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The Keller–Osserman-type conditions for the study of a semilinear elliptic system

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Abstract

We study the following system of equations:

$$\begin{cases} \Delta u_1 = p_1(|x|)f_1(u_1,u_2) & \text{in } \mathbb{R}^N, \\ \Delta u_2 = p_2(|x|)f_2(u_1,u_2) & \text{in } \mathbb{R}^N. \end{cases} \tag{0.1}$$

Here f_1, f_2 are continuous nonlinear functions that satisfy Keller–Osserman-type conditions, and p_1 and p_2 are continuous weight functions. We establish the existence of radial solutions for this system under various boundary conditions.

Keywords: Entire solutions; Large solution; Systems of elliptic equations

1 Introduction

In this paper, we study the existence and asymptotic behavior of positive radial solutions of the following semilinear elliptic system:

$$\begin{cases} \Delta u_1 = p_1(|x|)f_1(u_1,u_2), \\ \Delta u_2 = p_2(|x|)f_2(u_1,u_2), \end{cases} \quad \text{for } x \in \mathbb{R}^N\ (N \geq 3). \tag{1.1}$$

Systems of type (1.1) arise from the study of Lotka–Volterra equations of predator–prey and competitive type under a zero Dirichlet-type condition and variable coefficients (possibly vanishing on subdomains of $\mathbb{R}^N$, for more on this, see [3, 4, 13, 16, 17]). Moreover, there are several connections between the diffusion–reaction system we consider and the modeling of some problems in physics; see [12].

We study system (1.1) under three different sets of boundary conditions:

- *Finite Case:* Both components (u_1, u_2) are bounded, that is,

$$\lim_{|x|\to\infty} u_1(|x|) < \infty \quad \text{and} \quad \lim_{|x|\to\infty} u_2(|x|) < \infty. \tag{1.2}$$

- *Infinite Case:* Both components (u_1, u_2) are large, that is,

$$\lim_{|x|\to\infty} u_1(|x|) = \infty \quad \text{and} \quad \lim_{|x|\to\infty} u_2(|x|) = \infty. \tag{1.3}$$

- *Semifinite Case:* One of the components is bounded, whereas the other is large, that is,

$$\lim_{|x|\to\infty} u_1(|x|) < \infty \quad \text{and} \quad \lim_{|x|\to\infty} u_2(|x|) = \infty, \tag{1.4}$$

or

$$\lim_{|x|\to\infty} u_1(|x|) = \infty \quad \text{and} \quad \lim_{|x|\to\infty} u_2(|x|) < \infty. \tag{1.5}$$

Let us present some existing literature on this topic. The works of García-Melián, Rossi, and Sabina de Lis [5] and Lair and Mohammed [10] deal with the existence of solutions to the following system:

$$\begin{cases} \Delta u_1 = p_1(x)u_1^{\alpha}u_2^{\beta} & \text{in } B_R, \\ \Delta u_2 = p_2(x)u_1^{\gamma}u_2^{\nu} & \text{in } B_R, \\ u_1(x) \to \infty & \text{as } |x| \to R, \\ u_2(x) \to \infty & \text{as } |x| \to R, \end{cases} \tag{1.6}$$

where B_R is the ball of radius R in $\mathbb{R}^N$ (bounded or unbounded) centered at the origin, p_1 and p_2 are Hölder continuous positive functions, and $\alpha, \beta, \gamma, \nu$ are nonnegative constants. If $R = \infty$, then $B_R = \mathbb{R}^N$, and the limit in (1.6) should be taken as $|x| \to \infty$. In the particular case of $R = \infty$ and $p_1(x) = p_2(x) = 1$, Lair and Mohammed [10] proved that system (1.6) has a positive entire large radial solution if and only if

$$0 \le \max\{\alpha, \nu\} \le 1 \quad \text{and} \quad \beta\gamma \le (1-\alpha)(1-\nu). \tag{1.7}$$

Let us point out that our system (1.1) is more general than system (1.6) considered in the aforementioned works. The goal of our paper is to obtain necessary and sufficient conditions for the existence of positive solutions to system (1.1) under conditions of the Keller–Osserman type [6, 14]. Another contribution of our work is estimates of the solutions, which generalizes similar results obtained in [7–9, 11, 15]. Let us finish this introduction by mentioning that some of the basic ideas underlying the present paper were already developed in our earlier works [1, 2].

2 The mathematical results

Let us start with the following formal definition.

Definition 1 A solution $(u_1, u_2) \in C^2([0,\infty)) \times C^2([0,\infty))$ of system (1.1) is called an *entire bounded solution* if condition (1.2) holds; it is called an *entire large solution* if condition (1.3) holds; it is called a *semifinite entire large solution* when (1.4) or (1.5) hold.

For clarity and ease of presentation, we introduce the following notations:

$$r = |x| \quad \text{and} \quad a, b, \overline{c}_1, \overline{c}_2, \varepsilon_1, \varepsilon_2 \in (0,\infty) \quad \text{are suitably chosen,}$$

$$G_1(r) = \int_0^r t^{1-N} \int_0^t s^{N-1} p_1(s)\,ds\,dt,$$

$$G_2(r) = \int_0^r t^{1-N} \int_0^t s^{N-1} p_2(s)\, ds\, dt,$$

$$H(r) = \int_{a+b}^r \frac{1}{(f_1+f_2)(t,t)}\, dt, \qquad H(\infty) = \lim_{r\to\infty} H(r),$$

$$G(r) = a + b + G_1(r) + G_2(r), \qquad G_i(\infty) = \lim_{r\to\infty} G_i(r), \quad i = 1, 2,$$

$$\omega_1(r) = f_1\left(1, \frac{b}{f_2(a,a)} + \overline{c}_2 f_2\left(1, 1 + \frac{1}{a} H^{-1}\bigl(G(r)\bigr)\right) G_2(r)\right),$$

$$\omega_2(r) = f_2\left(\frac{a}{f_1(b,b)} + \overline{c}_1 f_1\left(1 + \frac{1}{b} H^{-1}\bigl(G(r)\bigr), 1\right) G_1(r), 1\right),$$

$$P_1(r) = \int_0^r y^{1-N} \int_0^y t^{N-1} p_1(t) f_1\bigl(a, b + f_2(a,b) G_2(t)\bigr)\, dt\, dy,$$

$$Q_1(r) = \int_0^r y^{1-N} \int_0^y t^{N-1} p_2(t) f_2\bigl(a + f_1(a,b) G_1(t), b\bigr)\, dt\, dy,$$

$$P_2(r) = \int_0^r \overline{c}_1 z^{1+\varepsilon_1} p_1(z) \omega_1(z)\, dz,$$

$$Q_2(r) = \int_0^r \overline{c}_2 z^{1+\varepsilon_2} p_2(z) \omega_2(z)\, dz,$$

$$P_3(r) = \int_0^r \sqrt{2\overline{c}_1 \phi_1(z) \omega_1(z)}\, dz, \quad \phi_1(z) = \max\bigl\{p_1(t) | 0 \le t \le z\bigr\},$$

$$Q_3(r) = \int_0^r \sqrt{2\overline{c}_2 \phi_2(z) \omega_2(z)}\, dz, \quad \phi_2(z) = \max\bigl\{p_2(t) | 0 \le t \le z\bigr\},$$

$$F_1(r) = \int_a^r \frac{1}{\sqrt{\int_0^s f_1(t, f_2(t,t))\, dt}}\, ds, \qquad F_1(\infty) = \lim_{r\to\infty} F_1(r),$$

$$F_2(r) = \int_b^r \frac{1}{\sqrt{\int_0^s f_2(f_1(t,t), t)\, dt}}\, ds, \qquad F_2(\infty) = \lim_{r\to\infty} F_2(r),$$

$$P_i(\infty) = \lim_{r\to\infty} P_i(r), \qquad Q_i(\infty) = \lim_{r\to\infty} Q_i(r) \quad \text{for } i = 1, 2, 3.$$

Next, we state our working assumptions.

Standing Assumption The weight functions p_1, p_2 and the nonlinearities f_1, f_2 satisfy:

(P1) $p_1, p_2 : [0,\infty) \to [0,\infty)$ are nontrivial radial continuous functions (radial, i.e., $p_1(x) = p_1(|x|)$ and $p_2(x) = p_2(|x|)$);

(C1) $f_1, f_2 : [0,\infty) \times [0,\infty) \to [0,\infty)$ are continuous and nondecreasing in both arguments, and $f_1(x,y) > 0, f_2(x,y) > 0$ for all $x, y > 0$;

(C2) for fixed parameters $a, b \in (0,\infty)$, there exist $\overline{c}_1, \overline{c}_2 \in (0,\infty)$ such that:

- for all $t \ge \min\{a,b\}$, $w \ge 1$, $u \ge \min\{b, f_2(a,a)\}$, $v \ge \min\{1, \frac{b}{f_2(a,a)}\}$, we have

$$f_1(tw, uv) \le \overline{c}_1 f_1(t,u) f_1(w,v) \quad \text{and} \quad f_1(a,b) \ge 1; \tag{2.1}$$

- for all $t \ge \min\{a,b\}$, $w \ge 1$, $u \ge \min\{a, f_1(b,b)\}$, $v \ge \min\{1, \frac{a}{f_1(b,b)}\}$, we have

$$f_2(tw, uv) \le \overline{c}_2 f_2(t,u) f_2(w,v) \quad \text{and} \quad f_2(a,b) \ge 1. \tag{2.2}$$

At this point we are ready to state our main results. The first result concerns the existence of entire solutions of (1.1) in the case $F_1(\infty) = F_2(\infty) = \infty$. Our findings here are summarized in the next theorem.

Theorem 2 *Assume that $F_1(\infty) = F_2(\infty) = \infty$. Then there exists an entire positive radial solution $(u_1, u_2) \in C^2([0,\infty)) \times C^2([0,\infty))$ of (1.1) such that $u_1(0) = a$, $u_2(0) = b$. Moreover, the following properties hold*:

(1) *If $r^{2N-2}p_1(r)$, $r^{2N-2}p_2(r)$ are nondecreasing for large r and there exist ε_1, $\varepsilon_2 \in (0,\infty)$ such that p_1, p_2 satisfy*

$$P_2(\infty) < \infty \quad \text{and} \quad Q_2(\infty) < \infty, \tag{2.3}$$

then the nonnegative radial solution (u_1, u_2) of (1.1) satisfies condition (1.2).

(2) *If p_1 and p_2 satisfy*

$$P_1(\infty) = Q_1(\infty) = \infty, \tag{2.4}$$

then the nonnegative radial solution (u_1, u_2) of (1.1) satisfies condition (1.3).

(3) *If $r^{2N-2}p_1(r)$ is nondecreasing for large r and there exists $\varepsilon_1 \in (0,\infty)$ such that p_1, p_2 satisfy*

$$P_2(\infty) < \infty \quad \text{and} \quad Q_1(\infty) = \infty, \tag{2.5}$$

then the nonnegative radial solution (u_1, u_2) of (1.1) satisfies condition (1.4).

(4) *If $r^{2N-2}p_2(r)$ is nondecreasing for large r and there exists $\varepsilon_2 \in (0,\infty)$ such that p_1, p_2 satisfy*

$$P_1(\infty) = \infty \quad \text{and} \quad Q_2(\infty) < \infty, \tag{2.6}$$

then the nonnegative radial solution (u_1, u_2) of (1.1) satisfies condition (1.5).

(5) *If (1.1) has a nonnegative entire large solution (u_1, u_2) such that $u_1(0) = a$, $u_2(0) = b$ and $r^{2N-2}p_1(r)$, $r^{2N-2}p_2(r)$ are nondecreasing for large r, then p_1 and p_2 satisfy*

$$P_2(\infty) = \infty \quad \text{and} \quad Q_2(\infty) = \infty, \tag{2.7}$$

for all $\varepsilon_1, \varepsilon_2 > 0$.

Our next result concerns the existence of entire solutions (1.1) in the case $F_1(\infty) \le \infty$ and $F_2(\infty) \le \infty$. Our findings are summarized in the next theorem.

Theorem 3 *The following statements hold*:

(i) *If $P_3(\infty) < F_1(\infty) < \infty$ and $Q_3(\infty) < F_2(\infty) < \infty$, then system (1.1) has a positive bounded radial solution $(u_1, u_2) \in C^2([0,\infty)) \times C^2([0,\infty))$ with $u_1(0) = a$ and $u_2(0) = b$ such that*

$$\begin{cases} a + P_1(r) \le u_1(r) \le F_1^{-1}(P_3(r)), \\ b + Q_1(r) \le u_2(r) \le F_2^{-1}(Q_3(r)). \end{cases}$$

(ii) *If* $F_1(\infty) = \infty$, $P_1(\infty) = \infty$, *and* $Q_3(\infty) < F_2(\infty) < \infty$, *then system* (1.1) *has a positive radial solution*

$$(u_1, u_2) \in C^2\big([0,\infty)\big) \times C^2\big([0,\infty)\big), \tag{2.8}$$

such that $u_1(0) = a$, $u_2(0) = b$, *and* (1.5) *holds.*

(iii) *If* $P_3(\infty) < F_1(\infty) < \infty$ *and* $F_2(\infty) = \infty$, $Q_1(\infty) = \infty$, *then system* (1.1) *has a positive radial solution*

$$(u_1, u_2) \in C^2\big([0,\infty)\big) \times C^2\big([0,\infty)\big), \tag{2.9}$$

such that $u_1(0) = a$, $u_2(0) = b$, *and* (1.4) *holds.*

(iv) *If* $r^{2N-2}p_1(r)$ *is nondecreasing for large* r, $F_1(\infty) = \infty$, *and there exists* $\varepsilon_1 \in (0,\infty)$ *such that* $P_2(\infty) < \infty$ *and* $Q_3(\infty) < F_2(\infty) < \infty$, *then system* (1.1) *has a positive radial solution*

$$(u_1, u_2) \in C^2\big([0,\infty)\big) \times C^2\big([0,\infty)\big), \tag{2.10}$$

such that $u_1(0) = a$, $u_2(0) = b$, *and* (1.2) *holds.*

(v) *If* $r^{2N-2}p_2(r)$ *is nondecreasing for large* r, $P_3(\infty) < F_1(\infty) < \infty$, $F_2(\infty) = \infty$, *and there exists* $\varepsilon_2 \in (0,\infty)$ *such that* $Q_2(\infty) < \infty$, *then system* (1.1) *has a positive radial solution*

$$(u_1, u_2) \in C^2\big([0,\infty)\big) \times C^2\big([0,\infty)\big), \tag{2.11}$$

such that $u_1(0) = a$, $u_2(0) = b$, *and* (1.2) *holds.*

Remark 4 If $f_1(u_1,u_2) = u_1^{\alpha} u_2^{\beta}$ and $f_2(u_1,u_2) = u_1^{\gamma} u_2^{\nu}$, where α, β, γ, ν are nonnegative constants such that $\alpha + (\gamma+\nu)\beta \le 1$ and $(\alpha+\beta)\gamma + \upsilon \le 1$, then $F_1(\infty) = F_2(\infty) = \infty$. If $G_1(\infty) = G_2(\infty) = \infty$, then $P_1(\infty) = Q_1(\infty) = \infty$, but the converse is not true. If $G_1(\infty) < \infty$ and $G_2(\infty) < \infty$, then $P_1(\infty) < \infty$ and $Q_1(\infty) < \infty$, but the converse is not true.

3 Proofs of theorems

The main idea in proving our results is reducing system (1.1) to a system of second-order ODEs and giving a complete classification of its solutions. Among many possible methods to establish the existence of radial solutions to system (1.1), we will follow here the one based on a successive approximation as in [2]. In the radial setting, system (1.1) becomes a system of differential equations of the form

$$\begin{cases} (r^{N-1}u_1'(r))' = r^{N-1}p_1(r)f_1(u_1(r), u_2(r)), & r \in [0,\infty), \\ (r^{N-1}u_2'(r))' = r^{N-1}p_2(r)f_2(u_1(r), u_2(r)), & r \in [0,\infty), \end{cases} \tag{3.1}$$

which can be solved subject to the initial boundary conditions $u_1(0) = a$, $u_2(0) = b$, and $u_1'(0) = u_2'(0) = 0$. The differential equations and initial conditions in (3.1) are equivalent to the integral equations

$$\begin{cases} u_1(r) = a + \int_0^r t^{1-N} \int_0^t s^{N-1}p_1(s)f_1(u_1(s), u_2(s))\,ds\,dt, \\ u_2(r) = b + \int_0^r t^{1-N} \int_0^t s^{N-1}p_2(s)f_2(u_1(s), u_2(s))\,ds\,dt. \end{cases} \tag{3.2}$$

To construct a solution to this system, we define the sequences $\{u_1^k(r)\}_{k\geq 0}$ and $\{u_2^k(r)\}_{k\geq 0}$ on $[0,\infty)$ by

$$\begin{cases} u_1^0 = a \quad \text{and} \quad u_2^0 = b, \quad r \geq 0, \\ u_1^k(r) = a + \int_0^r t^{1-N} \int_0^t s^{N-1} p_1(s) f_1(u_1^{k-1}(s), u_2^{k-1}(s))\, ds\, dt, \\ u_2^k(r) = b + \int_0^r t^{1-N} \int_0^t s^{N-1} p_2(s) f_2(u_1^{k-1}(s), u_2^{k-1}(s))\, ds\, dt. \end{cases} \tag{3.3}$$

Obviously, for all $r \geq 0$ and $k \in \mathbb{N}$, we have $u_1^k(r) \geq a$ and $u_2^k(r) \geq b$. Our assumptions yield $u_1^0(r) \leq u_1^1(r)$ and $u_2^0(r) \leq u_2^1(r)$ for all $r \geq 0$. From these inequalities we can easily get $u_1^1(r) \leq u_1^2(r)$ and $u_2^1(r) \leq u_2^2(r)$ for all $r \geq 0$. Continuing this reasoning, we obtain that the sequences $\{u_1^k(r)\}_{k\geq 0}$ and $\{u_2^k(r)\}_{k\geq 0}$ are nondecreasing on $[0,\infty)$. Thus there exist

$$u_1(r) = \lim_{k\to\infty} u_1^k(r) \leq \infty \quad \text{and} \quad u_2(r) = \lim_{k\to\infty} u_2^k(r) \leq \infty, \quad r \in [0,\infty). \tag{3.4}$$

We will next establish "upper bounds" for this sequences. To do this, we note that $\{u_1^k(r)\}_{k\geq 0}$ and $\{u_2^k(r)\}_{k\geq 0}$ satisfy

$$\begin{cases} [r^{N-1}(u_1^k(r))']' = r^{N-1} p_1(r) f_1(u_1^{k-1}(r), u_2^{k-1}(r)), \\ [r^{N-1}(u_2^k(r))']' = r^{N-1} p_2(r) f_2(u_1^{k-1}(r), u_2^{k-1}(r)). \end{cases} \tag{3.5}$$

Using the monotonicity of $\{u_1^k(r)\}_{k\geq 0}$ and $\{u_2^k(r)\}_{k\geq 0}$, we obtain the inequalities

$$\begin{cases} [r^{N-1}(u_1^k(r))']' = r^{N-1} p_1(r) f_1(u_1^{k-1}(r), u_2^{k-1}(r)) \\ \qquad\qquad \leq r^{N-1} p_1(r) f_1(u_1^k(r), u_2^k(r)), \\ [r^{N-1}(u_2^k(r))']' = r^{N-1} p_2(r) f_2(u_1^{k-1}(r), u_2^{k-1}(r)) \\ \qquad\qquad \leq r^{N-1} p_2(r) f_2(u_1^k(r), u_2^k(r)). \end{cases} \tag{3.6}$$

Taking into account (3.6), we easily to see that

$$\begin{aligned} &\bigl[r^{N-1}\bigl(u_1^k(r) + u_2^k(r)\bigr)'\bigr]' \\ &\quad \leq r^{N-1}\bigl(p_1(r) f_1\bigl(u_1^k(r), u_2^k(r)\bigr) + p_2(r) f_2\bigl(u_1^k(r), u_2^k(r)\bigr)\bigr) \\ &\quad \leq r^{N-1}\bigl(p_1(r) + p_2(r)\bigr)\bigl((f_1 + f_2)\bigl(u_1^k(r) + u_2^k(r), u_1^k(r) + u_2^k(r)\bigr)\bigr). \end{aligned} \tag{3.7}$$

Integrating this inequality yields

$$\frac{(u_1^k(r) + u_2^k(r))'}{(f_1 + f_2)(u_1^k(r) + u_2^k(r), u_1^k(r) + u_2^k(r))} \leq G_1'(r) + G_2'(r). \tag{3.8}$$

Integrating (3.8) between 0 and r, we get

$$\int_{a+b}^{u_1^k(r)+u_2^k(r)} \frac{dt}{(f_1 + f_2)(t,t)} \leq G(r), \tag{3.9}$$

or, in the H notation,

$$H\bigl(u_1^k(r) + u_2^k(r)\bigr) \leq G(r). \tag{3.10}$$

It follows from this estimate and the fact that H is a bijection (with the inverse denoted H^{-1}) that

$$u_1^k(r) + u_2^k(r) \leq H^{-1}\bigl(G(r)\bigr). \tag{3.11}$$

This occurs on bounded intervals, since

$$\begin{aligned} u_1^k(r) + u_2^k(r) &\geq a + b + f_1(a,b)G_1(r) + f_2(a,b)G_2(r) \\ &\geq a + b + G_1(r) + G_2(r) = G(r) \end{aligned} \tag{3.12}$$

by (3.3) and (C1). Recalling that $\{u_1^k(r)\}_{k\geq 0}$ and $\{u_2^k(r)\}_{k\geq 0}$ are nondecreasing sequences on $[0,\infty)$, the above estimate yields

$$\begin{aligned} u_1^k(r) &\leq a + \int_0^r t^{1-N}\int_0^t s^{N-1}p_1(s)f_1\bigl(u_1^k(s),u_2^k(s)\bigr)\,ds\,dt \\ &\leq a + f_1\bigl(u_1^k(r),u_2^k(r)\bigr)G_1(r) \\ &\leq a + f_1\bigl(u_2^k(r) + H^{-1}\bigl(G(r)\bigr),u_2^k(r)\bigr)G_1(r) \\ &= a + f_1\left(u_2^k(r)\left(1 + \frac{1}{u_2^k(r)}H^{-1}\bigl(G(r)\bigr)\right),u_2^k(r)\right)G_1(r) \\ &\leq a + f_1\left(u_2^k(r)\left(1 + \frac{1}{b}H^{-1}\bigl(G(r)\bigr)\right),u_2^k(r)\right)G_1(r) \\ &\leq a + \overline{c}_1 f_1\bigl(u_2^k(r),u_2^k(r)\bigr)f_1\left(1 + \frac{1}{b}H^{-1}\bigl(G(r)\bigr),1\right)G_1(r) \\ &= f_1\bigl(u_2^k(r),u_2^k(r)\bigr)\left(\frac{a}{f_1(u_2^k(r),u_2^k(r))} + \overline{c}_1 f_1\left(1 + \frac{1}{b}H^{-1}\bigl(G(r)\bigr),1\right)G_1(r)\right) \\ &\leq f_1\bigl(u_2^k(r),u_2^k(r)\bigr)\left(\frac{a}{f_1(b,b)} + \overline{c}_1 f_1\left(1 + \frac{1}{b}H^{-1}\bigl(G(r)\bigr),1\right)G_1(r)\right) \end{aligned} \tag{3.13}$$

and

$$\begin{aligned} u_2^k(r) &\leq b + \int_0^r t^{1-N}\int_0^t s^{N-1}p_2(s)f_2\bigl(u_1^k(s),u_2^k(s)\bigr)\,ds\,dt \\ &\leq b + f_2\bigl(u_1^k(r),u_2^k(r)\bigr)G_2(r) \\ &\leq b + f_2\bigl(u_1^k(r),u_1^k(r) + H^{-1}\bigl(G(r)\bigr)\bigr)G_2(r) \\ &= b + f_2\left(u_1^k(r),u_1^k(r)\left(1 + \frac{1}{u_1^k(r)}H^{-1}\bigl(G(r)\bigr)\right)\right)G_2(r) \\ &\leq b + f_2\left(u_1^k(r),u_1^k(r)\left(1 + \frac{1}{a}H^{-1}\bigl(G(r)\bigr)\right)\right)G_2(r) \\ &\leq b + \overline{c}_2 f_2\bigl(u_1^k(r),u_1^k(r)\bigr)G_2(r)f_2\left(1,1 + \frac{1}{a}H^{-1}\bigl(G(r)\bigr)\right) \\ &\leq f_2\bigl(u_1^k(r),u_1^k(r)\bigr)\left(\frac{b}{f_2(a,a)} + \overline{c}_2 f_2\left(1,1 + \frac{1}{a}H^{-1}\bigl(G(r)\bigr)\right)G_2(r)\right). \end{aligned} \tag{3.14}$$

Substituting (3.13) and (3.14) into (3.6), we obtain

$$\begin{aligned}
\left[r^{N-1}\left(u_1^k(r)\right)'\right]' &\le r^{N-1}p_1(r)f_1\left(u_1^k(r),u_2^k(r)\right)\\
&\le r^{N-1}p_1(r)f_1(u_1^k(r),f_2\left(u_1^k(r),u_1^k(r)\right)\left(\frac{b}{f_2(a,a)}\right.\\
&\quad\left.+\bar{c}_2f_2\left(1,\frac{a+H^{-1}(G(r))}{a}\right)G_2(r)\right)\\
&\le r^{N-1}p_1(r)\bar{c}_1f_1\left(u_1^k(r),f_2\left(u_1^k(r),u_1^k(r)\right)\right)\omega_1(r)
\end{aligned}\tag{3.15}$$

and

$$\begin{aligned}
\left[r^{N-1}\left(u_2^k(r)\right)'\right]' &\le r^{N-1}p_2(r)f_2\left(u_1^k(r),u_2^k(r)\right)\\
&\le r^{N-1}p_2(r)f_2\left(f_1\left(u_2^k(r),u_2^k(r)\right)\left(\frac{a}{f_1(b,b)}\right.\right.\\
&\quad\left.\left.+\bar{c}_1f_1\left(\frac{b+H^{-1}(G(r))}{b},1\right)G_1(r)\right),u_2^k(r)\right)\\
&\le r^{N-1}p_2(r)\bar{c}_2f_2\left(f_1\left(u_2^k(r),u_2^k(r)\right),u_2^k(r)\right)\omega_2(r).
\end{aligned}\tag{3.16}$$

In summary, we get

$$\begin{cases}
r^{N-1}(u_1^k)'' \le (N-1)r^{N-2}(u_1^k)' + r^{N-1}(u_1^k)'' = [r^{N-1}(u_1^k)']'\\
\qquad\qquad \le r^{N-1}p_1(r)\bar{c}_1f_1(u_1^k(r),f_2(u_1^k(r),u_1^k(r)))\omega_1(r),\\
r^{N-1}(u_2^k)'' \le [r^{N-1}(u_2^k)']'\\
\qquad\qquad \le r^{N-1}p_2(r)\bar{c}_2f_2(f_1(u_2^k(r),u_2^k(r)),u_2^k(r))\omega_2(r).
\end{cases}\tag{3.17}$$

Multiplying the first inequality in (3.17) by $(u_1^k(r))'$ and the second by $(u_2^k(r))'$, we arrive at

$$\begin{cases}
\{[(u_1^k(r))']^2\}' \le 2p_1(r)\bar{c}_1f_1(u_1^k(r),f_2(u_1^k(r),u_1^k(r)))(u_1^k(r))'\omega_1(r),\\
\{[(u_2^k(r))']^2\}' \le 2p_2(r)\bar{c}_2f_2(f_1(u_2^k(r),u_2^k(r)),u_2^k(r))(u_2^k(r))'\omega_2(r).
\end{cases}\tag{3.18}$$

Integrating in (3.18) from 0 to r, we get

$$\begin{cases}
[(u_1^k(r))']^2 \le \int_0^r 2p_1(z)\bar{c}_1f_1(u_1^k(z),f_2(u_1^k(z),u_1^k(z)))(u_1^k(z))'\omega_1(z)\,dz,\\
[(u_2^k(r))']^2 \le \int_0^r 2p_2(z)\bar{c}_2f_2(f_1(u_2^k(z),u_2^k(z)),u_2^k(z))(u_2^k(z))'\omega_2(z)\,dz.
\end{cases}\tag{3.19}$$

Now set

$$\begin{aligned}
\phi_1(r) &= \max\left\{p_1(z)|0\le z\le r\right\},\\
\phi_2(r) &= \max\left\{p_2(z)|0\le z\le r\right\}.
\end{aligned}\tag{3.20}$$

By the definition of $\phi_1(r)$ and $\phi_2(r)$ we get from inequalities (3.19) that

$$\begin{cases}
[(u_1^k(r))']^2 \le 2\bar{c}_1\phi_1(r)\omega_1(r)\int_0^r f_1(u_1^k(z),f_2(u_1^k(z),u_1^k(z)))(u_1^k(z))'\,dz,\\
[(u_2^k(r))']^2 \le 2\bar{c}_2\phi_2(r)\omega_2(r)\int_0^r f_2(f_1(u_2^k(z),u_2^k(z)),u_2^k(z))(u_2^k(z))'\,dz.
\end{cases}\tag{3.21}$$

As a consequence of (3.21), we get

$$\begin{cases} (u_1^k(r))' \le \sqrt{2\phi_1(r)\omega_1(r)}(\int_a^{u_1^k(r)} \overline{c}_1 f_1(z,f_2(z,z))\,dz)^{1/2}, \\ (u_2^k(r))' \le \sqrt{2\phi_2(r)\omega_2(r)}(\int_b^{u_2^k(r)} \overline{c}_2 f_2(f_1(z,z),z)\,dz)^{1/2}. \end{cases} \tag{3.22}$$

From this we easily deduce

$$\begin{cases} \frac{(u_1^k(r))'}{(\int_a^{u_1^k(r)} f_1(z,f_2(z,z))\,dz)^{1/2}} \le \overline{c}_1^{1/2}\sqrt{2\phi_1(r)\omega_1(r)}, \\ \frac{(u_2^k(r))'}{(\int_b^{u_2^k(r)} f_2(f_1(z,z),z)\,dz)^{1/2}} \le \overline{c}_2^{1/2}\sqrt{2\phi_2(r)\omega_2(r)}. \end{cases} \tag{3.23}$$

Integrating (3.23), we arrive at

$$\begin{cases} \int_a^{u_1^k(r)} \frac{1}{\sqrt{\int_0^s f_1(t,f_2(t,t))\,dt}}\,ds \le \int_0^r \sqrt{2\overline{c}_1\phi_1(z)\omega_1(z)}\,dz, \\ \int_b^{u_2^k(r)} \frac{1}{\sqrt{\int_0^s f_2(f_1(t,t),t)\,dt}}\,ds \le \int_0^r \sqrt{2\overline{c}_2\phi_2(z)\omega_2(z)}\,dz. \end{cases} \tag{3.24}$$

Now (3.24) can be written as

$$\begin{cases} F_1(u_1^k(r)) \le P_3(r), \\ F_2(u_2^k(r)) \le Q_3(r). \end{cases} \tag{3.25}$$

Finally, using the fact that F_1^{-1}, F_2^{-1} are strictly increasing on $[0, F_1(\infty))$ and $[0, F_2(\infty))$, we get

$$\begin{cases} u_1^k(r) \le F_1^{-1}(P_3(r)), \\ u_2^k(r) \le F_2^{-1}(Q_3(r)). \end{cases} \tag{3.26}$$

These inequalities are independent of k. We claim that the sequences $\{u_1^k(r)\}_{k\ge0}$ and $\{u_2^k(r)\}_{k\ge0}$ are bounded on $[0, c_0]$ for arbitrary $c_0 > 0$. Indeed, since

$$\left(u_1^k(r)\right)' \ge 0 \quad \text{and} \quad \left(u_2^k(r)\right)' \ge 0, \quad r \ge 0, \tag{3.27}$$

it follows that

$$u_1^k(r) \le u_1^k(c_0) \le C_1 \quad \text{and} \quad u_2^k(r) \le u_2^k(c_0) \le C_2 \quad \text{on } [0, c_0]. \tag{3.28}$$

Here $C_1 = F_1^{-1}(P_3(c_0))$ and $C_2 = F_2^{-1}(Q_3(c_0))$ are positive constants. Thus the sequences $\{u_1^k(r)\}_{k\ge0}$ and $\{u_2^k(r)\}_{k\ge0}$ are bounded and equicontinuous on $[0, c_0]$ for arbitrary $c_0 > 0$. By the Arzelà–Ascoli theorem there exist subsequences of $\{u_1^k(r)\}_{k\ge0}$ and $\{u_2^k(r)\}_{k\ge0}$ converging uniformly to $u_1(r)$ and $u_2(r)$ on $[0, c_0]$. Since $\{u_1^k(r)\}_{k\ge0}$ and $\{u_2^k(r)\}_{k\ge0}$ are nondecreasing on $[0,\infty)$, we see that $\{u_1^k(r)\}_{k\ge0}$ and $\{u_2^k(r)\}_{k\ge0}$ converge uniformly to $u_1(r)$ and $u_2(r)$ on $[0,\infty)$. By the arbitrariness of c_0 we deduce that (u_1, u_2) is the desired solution of (3.2). Since (u_1^k, u_2^k) is spherically symmetric, then (u_1, u_2), obtained as a limit, is also spherically symmetric. Then

$$u_1(r) = \lim_{k\to\infty} u_1^k(r) \quad \text{and} \quad u_2(r) = \lim_{k\to\infty} u_2^k(r), \quad r \geq 0, \tag{3.29}$$

are well defined. A straightforward calculation shows that radial solutions of (1.1) are solutions of the ordinary differential equation system (3.1). Then it follows that the radial solutions of (1.1) with $u_1(0) = a$ and $u_2(0) = b$ satisfy

$$u_1(r) = a + \int_0^r \frac{1}{t^{N-1}} \int_0^t s^{N-1} p_1(s) f_1\bigl(u_1(s), u_2(s)\bigr)\, ds\, dt, \quad r \geq 0, \tag{3.30}$$

$$u_2(r) = b + \int_0^r \frac{1}{t^{N-1}} \int_0^t s^{N-1} p_2(s) f_2\bigl(u_1(s), u_2(s)\bigr)\, ds\, dt, \quad r \geq 0. \tag{3.31}$$

Setting

$$\begin{aligned}
L_1(r) &= r^{1-N} \int_0^r s^{N-1} p_1(s) f_1\bigl(u_1(s), u_2(s)\bigr)\, ds, \\
L_2(r) &= r^{-N}(1-N) \int_0^r s^{N-1} p_1(s) f_1\bigl(u_1(s), u_2(s)\bigr)\, ds, \\
L_3(r) &= p_1(r) f_1\bigl(u_1(s), u_2(s)\bigr)
\end{aligned}$$

and repeating the proof in [1], we can see that

$$\begin{aligned}
&\lim_{r\to 0} u_1'(r) = \lim_{r\to 0} L_1(r) = u_1'(0) = 0, \\
&\lim_{r\to 0} u_1''(r) = \lim_{r\to 0}\bigl(L_2(r) + L_3(r)\bigr) = u_1''(0) = \frac{p_1(0) f_1(u_1(0), u_2(0))}{N},
\end{aligned}$$

from which it follows that $u_1'(r)$ and $u_1''(r)$ are continuous at $r = 0$. In the same fashion, $u_2'(r)$ and $u_2''(r)$ are continuous at $r = 0$. Clearly, $(u_1, u_2) \in C^2[0,\infty) \times C^2[0,\infty)$.

Proof of Theorem 2 completed Choose $R > 0$ such that $r^{2N-2}p_1(r)$ and $r^{2N-2}p_2(r)$ are nondecreasing for $r \geq R$. Using the same arguments as in (3.15) and (3.16), we can see that

$$\begin{cases}
[r^{N-1}(u_1(r))']' \leq r^{N-1} p_1(r) \overline{c}_1 f_1(u_1(r), f_2(u_1(r), u_1(r)))\omega_1(r), \\
[r^{N-1}(u_2(r))']' \leq r^{N-1} p_2(r) \overline{c}_2 f_2(f_1(u_2(r), u_2(r)), u_2(r))\omega_2(r).
\end{cases} \tag{3.32}$$

Multiplying the first equation in (3.32) by $r^{N-1}(u_1)'$ and the second by $r^{N-1}(u_2)'$ and integrating from R to r yield, for $r \geq R$,

$$\begin{cases}
[r^{N-1}(u_1(r))']^2 \leq [R^{N-1}(u_1(R))']^2 \\
\qquad\qquad + 2\int_R^r z^{2N-2} p_1(z) \overline{c}_1 \omega_1(z) \frac{d}{dz} \int_a^{u_1(z)} f_1(s, f_2(s,s))\, ds\, dz, \\
[r^{N-1}(u_2(r))']^2 \leq [R^{N-1}(u_2(R))']^2 \\
\qquad\qquad + 2\int_R^r z^{2N-2} p_2(z) \overline{c}_2 \omega_2(z) \frac{d}{dz} \int_b^{u_2(z)} f_2(f_1(s,s), s)\, ds\, dz,
\end{cases} \tag{3.33}$$

from the monotonicity of $z^{2N-2}p_1(z)$ and $z^{2N-2}p_2(z)$ for $r \geq z \geq R$ we get that

$$\begin{cases} [r^{N-1}(u_1(r))']^2 \leq C_1 + 2\overline{c}_1 r^{2N-2} p_1(r)\omega_1(r)\overline{F}_1(u_1(r)), \\ [r^{N-1}(u_2(r))']^2 \leq C_2 + 2\overline{c}_2 r^{2N-2} p_2(r)\omega_2(r)\overline{F}_2(u_2(r)), \end{cases} \tag{3.34}$$

where

$$\begin{aligned} &C_1 = \left[R^{N-1}\big(u_1(R)\big)'\right]^2, \qquad C_2 = \left[R^{N-1}\big(u_2(R)\big)'\right]^2, \\ &\overline{F}_1\big(u_1(r)\big) = \int_0^{u_1(r)} f_1\big(s, f_2(s,s)\big)\,ds, \\ &\overline{F}_2\big(u_2(r)\big) = \int_0^{u_2(r)} f_2\big(f_1(s,s),s\big)\,ds. \end{aligned} \tag{3.35}$$

This implies that

$$\begin{cases} \frac{(u_1(r))'}{\sqrt{\overline{F}_1(u_1(r))}} \leq \frac{\sqrt{C_1}r^{1-N}}{\sqrt{\overline{F}_1(u_1(r))}} + \sqrt{2\overline{c}_1 p_1(r)\omega_1(r)}, \\ \frac{(u_2(r))'}{\sqrt{\overline{F}_2(u_2(r))}} \leq \frac{\sqrt{C_2}r^{1-N}}{\sqrt{\overline{F}_2(u_2(r))}} + \sqrt{2\overline{c}_2 p_2(r)\omega_2(r)}. \end{cases} \tag{3.36}$$

In particular, integrating (3.36) from R to r, we arrive at the following inequality:

$$\begin{aligned} &\int_{u_1(R)}^{u_1(r)} \left[\int_0^t f_1\big(z, f_2(z,z)\big)\,dz\right]^{-1/2} dt \\ &\quad = F_1\big(u_1(r)\big) - F_1\big(u_1(R)\big) \\ &\quad \leq \int_R^r \frac{\sqrt{C_1}t^{1-N}}{(\int_0^{u_1(t)} f_1(s, f_2(s,s))\,ds)^{-1/2}}\,dt + \int_R^r \sqrt{2\overline{c}_1 p_1(z)\omega_1(z)}\,dz \\ &\quad = \int_R^r \frac{\sqrt{C_1}t^{1-N}}{(\int_0^{u_1(t)} f_1(s, f_2(s,s))\,dz)^{-1/2}}\,dt + \int_R^r z^{\frac{-1-\varepsilon_1}{2}} z^{\frac{1+\varepsilon_1}{2}} \big(2\overline{c}_1 p_1(z)\omega_1(z)\big)^{\frac{1}{2}}\,dz \\ &\quad \leq \frac{\sqrt{C_1}\int_R^r t^{1-N}\,dt}{(\int_0^{u_1(R)} f_1(z, f_2(z,z))\,dz)^{1/2}} \\ &\qquad + \left(\int_R^r z^{-1-\varepsilon_1}\,dz\right)^{\frac{1}{2}} \left(\int_R^r z^{1+\varepsilon_1} 2\overline{c}_1 p_1(z)\omega_1(z)\,dz\right)^{\frac{1}{2}} \\ &\quad \leq \frac{\sqrt{C_1}\int_R^r t^{1-N}\,dt}{(\int_0^{u_1(R)} f_1(z, f_2(z,z))\,dz)^{1/2}} + \left(\frac{2P_2(r)}{\varepsilon_1 R^{\varepsilon_1}}\right)^{\frac{1}{2}}. \end{aligned} \tag{3.37}$$

We next turn to estimating the second solution. A similar calculation yields

$$F_2\big(u_2(r)\big) - F_2\big(u_2(R)\big) \leq \frac{\sqrt{C_2}\int_R^r t^{1-N}\,dt}{(\int_0^{u_2(R)} f_2(f_1(z,z),z)\,dz)^{1/2}} + \left(\frac{2Q_2(r)}{\varepsilon_2 R^{\varepsilon_2}}\right)^{\frac{1}{2}}. \tag{3.38}$$

Inequalities (3.37) and (3.38) are needed in proving the "boundedness" of the functions u_1 and u_2. Indeed, they can be written as

$$\begin{cases} u_1(r) \le F_1^{-1}(F_1(u_1(R)) + \frac{\sqrt{C_1}\int_R^r t^{1-N}\,dt}{(\int_0^{u_1(R)} f_1(z,f_2(z,z))\,dz)^{1/2}} + (\frac{2P_2(r)}{\varepsilon_1 R^{\varepsilon_1}})^{\frac{1}{2}}), \\ u_2(r) \le F_2^{-1}(F_2(u_2(R)) + \frac{\sqrt{C_2}\int_R^r t^{1-N}\,dt}{(\int_0^{u_2(R)} f_2(f_1(z,z),z)\,dz)^{1/2}} + (\frac{2Q_2(r)}{\varepsilon_2 R^{\varepsilon_2}})^{\frac{1}{2}}). \end{cases} \tag{3.39}$$

Having discussed the "bounded" case, we now turn to the claims of the theorem.

Claim 1: When $P_2(\infty) < \infty$ and $Q_2(\infty) < \infty$, from (3.39) we find that

$$\begin{cases} \lim_{r\to\infty} u_1(r) < \infty \\ \lim_{r\to\infty} u_2(r) < \infty \end{cases} \quad \text{for all } r \ge 0, \tag{3.40}$$

and so (u_1, u_2) is bounded. We next consider:

Claim 2: Let (u_1, u_2) be a solution of (3.2). The case $P_1(\infty) = Q_1(\infty) = \infty$ is proved as follows:

$$\begin{aligned} u_1(r) &= a + \int_0^r t^{1-N}\int_0^t s^{N-1}p_1(s)f_1\bigl(u_1(s),u_2(s)\bigr)\,ds\,dt \\ &= a + \int_0^r y^{1-N}\int_0^y t^{N-1}p_1(t)f_1\biggl(a, b \\ &\quad + \int_0^t z^{1-N}\int_0^z s^{N-1}p_2(s)f_2\bigl(u_1(s),u_2(s)\bigr)\,ds\biggr)\,dz)\,dt\,dy \\ &\ge a + \int_0^r y^{1-N}\int_0^y t^{N-1}p_1(t)f_1\bigl(a, b + f_2(a,b)G_2(t)\bigr)\,dt\,dy \\ &\ge \int_0^r y^{1-N}\int_0^y t^{N-1}p_1(t)f_1\bigl(a, b + f_2(a,b)G_2(t)\bigr)\,dt\,dy = P_1(r). \end{aligned} \tag{3.41}$$

As in the preceding lines, we can prove

$$u_2(r) \ge Q_1(r). \tag{3.42}$$

Passing to the limit in (3.41) and in the last inequality, we get

$$\lim_{r\to\infty} u_1(r) = \lim_{r\to\infty} u_2(r) = \infty, \tag{3.43}$$

which yields the claim.

Claim 3: In a similar way as in Claim 1 and Claim 2, we have the estimates

$$\begin{aligned} &\lim_{r\to\infty} u_1(r) \\ &\quad \le F_1^{-1}\biggl(F_1\bigl(u_1(R)\bigr) + \frac{\sqrt{C_1}\int_R^\infty t^{1-N}\,dt}{(\int_0^{u_1(R)} f_1(z,f_2(z,z))\,dz)^{1/2}} + \sqrt{\frac{2P_2(\infty)}{\varepsilon_1 R^{\varepsilon_1}}}\biggr) \\ &\quad \le F_1^{-1}\biggl(F_1\bigl(u_1(R)\bigr) + \frac{R^{N-2}\sqrt{C_1}}{(N-2)(\int_0^{u_1(R)} f_1(z,f_2(z,z))\,dz)^{1/2}} + \sqrt{\frac{2P_2(\infty)}{\varepsilon_2 R^{\varepsilon_2}}}\biggr) < \infty. \end{aligned} \tag{3.44}$$

Arguing as in (3.42) we have

$$\lim_{r\to\infty} u_2(r) = \infty. \tag{3.45}$$

Finally, since we know that

$$P_2(\infty) < \infty \quad \text{and} \quad Q_1(\infty) = \infty, \tag{3.46}$$

the claim yields

$$\lim_{r\to\infty} u_1(r) < \infty \quad \text{and} \quad \lim_{r\to\infty} u_2(r) = \infty. \tag{3.47}$$

Claim 4: By a straightforward modification of the proof presented in the Claim 3 the results hold since any statement about $P_2(\infty)$ can be translated into a statement about $Q_2(\infty)$.

Claim 5: If (u_1, u_2) is a nonnegative non-trivial entire large solution of (1.1), then (u_1, u_2) satisfy

$$u_1(r) \le F_1^{-1}\left(F_1\big(u_1(R)\big) + \frac{\sqrt{C_1}\int_R^r t^{1-N}\,dt}{(\int_0^{u_1(R)} f_1(z, f_2(z,z))\,dz)^{1/2}} + \sqrt{\frac{2P_2(r)}{\varepsilon_1 R^{\varepsilon_1}}}\right), \tag{3.48}$$

$$u_2(r) \le F_2^{-1}\left(F_2\big(u_2(R)\big) + \frac{\sqrt{C_2}\int_R^r t^{1-N}\,dt}{(\int_0^{u_2(R)} f_2(f_1(z,z),z)\,dz)^{1/2}} + \sqrt{\frac{2Q_2(r)}{\varepsilon_2 R^{\varepsilon_2}}}\right), \tag{3.49}$$

where

$$C_1 = \left[R^{N-1}\big(u_1(R)\big)'\right]^2 \quad \text{and} \quad C_2 = \left[R^{N-1}\big(u_2(R)\big)'\right]^2. \tag{3.50}$$

Next, assuming to the contrary that

$$P_2(\infty) < \infty \quad \text{and} \quad Q_2(\infty) < \infty, \tag{3.51}$$

then (2.7) follows by taking $r \to \infty$ in (3.48) and (3.49). This concludes the last claim and concludes the proof of the theorem. □

Proof of Theorem 3 completed It follows from (3.25) and the conditions of the theorem that

$$F_1\big(u_1^k(r)\big) \le P_3(\infty) < F_1(\infty) < \infty, \tag{3.52}$$

$$F_2\big(u_2^k(r)\big) \le Q_3(\infty) < F_2(\infty) < \infty. \tag{3.53}$$

On the other hand, since F_1^{-1}, respectively F_2^{-1}, is strictly increasing on $[0, F_1(\infty))$ respectively $[0, F_2(\infty))$, we find that

$$u_1^k(r) \le F_1^{-1}\big(P_3(\infty)\big) < \infty \quad \text{and} \quad u_2^k(r) \le F_2^{-1}\big(Q_3(\infty)\big) < \infty, \tag{3.54}$$

and then the nondecreasing sequences $\{u_1^k(r)\}_{k\geq 0}$ and $\{u_2^k(r)\}_{k\geq 0}$ are bounded above for all $r \geq 0$ and k. Combining these two facts, we conclude that $(u_1^k(r), u_2^k(r)) \to (u_1(r), u_2(r))$ as $k \to \infty$ and the limit functions u_1 and u_2 are positive entire bounded radial solutions of system (1.1). The remainder of the proof is similar to that of Theorem 2. □

4 Conclusion

In this paper we investigate the existence of solutions on $\mathbb{R}^N$ for a system of partial differential equations under new Keller–Osserman-type conditions.

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