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Life span of solutions with large initial data for a semilinear parabolic system coupling exponential reaction terms

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Abstract

In this paper, we study a coupled systems of parabolic equations subject to large initial data. By using comparison principle and Kaplan's method, we get the upper and lower bound for the life span of the solutions.

MSC: 35K45; 35K51

Keywords: Parabolic system; Blow-up; Life span

1 Introduction

In this paper, we consider the following nonlinear parabolic system:

$$\begin{cases} u_t = \Delta u + e^{mu+pv}, & x \in \Omega, t > 0, \\ v_t = \Delta v + e^{qu+nv}, & x \in \Omega, t > 0, \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = \lambda\varphi(x), \quad v(x,0) = \lambda\psi(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $n > m > 1, p > q > 1, pm > qn$; Ω is a bounded domain in R^N with a smooth boundary $\partial\Omega$; $\lambda > 0$ is a parameter, φ and ψ are nonnegative continuous functions on $\bar{\Omega}$.

The existence and the uniqueness of local classical solutions to problem of semilinear parabolic systems are well known (see, e.g., [1]). We denote by T_λ^* the maximal existence time of a classical solution (u,v) of problem (1.1), that is,

$$T_\lambda^* = \sup\Big\{T > 0, \sup_{0\le t\le T} \big(\|u(\cdot,t)\|_\infty + \|v(\cdot,t)\|_\infty\big) < \infty\Big\},$$

and we call T_λ^* the life span of (u,v). If $T_\lambda^* < \infty$, then we have

$$\lim_{t\to T_\lambda^*} \sup\|u(\cdot,t)\|_\infty = \lim_{t\to T_\lambda^*} \sup\|v(\cdot,t)\|_\infty = \infty.$$

We are interested in T_λ^* and aim to give some properties of the T_λ^*.

Since Fujita's classic work [2], the single equation

$$u_t = \Delta u + u^p \tag{1.2}$$

has been studied extensively in various directions. Friedman and Lacey [3] gave a result on the life span of solutions of (1.2) in the case of small diffusion. Subsequently, Gui and Wang [4], Lee and Ni [5] obtained the leading term of the expansion of the life span T_λ of the solution for (1.2) with the initial data $\lambda\varphi(x)$, and later, Mizoguchi and Yanagida [6] extended the result and determined the second term of the expansion of T_λ, the proved that φ attains the maximum at only one point as $\lambda \to \infty$. Moreover, Mizoguchi and Yanagida [7] extended the result on the life span of solutions of (1.2) in the case of small diffusion. In [8], Sato extended the results to general nonlinearities $f(u)$ in the case of large initial data. Parabolic systems of the following form:

$$u_t = \Delta u + f(v), \qquad v_t = \Delta v + g(u) \tag{1.3}$$

have also been studied in several directions. In [9], Sato investigated (1.3) with $f(v)$ and $g(u)$ replaced by v^p and u^q, in this article the life span of (u, v) with large initial data was obtained. The present author studied the case $f(v) = e^{pv}$, $g(u) = e^{qu}$ in [10] and got some properties of the life span of the solution when initial data is large enough. For other results on system (1.3), we refer the reader to the survey [11], the monograph [12], as well as [13], and the references therein.

On the other hand, much effort has been devoted to the study of coupled parabolic systems, local and global existence, finite time blowup and blowup rate estimates, etc. We recommend reading the latest results [14, 15]. In [16], Zheng and Zhao considered the radially symmetric solutions for the parabolic system

$$u_t = \Delta u + \lambda e^{mu+pv}, \qquad v_t = \Delta v + \mu e^{qu+nv}.$$

And Zhang and Zheng in [17] investigated the above system with nonlocal sources $\lambda \int_\Omega e^{mu+pv}$ and $\mu \int_\Omega e^{qu+nv}$. Also the case with localized sources was studied by Li and Wang in [18].

Parabolic equations (1.3) with the nonlinearities $f(v) = u^m e^{pv}$, $g(u) = u^q e^{nv}$ subject to null Dirichlet boundary conditions were considered in [19] by Liu and Li.

However, to the best of our knowledge, there is little literature on the study of the life span of solutions for problem (1.1). The aim of this paper is to obtain some properties of the life span T_λ^* as λ is large enough. We give a quantitative characterization of life span for the solutions. In the following, we denote by M_φ and M_ψ the maximum of φ and ψ on $\overline{\Omega}$. Then our main results can be summarized as the following theorem.

Theorem 1.1 *Suppose $\varphi, \psi \in C(\bar{\Omega})$ satisfy $\varphi, \psi \geq 0$ in Ω, $\varphi = \psi = 0$ on $\partial\Omega$, $\varphi + \psi \not\equiv 0$.*

(i) *If $p - n > q - m > 0$ and $(q - m)M_\varphi > (p - n)M_\psi$, then we have*

$$\liminf_{\lambda\to\infty} T_\lambda^* e^{\frac{(pq-mn)M_\varphi\lambda}{p-n}} \geq \left(\frac{q-m}{p-n}\right)^{\frac{p}{p-n}} \frac{p-n}{pq-mn}. \tag{1.4}$$

(ii) *If $q - m > p - n > 0$ and $(p - n)M_\psi > (q - m)M_\varphi$, then we have*

$$\liminf_{\lambda\to\infty} T_\lambda^* e^{\frac{(pq-mn)M_\psi\lambda}{q-m}} \geq \left(\frac{p-n}{q-m}\right)^{\frac{q}{q-m}} \frac{p-n}{pq-mn}. \tag{1.5}$$

Theorem 1.2 *Suppose $\varphi, \psi \in C(\bar{\Omega})$ satisfy $\varphi, \psi \geq 0$ in Ω, $\varphi = \psi = 0$ on $\partial\Omega$, $\varphi + \psi \not\equiv 0$.*

(i) *If $p - n > q - m > 0$ and $(q - m)M_\varphi > (p - n)M_\psi$, then we have*

$$\limsup_{\lambda\to\infty} T_\lambda^* \frac{e^{qM_\varphi\lambda}}{\lambda} \leq \frac{qM_\varphi - pM_\psi}{p}. \tag{1.6}$$

(ii) *If $q - m > p - n > 0$ and $(p - n)M_\psi > (q - m)M_\varphi$, then we have*

$$\limsup_{\lambda\to\infty} T_\lambda^* \frac{e^{pM_\psi\lambda}}{\lambda} \leq \frac{pM_\psi - qM_\varphi}{q}. \tag{1.7}$$

2 Preliminaries

In this section we first consider the ODE system

$$\begin{cases} z_t = e^{mz+pw}, & w_t = e^{qz+nw}, \quad t > 0, \\ z(0) = \alpha, & w(0) = \beta, \end{cases} \tag{2.1}$$

where α and β are nonnegative constants.

Here, for constants α and β with $(\alpha, \beta) \not\equiv (0,0)$, we define by $(z(t;\alpha,\beta), w(t;\alpha,\beta))$ the solution for problem (2.1). It is well known that $(z(t;\alpha,\beta), w(t;\alpha,\beta))$ exists and blows up in finite time. We then give the following lemma.

Lemma 2.1 *Suppose that α, β are nonnegative constants and $(\alpha, \beta) \not\equiv (0,0)$. Then the life span of the solution (z, w) for problem (2.1) is*

$$\begin{aligned} T_{\alpha,\beta}^* &= \int_\alpha^\infty \frac{d\xi}{e^{m\xi}\{\frac{p-n}{q-m}[e^{(q-m)\xi} - e^{(q-m)\alpha}] + e^{(p-n)\beta}\}^{\frac{p}{p-n}}} \\ &= \int_\beta^\infty \frac{d\eta}{e^{n\eta}\{\frac{q-m}{p-n}[e^{(p-n)\eta} - e^{(p-n)\beta}] + e^{(q-m)\alpha}\}^{\frac{q}{q-m}}}. \end{aligned} \tag{2.2}$$

Proof Multiplying the first equation in (2.1) by e^{qz+nw} and the second equation by e^{mz+pw}, we obtain the equality

$$e^{(q-m)z} z_t = e^{(p-n)w} w_t.$$

Integrating this equality over $(0, t)$, we have

$$\frac{1}{q-m}\bigl[e^{(q-m)z} - e^{(q-m)\alpha}\bigr] = \frac{1}{p-n}\bigl[e^{(p-n)w} - e^{(p-n)\beta}\bigr].$$

Hence we get

$$\begin{aligned} e^{(q-m)z} &= \frac{q-m}{p-n}\bigl[e^{(p-n)w} - e^{(p-n)\beta}\bigr] + e^{(q-m)\alpha}, \\ e^{(p-n)w} &= \frac{p-n}{q-m}\bigl[e^{(q-m)z} - e^{(q-m)\alpha}\bigr] + e^{(p-n)\beta}. \end{aligned}$$

Substituting these equalities into the equations of (2.1), we see that (z,w) satisfies the initial-value problem

$$z_t = e^{mz}\left\{\frac{p-n}{q-m}\left[e^{(q-m)z} - e^{(q-m)\alpha}\right] + e^{(p-n)\beta}\right\}^{\frac{p}{p-n}}, \quad t > 0, z(0) = \alpha, \tag{2.3}$$

$$w_t = e^{nz}\left\{\frac{q-m}{p-n}\left[e^{(p-n)w} - e^{(p-n)\beta}\right] + e^{(q-m)\alpha}\right\}^{\frac{q}{q-m}}, \quad t > 0, w(0) = \beta. \tag{2.4}$$

Integrating equations in (2.3), (2.4) over $(0,t)$ yields

$$\int_\alpha^{z(t)} \frac{d\xi}{e^{m\xi}\{\frac{p-n}{q-m}[e^{(q-m)\xi} - e^{(q-m)\alpha}] + e^{(p-n)\beta}\}^{\frac{p}{p-n}}} = t,$$

$$\int_\beta^{w(t)} \frac{d\eta}{e^{n\eta}\{\frac{q-m}{p-n}[e^{(p-n)\eta} - e^{(p-n)\beta}] + e^{(q-m)\alpha}\}^{\frac{q}{q-m}}} = t.$$

This implies that the life span of (z,w) is

$$T^*_{\alpha,\beta} = \min\left\{\int_\alpha^\infty \frac{d\xi}{e^{m\xi}\{\frac{p-n}{q-m}[e^{(q-m)\xi} - e^{(q-m)\alpha}] + e^{(p-n)\beta}\}^{\frac{p}{p-n}}},\right.$$
$$\left.\int_\beta^\infty \frac{d\eta}{e^{n\eta}\{\frac{q-m}{p-n}[e^{(p-n)\eta} - e^{(p-n)\beta}] + e^{(q-m)\alpha}\}^{\frac{q}{q-m}}}\right\}.$$

By using the change of variables

$$e^{(q-m)\xi} = \frac{q-m}{p-n}\left(e^{(p-n)\eta} - e^{(p-n)\beta}\right) + e^{(q-m)\alpha},$$

we see that

$$\int_\alpha^\infty \frac{d\xi}{e^{m\xi}\{\frac{p-n}{q-m}[e^{(q-m)\xi} - e^{(q-m)\alpha}] + e^{(p-n)\beta}\}^{\frac{p}{p-n}}}$$
$$= \int_\beta^\infty \frac{d\eta}{e^{n\eta}\{\frac{q-m}{p-n}[e^{(p-n)\eta} - e^{(p-n)\beta}] + e^{(q-m)\alpha}\}^{\frac{q}{q-m}}}.$$

□

3 Proof of main results

We first give a lower bound of life span to the solutions and prove Theorem 1.1.

Proof We give the proof of (i). It is obvious that the solution $(z(t;\lambda M_\varphi,\lambda M_\psi), w(t;\lambda M_\varphi,\lambda M_\psi))$ is a supersolution of problem (1.1), so we have

$$u(x,t) \le z(t;\lambda M_\varphi,\lambda M_\psi), \qquad v(x,t) \le w(t;\lambda M_\varphi,\lambda M_\psi)$$

for $x \in \Omega$ and $0 < t < \min\{T^*_{\lambda M_\varphi,\lambda M_\psi}, T^*_\lambda\}$. This implies

$$T^*_\lambda \ge T^*_{\lambda M_\varphi,\lambda M_\psi}. \tag{3.1}$$

First we assume that $\varphi \not\equiv 0$. Then by (3.1) and Lemma 2.1, a routine computation shows

$$\begin{aligned} T_\lambda^* &\geq \int_{\lambda M_\varphi}^{\infty} \frac{d\xi}{e^{m\xi}\{\frac{p-n}{q-m}(e^{(q-m)\xi} - e^{(q-m)\lambda M_\varphi}) + e^{(p-n)\lambda M_\psi}\}^{\frac{p}{p-n}}} \\ &= \int_{1}^{\infty} \frac{\lambda M_\varphi \, d\xi}{e^{m\lambda M_\varphi \xi}\{\frac{p-n}{q-m}[e^{(q-m)\lambda M_\varphi \xi} - e^{(q-m)\lambda M_\varphi}] + e^{(p-n)\lambda M_\psi}\}^{\frac{p}{p-n}}} \\ &\geq \frac{\lambda M_\varphi}{(\frac{p-n}{q-m})^{\frac{p}{p-n}}} \int_{1}^{\infty} \frac{d\xi}{e^{[m+\frac{p(q-m)}{p-n}]\lambda M_\varphi \xi}} \\ &= \left(\frac{q-m}{p-n}\right)^{\frac{p}{p-n}} \frac{p-n}{pq-mn} e^{-\frac{(pq-mn)M_\varphi \lambda}{p-n}}, \end{aligned}$$

and this yields

$$T_\lambda^* \geq \left(\frac{q-m}{p-n}\right)^{\frac{p}{p-n}} \frac{p-n}{pq-mn} e^{-\frac{(pq-mn)M_\varphi \lambda}{p-n}},$$

so we get

$$\liminf_{\lambda \to \infty} T_\lambda^* e^{\frac{(pq-mn)M_\varphi \lambda}{p-n}} \geq \left(\frac{q-m}{p-n}\right)^{\frac{p}{p-n}} \frac{p-n}{pq-mn}.$$

One can prove (ii) by using similar arguments. □

Next, we give an upper estimate of T_λ^* and prove Theorem 1.2.

Proof We prove by using Kaplan's method [20]. We only give the proof of (i); case (ii) can be proved similarly. Without loss of generality, we may assume that $\varphi(0) = M_\varphi$. We define by μ_R the first eigenvalue of $-\Delta$ in the ball $B_R = B_R(0)$, ϕ_R being the corresponding eigenfunction which satisfies $\int_{B_R} \phi_R(x)\, dx = 1$. Thus, we have

$$\begin{cases} -\Delta\phi_R = \mu_R \phi_R & \text{in } B_R, \\ \phi_R = 0 & \text{on } \partial B_R. \end{cases} \tag{3.2}$$

It is easy to check that

$$\mu_R = \frac{\mu_1}{R^2}, \qquad \phi_R(x) = R^{-N}\phi_1\left(\frac{x}{R}\right).$$

Let R be small enough such that $B_R \subset \Omega$ and set

$$z(t) = \int_{B_R} u(x,t)\phi_R(x)\, dx, \qquad w(t) = \int_{B_R} v(x,t)\phi_R(x)\, dx, \tag{3.3}$$

$$\alpha(R) = \int_{B_R} \varphi(x)\phi_R(x)\, dx, \qquad \beta(R) = \int_{B_R} \psi(x)\phi_R(x)\, dx. \tag{3.4}$$

For $\varphi, \psi \in C(\bar{\Omega})$, $\int_{B_1} \phi_1(x)\,dx = 1$, we have

$$\lim_{R\to 0} \alpha(R) = \varphi(0), \qquad \lim_{R\to 0} \beta(R) = \psi(0).$$

Multiplying the equations in (1.1) by ϕ_R, integrating by parts and using Jensen's inequality, we obtain

$$z_t \geq -\mu_R z + e^{mz+pw}, \quad t > 0, \tag{3.5}$$

$$w_t \geq -\mu_R w + e^{qz+nw}, \quad t > 0, \tag{3.6}$$

$$z(0) = \lambda\alpha(R), \qquad w(0) = \lambda\beta(R). \tag{3.7}$$

Hence, we have

$$\left(e^{\mu_R t} z\right)_t \geq e^{\mu_R t + mz + pw}, \qquad \left(e^{\mu_R t} w\right)_t \geq e^{\mu_R t + qz + nw}.$$

Integrating these inequalities over $(0, t)$, we see that

$$e^{\mu_R t} z - \lambda\alpha \geq \int_0^t e^{\mu_R s + mz(s) + pw(s)}\,ds,$$

$$e^{\mu_R t} w - \lambda\beta \geq \int_0^t e^{\mu_R s + qz(s) + nw(s)}\,ds.$$

Substituting the second inequality into the first, it follows that

$$e^{\mu_R t} z - \lambda\alpha \geq \int_0^t \exp\left\{\mu_R s + mz(s) + \lambda p\beta e^{-\mu_R s} + p e^{-\mu_R s} \int_0^s e^{\mu_R y + qz(y)}\,dy\right\} ds,$$

thus we have

$$z(t) \geq \lambda\alpha e^{-\mu_R t} + e^{-\mu_R t} \int_0^t \exp\left\{\mu_R s + mz(s) + \lambda p\beta e^{-\mu_R s} + p e^{-\mu_R s} \int_0^s e^{\mu_R y + qz(y)}\,dy\right\} ds.$$

We fix $0 < \epsilon < 1$ and take $T_R > 0$ such that $e^{-\mu_R T_R} > 1 - \epsilon$. Then we have

$$z(t) \geq (1-\epsilon)\lambda\alpha + (1-\epsilon) \int_0^t \exp\left\{(1-\epsilon)\lambda p\beta + p(1-\epsilon) \int_0^s e^{qz(y)}\,dy\right\} ds.$$

We set

$$h(t) = (1-\epsilon)\lambda\alpha + (1-\epsilon) \int_0^t \exp\left\{(1-\epsilon)\lambda p\beta + p(1-\epsilon) \int_0^s e^{qz(y)}\,dy\right\} ds,$$

then we have

$$h'(t) = (1-\epsilon)\exp\left\{(1-\epsilon)\lambda p\beta + p(1-\epsilon)\int_0^t e^{qz(s)}\,ds\right\},$$
$$h''(t) = (1-\epsilon)\exp\left\{(1-\epsilon)\lambda p\beta + p(1-\epsilon)\int_0^t e^{qz(s)}\,ds\right\}\cdot p(1-\epsilon)e^{qz}.$$

After a careful computation, we see that

$$h''(t) \geq h'(t)p(1-\epsilon)e^{qh(t)}.$$

Integrating this inequality over $(0,t)$, it follows that

$$h'(t) \geq \frac{p}{q}(1-\epsilon)e^{qh(t)} + (1-\epsilon)e^{(1-\epsilon)\lambda p\beta} - \frac{p}{q}(1-\epsilon)e^{(1-\epsilon)\lambda q\alpha}.$$

Dividing the left-hand side by the right-hand side and integrating over $(0,t)$, we obtain

$$\int_{(1-\epsilon)\lambda\alpha}^{h(t)} \frac{ds}{\frac{p}{q}(1-\epsilon)e^{qs} + (1-\epsilon)e^{(1-\epsilon)\lambda p\beta} - \frac{p}{q}(1-\epsilon)e^{(1-\epsilon)\lambda q\alpha}} \geq t,$$

we then take λ large enough such that

$$T_{\epsilon,R} = \int_{(1-\epsilon)\lambda\alpha}^{\infty} \frac{ds}{\frac{p}{q}(1-\epsilon)e^{qs} + (1-\epsilon)e^{(1-\epsilon)\lambda p\beta} - \frac{p}{q}(1-\epsilon)e^{(1-\epsilon)\lambda q\alpha}} \leq T_R.$$

Then z blows up at some $T \leq T_{\epsilon,R}$, and a careful computation yields

$$T_{\epsilon,R} = \frac{\ln p - \ln q + \lambda(1-\epsilon)(q\alpha - p\beta)}{p(1-\epsilon)e^{(1-\epsilon)\lambda q\alpha} - q(1-\epsilon)e^{(1-\epsilon)\lambda p\beta}},$$

hence we get

$$T_\lambda^* \leq \frac{\ln p - \ln q + \lambda(1-\epsilon)(q\alpha - p\beta)}{p(1-\epsilon)e^{(1-\epsilon)\lambda q\alpha} - q(1-\epsilon)e^{(1-\epsilon)\lambda p\beta}}.$$

Therefore, taking $R \to 0$ and then $\epsilon \to 0$, paying attention to $(q-m)M_\varphi > (p-n)M_\psi$ and $pm > qn$, it follows that

$$T_\lambda^* \leq \frac{\ln p - \ln q + \lambda(qM_\varphi - pM_\psi)}{pe^{qM_\varphi\lambda} - qe^{pM_\psi\lambda}},$$

so we get

$$\limsup_{\lambda\to\infty} T_\lambda^* \frac{e^{qM_\varphi\lambda}}{\lambda} \leq \frac{qM_\varphi - pM_\psi}{p},$$

which is the inequality in (1.6). By a similar argument, we can prove (1.7), and thus Theorem 1.2 is proved. □

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