ds \le C\int_{t_1}^{t_2}\|\nabla w\|^2\,ds. \tag{87}$$

Assumption (14) on $f_1(\cdot)$ and the estimate of (40) imply that

$$\int_{t_1}^{t_2}\int_\Omega J_2 w\,dx\,ds \le C\int_{t_1}^{t_2}\|\nabla w\|^2\,ds. \tag{88}$$

Using the mean value theorem and considering the assumption on $\phi(\cdot)$ and the estimate of (40), we have

$$\begin{aligned}
&\int_{t_1}^{t_2}\int_\Omega\left[\phi\left(\int_\Omega \nabla u\nabla u_t\,dx\right) - \phi\left(\int_\Omega \nabla v\nabla v_t\,dx\right)\right]\triangle v w\,dx\,ds \\
&\quad= \int_{t_1}^{t_2}\phi'(\xi_4)\int_\Omega \nabla w\nabla w_t\,dx\int_\Omega \triangle v w\,dx\,ds \\
&\quad\le C\int_{t_1}^{t_2}\|w\|\|w_t\|\,ds \\
&\quad\le C\int_{t_1}^{t_2}\|\nabla w\|^2\,ds + C\int_{t_1}^{t_2}\|w_t\|^2\,ds \\
&\quad\le C\int_{t_1}^{t_2}\|\nabla w\|^2\,ds + \frac{4C}{\lambda_1}F^2(t)
\end{aligned} \tag{89}$$

and

$$\int_{t_1}^{t_2} \phi\left(\int_\Omega \nabla u \nabla u_t\,dx\right)\|\nabla w\|^2\,ds \le C\int_{t_1}^{t_2}\|\nabla w\|^2\,ds, \tag{90}$$

where ξ_4 is among $\int_\Omega \nabla u\nabla u_t\,dx$ and $\int_\Omega \nabla v\nabla v_t\,dx$.

Finally, using twice Holder's inequality with $\frac{\varrho+1}{\varrho+2}+\frac{1}{\varrho+2}=1$, combined with the third inequality of (80), we have

$$\begin{aligned}
&\int_t^{t+1}\|\vartheta\|^2\,ds \\
&\quad\le \int_t^{t+1}\left(\int_\Omega 1^{\frac{\varrho+2}{\varrho}}\,dx\right)^{\frac{\varrho}{\varrho+2}}\left(\int_\Omega \vartheta^{\varrho+2}\,dx\right)^{\frac{\varrho+2}{2}}\,ds \\
&\quad\le |\Omega|^{\frac{\varrho}{\varrho+2}}\left(\int_t^{t+1}1^{\frac{\varrho+2}{\varrho}}\,ds\right)^{\frac{\varrho}{\varrho+2}}\left(\int_t^{t+1}\int_\Omega \vartheta^{\varrho+2}\,dx\,ds\right)^{\frac{2}{\varrho+2}} \\
&\quad\le |\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{1}{k_2}\right)^{\frac{2}{\varrho+2}}F(t)^{\frac{4}{\varrho+2}}.
\end{aligned}\tag{91}$$

Thus, by Schwarz's inequality and Young's inequality, we get

$$\nu\int_{t_1}^{t_2}\int_\Omega \vartheta\triangle w\,dx\,ds \le \nu^2|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{1}{k_2}\right)^{\frac{2}{\varrho+2}}F(t)^{\frac{4}{\varrho+2}}+\frac{1}{4}\int_{t_1}^{t_2}\|\nabla w\|^2\,ds. \tag{92}$$

By inserting (83) and (85)-(90) and (92) into (82), we obtain that

$$\begin{aligned}
&\frac{1}{2}\int_{t_1}^{t_2}\left[\|\triangle w\|^2+\sigma\left(\|\nabla u\|^2\right)\|\nabla w\|^2\right]ds \\
&\quad\le C\int_t^{t+1}\|\nabla w\|^2\,ds+\frac{\nu^2}{2}|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{1}{k_2}\right)^{\frac{2}{\varrho+2}}F(t)^{\frac{4}{\varrho+2}} \\
&\qquad+\left(\frac{32}{\lambda_1}+2+\frac{2}{\lambda_1}+\frac{2C}{\lambda_1}\right)F^2(t)+\frac{3}{8}\sup_{t\le\sigma\le t+1}E_w(\sigma).
\end{aligned}\tag{93}$$

Then from the definition of $E_w(t)$ combined with (80), (83) and (91), we obtain that

$$\begin{aligned}
\frac{1}{2}\int_{t_1}^{t_2}E_w(s)\,ds &\le C\int_t^{t+1}\|\nabla w\|^2\,ds+\left(\frac{\nu^2}{2}+\frac{\iota}{2}\right)|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{1}{k_2}\right)^{\frac{2}{\varrho+2}}F(t)^{\frac{4}{\varrho+2}} \\
&\quad+\left(\frac{32}{\lambda_1}+2+\frac{4}{\lambda_1}+\frac{2C}{\lambda_1}+\frac{1}{\delta_1}\right)F^2(t)+\frac{3}{8}\sup_{t\le\sigma\le t+1}E_w(\sigma).
\end{aligned}\tag{94}$$

For (94), by using the mean value theorem, there exists $t^*\in[t_1,t_2]$ such that

$$\begin{aligned}
E_w\left(t^*\right) &\le C\int_t^{t+1}\|\nabla w\|^2\,ds+2\left(\frac{\nu^2}{2}+\frac{\iota}{2}\right)|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{1}{k_2}\right)^{\frac{2}{\varrho+2}}F(t)^{\frac{4}{\varrho+2}} \\
&\quad+2\left(\frac{32}{\lambda_1}+2+\frac{4}{\lambda_1}+\frac{2C}{\lambda_1}+\frac{1}{\delta_1}\right)F^2(t)+\frac{3}{4}\sup_{t\le\sigma\le t+1}E_w(\sigma).
\end{aligned}\tag{95}$$

From (79), we see that

$$E_w(t) \le E_w(t+1) + 2F^2(t). \tag{96}$$

Let $E_w(\tau) = \sup_{t\le\sigma\le t+1} E_w(\sigma)$ with $\tau \in [t, t+1]$, then integrate (78) over $[t, \tau]$ and over $[t^*, t+1]$ to have

$$\begin{aligned}\sup_{t\le\sigma\le t+1} E_w(\sigma) &= E_w(\tau) \\ &\le E_w(t+1) + 2F^2(t) + C\int_t^{t+1} \|\nabla w\|^2\,ds \\ &\le E_w(t^*) + 4F^2(t) + C\int_t^{t+1} \|\nabla w\|^2\,ds.\end{aligned} \tag{97}$$

Inserting (95) into (97), we obtain

$$\sup_{t\le\sigma\le t+1} E_w(\sigma) \le C\int_t^{t+1} \|\nabla w\|^2\,ds + 8\left(\frac{\nu^2}{2} + \frac{\iota}{2}\right)|\Omega|^{\frac{\varrho}{\varrho+2}}\left(\frac{1}{k_2}\right)^{\frac{2}{\varrho+2}} F(t)^{\frac{4}{\varrho+2}} + CF^2(t). \tag{98}$$

Therefore, from the boundary of $F(t)^{\frac{2\varrho}{\varrho+2}}$, we have

$$\sup_{t\le\sigma\le t+1} E_w(\sigma) \le CF(t)^{\frac{4}{\varrho+2}} + C\sup_{0\le\sigma\le T}\int_\sigma^{\sigma+1} \|\nabla w\|^2\,ds. \tag{99}$$

Therefore

$$\sup_{t\le\sigma\le t+1} E_w(\sigma)^{1+\frac{\varrho}{2}} \le C\big(E_w(t) - E_w(t+1)\big) + C\sup_{0\le\sigma\le T}\int_\sigma^{\sigma+1} \|\nabla w\|^2\,ds. \tag{100}$$

From Nakao's Lemma 11, there exist $C_B > 0$ and $C_T > 0$ such that

$$E_w(t) \le C_B\big[(t-1)^+\big]^{-\frac{2}{\varrho}} + C_T\left(\sup_{0\le\sigma\le T}\int_\sigma^{\sigma+1} \big(\|\nabla w\|^2\big)\,ds\right)^{\frac{2}{\varrho+2}}, \quad 0 \le t \le T. \tag{101}$$

From the definition of $E_w(t)$, we have

$$\big\|(w, w_t, \vartheta)\big\|_{H_0} \le C_B\big[(t-1)^+\big]^{-\frac{2}{\varrho}} + C_T\left(\sup_{0\le\sigma\le T}\int_\sigma^{\sigma+1} \big(\|\nabla w\|^2\big)\,ds\right)^{\frac{2}{\varrho+2}}. \tag{102}$$

Given $\varepsilon > 0$, we choose T large such that

$$C_B\big[(t-1)^+\big]^{-\frac{2}{\varrho}} \le \varepsilon, \tag{103}$$

and define $\varpi_T : \mathbb{H}_0 \times \mathbb{H}_0 \to R$ as

$$\varpi_T\big((u^0, u^1, \theta^0, \chi^0), (v^0, v^1, \tilde\theta^0, \tilde\chi^0)\big) = C_T\left(\sup\int_\sigma^{\sigma+1} \big(\|\nabla w\|^2\big)\,ds\right)^{\frac{2}{\varrho+2}}. \tag{104}$$

Then from (102)-(104) we get

$$\begin{aligned}&\|S(T)(u^0,u^1,\theta^0,\chi^0)-S(T)(v^0,v^1,\tilde{\theta}^0,\tilde{\chi}^0)\|_{\mathbb{H}_0}\\&\quad\leq\varepsilon+\varpi_T\big((u^0,u^1,\theta^0,\chi^0),(v^0,v^1,\tilde{\theta}^0,\tilde{\chi}^0)\big)\end{aligned}\tag{105}$$

for all $(u^0,u^1,\theta^0,\chi^0),(v^0,v^1,\tilde{\theta}^0,\tilde{\chi}^0)\in B$.

Let $(u_n^0,u_n^1,\theta_n^0,\chi_n^0)$ be a given sequence of initial data in B. Then the corresponding sequence $(u_n,u_{tn},\theta_n,\chi_n)$ of solutions of problem (28)-(33) is uniformly bounded in $\mathbb{H}_0$, because B is bounded and positively invariant. So $\{u_n\}$ is bounded in $C([0,\infty),H_0^2(\Omega))\cap C^1([0,\infty),L^2(\Omega))$. Since $H_0^2(\Omega)\hookrightarrow\hookrightarrow H_0^1(\Omega)$ compactly, there exists a subsequence u_{nk} which converges strongly in $C([0,T],H_0^1(\Omega))$. Therefore

$$\lim_{k\to\infty}\lim_{l\to\infty}\int_0^T\big(\|\nabla u_k(s)-\nabla u_{nl}(s)\|^2\big)\,ds=0\tag{106}$$

and

$$\lim_{k\to\infty}\lim_{l\to\infty}\varpi_T\big((u_{nk}^0,u_{nk}^1,\theta_{nk}^0,\chi_{nk}^0),(u_{nl}^0,u_{nl}^1,\theta_{nl}^0,\chi_{nk}^0)\big)=0.\tag{107}$$

So $S(t)$ is asymptotically smooth in $\mathbb{H}_0$. That is, Lemma 12 holds. Thus Theorem 14 is proved. □

Theorem 15 *The corresponding semigroup* $S(t)$ *of problem* (28)-(33) *has a compact global attractor in the phase space* $\mathbb{H}_0$.

Proof In view of Theorems 8, 13 and 14, we directly get Theorem 15. □

4 Conclusion

In this paper, we gave the existence and uniqueness of global solutions and the existence of a global attractor in $\mathbb{H}_0$ for an N-dimensional nonlinear thermoelastic coupled system with structural damping and past history thermal memory

$$\begin{aligned}&u_{tt}+\Delta^2u+\nu\Delta\theta+\Delta^2u_t-\bigg[\sigma\bigg(\int_\Omega(\nabla u)^2\,dx\bigg)+\phi\bigg(\int_\Omega\nabla u\nabla u_t\,dx\bigg)\bigg]\Delta u+f_1(u)\\&\quad=q_1(x),\quad\text{in }\Omega\times R^+,\\&\theta_t-\iota\Delta\theta-(1-\iota)\int_0^\infty k(s)\Delta\theta(t-s)\,ds-\nu\Delta u_t+f_2(\theta)=q_2(x),\quad\text{with }0\leq\iota<1,\\&u|_{\partial\Omega}=\frac{\partial u}{\partial\nu}\bigg|_{\partial\Omega}=0,\qquad\theta|_{\partial\Omega}=0,\\&u(x,0)=u^0(x),\qquad u_t(x,0)=u^1(x),\qquad\theta(x,0)=\theta^0(x).\end{aligned}$$

By considering the case where the internal (structural) damping is present, for $0\leq\iota<1$, we show that the thermal source term $f_2(\theta)$ is crucial and guarantees the existence of a global attractor for the above mentioned system in the present method. This main result may provide the design basis for the thermoelastic coupled structure in engineering. The relevant results have been mentioned in Introduction of [12] and [13].

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Competing interests
This paper does not involve conflict of interests between the authors, and all authors declare that they have no competing interests.

Authors' contributions
This paper is mainly completed by DX. JW deals with the structural damping term as proving the existence of a bounded absorbing set. All authors read and approved the final manuscript.

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