Lee and Kang *Boundary Value Problems* (2019) 2019:174
https://doi.org/10.1186/s13661-019-1277-y

Boundary Value Problems
a SpringerOpen Journal

RESEARCH Open Access

# Blow-up results for a quasilinear von Karman equation of memory type

Mi Jin Lee[1] and Jum-Ran Kang[2*]

*Correspondence:
pointegg@hanmail.net
[2]Department of Applied Mathematics, Pukyong National University, Busan, South Korea
Full list of author information is available at the end of the article

**Abstract**

In this paper, we consider the blow-up result of solution for a quasilinear von Karman equation of memory type with nonpositive initial energy as well as positive initial energy. For nonincreasing function $g > 0$ and nondecreasing function $f$, we prove a finite time blow-up result under suitable condition on the initial data.

**MSC:** 35B44; 37B25; 35L70; 74D10

**Keywords:** Blow-up result; Von Karman equation; Memory dissipation

## 1 Introduction

Let $\rho > 0, \alpha > 0$, and $p > 2$. Moreover, let us denote by $\Omega$ an open bounded set of $\mathbb{R}^2$ with sufficiently smooth boundary $\Gamma$. We assume that $\Gamma_0 \cup \Gamma_1 = \Gamma$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, $\Gamma_0 \neq \emptyset$, and $\Gamma_0$ and $\Gamma_1$ have positive measure. In this paper we investigate a blow-up result for the following quasilinear von Karman equation of memory type:

$$|y_t|^\rho y_{tt} - \alpha \Delta y_{tt} + \Delta^2 y - \int_0^t g(t-s)\Delta^2 y(s)\,ds = [y,z] \quad \text{in } \Omega \times (0,\infty), \tag{1.1}$$

$$\Delta^2 z = -[y,y] \quad \text{in } \Omega \times (0,\infty), \tag{1.2}$$

$$z = \frac{\partial z}{\partial \nu} = 0 \quad \text{on } \Gamma \times (0,\infty), \tag{1.3}$$

$$y = \frac{\partial y}{\partial \nu} = 0 \quad \text{on } \Gamma_0 \times (0,\infty), \tag{1.4}$$

$$\mathcal{B}_1 y - \mathcal{B}_1\left(\int_0^t g(t-s)y(s)\,ds\right) = 0 \quad \text{on } \Gamma_1 \times (0,\infty), \tag{1.5}$$

$$\alpha \frac{\partial y_{tt}}{\partial \nu} - \mathcal{B}_2 y + \mathcal{B}_2\left(\int_0^t g(t-s)y(s)\,ds\right) + f(y_t) = |y|^{p-2}y \quad \text{on } \Gamma_1 \times (0,\infty), \tag{1.6}$$

$$y(x,0) = y_0(x), \qquad y_t(x,0) = y_1(x) \quad \text{in } \Omega, \tag{1.7}$$

where $\nu = (\nu_1,\nu_2)$ is the outward unit normal vector on $\Gamma$. The relaxation function $g$ is a positive nonincreasing function and $f$ is a nondecreasing function. Here

$$\mathcal{B}_1 \varpi = \Delta \varpi + (1-\mu) B_1 \varpi, \qquad \mathcal{B}_2 \varpi = \frac{\partial \Delta \varpi}{\partial \nu} + (1-\mu)\frac{\partial B_2 \varpi}{\partial \tau},$$

where

$$B_1\varpi = 2\nu_1\nu_2\frac{\partial^2\varpi}{\partial x_1\partial x_2} - \nu_1^2\frac{\partial^2\varpi}{\partial x_2^2} - \nu_2^2\frac{\partial^2\varpi}{\partial x_1^2},$$
$$B_2\varpi = (\nu_1^2 - \nu_2^2)\frac{\partial^2\varpi}{\partial x_1\partial x_2} + \nu_1\nu_2\left(\frac{\partial^2\varpi}{\partial x_2^2} - \frac{\partial^2\varpi}{\partial x_1^2}\right)$$

and the constant $\mu \in (0, \frac{1}{2})$ represents Poisson's ratio. The von Karman bracket $[\varpi, \phi]$ is given by

$$[\varpi, \phi] = \frac{\partial^2\varpi}{\partial x_1^2}\frac{\partial^2\phi}{\partial x_2^2} + \frac{\partial^2\varpi}{\partial x_2^2}\frac{\partial^2\phi}{\partial x_1^2} - 2\frac{\partial^2\varpi}{\partial x_1\partial x_2}\frac{\partial^2\phi}{\partial x_1\partial x_2}.$$

The authors in [1–5] studied the asymptotic behavior of the solutions to a von Karman system with dissipative effects. The uniform decay rate for the von Karman system with frictional dissipative effect in the boundary has been proved by several authors [6–8]. For a von Karman equation with rotational inertia and memory of the form

$$y_{tt} - \alpha\Delta y_{tt} + \Delta^2 y - \int_0^t g(t-s)\Delta^2 y(s)\,ds = [y, z] \quad \text{in } \Omega \times (0, \infty),$$
$$\Delta^2 z = -[y, y] \quad \text{in } \Omega \times (0, \infty),$$

many authors [9–12] showed the existence and stability of solutions. Several authors [13–15] investigated the general stability for a von Karman system with memory boundary conditions. The stability for a von Karman system with acoustic boundary conditions was treated by [16, 17]. Some authors discussed the energy decay for a von Karman equation with time-varying delay (see [18, 19] and the reference therein).

On the other hand, many authors have considered the global existence, uniform decay rates, and blow-up of solutions for the wave equation with nonlinear damping and source terms:

$$y_{tt} - \Delta y + a|y_t|^{m-2}y_t = b|y|^{p-2}y \quad \text{in } \Omega \times (0, \infty),$$

where $a, b > 0$ and $p, m > 2$. When $a = 0$, Ball [20] showed that the source term $|u|^{p-2}u$ causes blow-up of solutions with negative initial energy in finite time. For $m = 2$, Levine [21, 22] proved that solutions with negative initial energy blow up in finite time. Georgiev and Todorova [23] extended Levin's result to the nonlinear damping case. Messaoudi [24] improved the blow-up result of [23] to the solutions with negative initial energy. Messaoudi [25] studied the blow-up property of solutions with negative initial energy for the following viscoelastic wave equation with $p > m$:

$$y_{tt} - \Delta y + \int_0^t g(t-s)\Delta y(s)\,ds + |y_t|^{m-2}y_t = |y|^{p-2}y \quad \text{in } \Omega \times (0, \infty). \tag{1.8}$$

Messaoudi [26] extended the blow-up result of [25] to the solution with positive initial energy. Song [27] proved the finite time blow-up of some solutions whose initial data have arbitrarily positive initial energy for (1.8). Recently, Park et al. [28] showed the blow-up of the solutions for a viscoelastic wave equation with weak damping. Liu and Yu [29] investigated the blow-up of the solutions for the following viscoelastic wave equation with boundary damping and source terms:

$$\begin{aligned}
&y_{tt} - \Delta y + \int_0^t g(t-s)\Delta y(s)\,ds = 0 \quad \text{in } \Omega \times (0,\infty),\\
&y = 0 \quad \text{in } \Gamma_0 \times [0,\infty),\\
&\frac{\partial y}{\partial \nu} - \int_0^t g(t-s)\frac{\partial y}{\partial \nu}(s)\,ds + |y_t|^{m-2}y_t = |y|^{p-2}y \quad \text{in } \Gamma_1 \times [0,\infty).
\end{aligned}$$

For more related works, we refer to [30–38] and the references therein.

To our best knowledge, there are no blow-up results of solution for the von Karman equation with memory. Motivated by the previous results, we consider the quasilinear von Karman equation with memory and boundary weak damping. We study a finite time blow-up result under suitable condition on the initial data.

The outline of the paper is the following. In Sect. 2, we give some notations and hypotheses for our work. In Sect. 3, we prove our main result.

## 2 Preliminary

In this section, we present some material needed in the proof of our result. Throughout this paper we denote

$$\begin{aligned}
&V = \{y \in H^1(\Omega) : y = 0 \text{ on } \Gamma_0\},\\
&W = \left\{y \in H^2(\Omega) : y = \frac{\partial y}{\partial \nu} = 0 \text{ on } \Gamma_0\right\},\\
&(y,z) = \int_\Omega y(x)z(x)\,dx, \qquad (y,z)_{\Gamma_1} = \int_{\Gamma_1} y(x)z(x)\,d\Gamma.
\end{aligned}$$

For a Banach space $X$, $\|\cdot\|_X$ denotes the norm of $X$. For simplicity, we denote $\|\cdot\|_{L^2(\Omega)}$ by the norm $\|\cdot\|$ and $\|\cdot\|_{L^2(\Gamma_1)}$ by $\|\cdot\|_{\Gamma_1}$, respectively. We define, for all $1 \le p < \infty$,

$$\|y\|_{p,\Gamma_1}^p = \int_{\Gamma_1} |y(x)|^p\,d\Gamma.$$

Let $0 < \mu < \frac{1}{2}$, we define the bilinear form $a(\cdot,\cdot)$ as follows:

$$\begin{aligned}
a(y,\kappa) = \int_\Omega \biggl\{ &\frac{\partial^2 y}{\partial x_1^2}\frac{\partial^2 \kappa}{\partial x_1^2} + \frac{\partial^2 y}{\partial x_2^2}\frac{\partial^2 \kappa}{\partial x_2^2} + \mu\left(\frac{\partial^2 y}{\partial x_1^2}\frac{\partial^2 \kappa}{\partial x_2^2} + \frac{\partial^2 y}{\partial x_2^2}\frac{\partial^2 \kappa}{\partial x_1^2}\right)\\
&+ 2(1-\mu)\frac{\partial^2 y}{\partial x_1 \partial x_2}\frac{\partial^2 \kappa}{\partial x_1 \partial x_2}\biggr\}\,dx. \qquad (2.1)
\end{aligned}$$

A simple calculation, based on the integration by parts formula, yields

$$\int_\Omega (\Delta^2 y)\kappa\,dx = a(y,\kappa) - \left(\mathcal{B}_1 y, \frac{\partial \kappa}{\partial \nu}\right)_\Gamma + (\mathcal{B}_2 y, \kappa)_\Gamma.$$

Thus, for $(y,\kappa) \in (H^4(\Omega) \cap W) \times W$, it holds

$$\int_\Omega (\Delta^2 y)\kappa \, dx = a(y,\kappa) - \left(\mathcal{B}_1 y, \frac{\partial \kappa}{\partial \nu}\right)_{\Gamma_1} + (\mathcal{B}_2 y, \kappa)_{\Gamma_1}. \tag{2.2}$$

Since $\Gamma_0 \neq \emptyset$, we have (see [39]) that $\sqrt{a(y,y)}$ is equivalent to the $H^2(\Omega)$ norm on $W$, that is,

$$C_1 \|\Delta y\|^2 \leq a(y,y) \leq C_2 \|\Delta y\|^2 \quad \text{for some } C_1, C_2 > 0. \tag{2.3}$$

Now we state the assumptions for problem (1.1)–(1.7). We will need the following assumptions.

(H1) Hypotheses on $g$.

Let $g : \mathbb{R}^+ \to \mathbb{R}^+$ be a nonincreasing $C^1$ function satisfying

$$g(0) > 0, \quad 1 - \int_0^\infty g(s)\, ds := l > 0. \tag{2.4}$$

(H2) Hypotheses on $f$.

Let $f : \mathbb{R} \to \mathbb{R}$ be a nondecreasing $C^1$ function with $f(0) = 0$. There exists an odd and strictly increasing function $\xi : [-1,1] \to \mathbb{R}$ such that

$$\left|\xi(s)\right| \leq \left|f(s)\right| \leq \left|\xi^{-1}(s)\right| \quad \text{for } |s| \leq 1, \tag{2.5}$$

$$c_1 |s|^{m-1} \leq \left|f(s)\right| \leq c_2 |s|^{m-1} \quad \text{for } |s| > 1, \tag{2.6}$$

where $c_1$ and $c_2$ are positive constants, $m > 2$, and $\xi^{-1}$ denotes the inverse function of $\xi$.

We state the well-posedness which can be established by the arguments of [11–13, 29, 40].

**Theorem 2.1** *Suppose that* (H1)–(H2) *hold and* $(y_0, y_1) \in (H^4(\Omega) \cap W) \times (H^3(\Omega) \cap V))$. *Then, for any* $T > 0$, *there exists a unique solution of problem* (1.1)–(1.7) *such that*

$$y \in C\big([0,T]; H^4(\Omega) \cap W\big) \cap C^1\big([0,T]; H^3(\Omega) \cap V\big) \cap C^2\big([0,T]; L^2(\Omega)\big).$$

A direct calculation gives

$$\begin{aligned} a\big((g * y)(t), y_t(t)\big) = &-\frac{1}{2}\frac{d}{dt}\left[\big(g \Box \partial^2 y\big)(t) - \left(\int_0^t g(s)\, ds\right) a\big(y(t), y(t)\big)\right] \\ &- \frac{1}{2} g(t) a\big(y(t), y(t)\big) + \frac{1}{2}\big(g' \Box \partial^2 y\big)(t), \end{aligned} \tag{2.7}$$

where

$$(g * y)(t) = \int_0^t g(t-s) y(s)\, ds, \big(g \Box \partial^2 y\big)(t) = \int_0^t g(t-s) a\big(y(t) - y(s), y(t) - y(s)\big)\, ds.$$

We recall the trace Sobolev embedding

$$W \hookrightarrow L^p(\Gamma_1) \quad \text{for } p \geq 2$$

and the embedding inequality

$$\|y\|_{p,\Gamma_1} \le B\|\Delta y\| \quad \text{for } y \in W, \tag{2.8}$$

where $B > 0$ is the optimal constant. We define the energy associated with problem (1.1)–(1.7) by

$$\begin{aligned} E(t) &:= E\big(y(t), z(t)\big) \\ &= \frac{1}{\rho+2}\big\|y_t(t)\big\|_{\rho+2}^{\rho+2} + \frac{\alpha}{2}\big\|\nabla y_t(t)\big\|^2 + \frac{1}{2}\left(1 - \int_0^t g(s)\,ds\right)a\big(y(t), y(t)\big) \\ &\quad + \frac{1}{2}\big(g\Box\partial^2 y\big)(t) - \frac{1}{p}\big\|y(t)\big\|_{p,\Gamma_1}^p + \frac{1}{4}\big\|\Delta z(t)\big\|^2, \end{aligned} \tag{2.9}$$

then

$$E'(t) = \frac{1}{2}\big(g'\Box\partial^2 y\big)(t) - \frac{g(t)}{2}a\big(y(t), y(t)\big) - \big(f\big(y_t(t)\big), y_t(t)\big)_{\Gamma_1} \le 0. \tag{2.10}$$

So the energy $E$ is a nonincreasing function. Next, we define the functionals

$$\begin{aligned} I(t) &:= I\big(y(t), z(t)\big) \\ &= \left(1 - \int_0^t g(s)\,ds\right)a\big(y(t), y(t)\big) + \big(g\Box\partial^2 y\big)(t) + \frac{1}{2}\big\|\Delta z(t)\big\|^2 - \big\|y(t)\big\|_{p,\Gamma_1}^p, \end{aligned} \tag{2.11}$$

$$\begin{aligned} H(t) &:= H\big(y(t), z(t)\big) \\ &= \frac{1}{2}\left[\left(1 - \int_0^t g(s)\,ds\right)a\big(y(t), y(t)\big) + \big(g\Box\partial^2 y\big)(t) + \frac{1}{2}\big\|\Delta z(t)\big\|^2\right] \\ &\quad - \frac{1}{p}\big\|y(t)\big\|_{p,\Gamma_1}^p. \end{aligned} \tag{2.12}$$

We define

$$e(t) = \inf_{(y,z)\in W\times H_0^2(\Omega), y\neq 0,\ \lambda\ge 0} \sup H(\lambda y, \lambda z), \quad t \ge 0.$$

**Lemma 2.1** *For $t \ge 0$, we get*

$$0 < e_0 \le e(t) \le \sup_{\lambda\ge 0} H(\lambda y, \lambda z),$$

*where $e_0 = \frac{p-2}{2p}(\frac{C_1 l}{B^2})^{\frac{p}{p-2}}$ and*

$$\sup_{\lambda\ge 0} H(\lambda y, \lambda z) = \frac{p-2}{2p}\left(\frac{(1 - \int_0^t g(s)\,ds)a(y(t), y(t)) + (g\Box\partial^2 y)(t) + \frac{1}{2}\|\Delta z(t)\|^2}{\|y(t)\|_{p,\Gamma_1}^2}\right)^{\frac{p}{p-2}}.$$

*Proof* We find that

$$H(\lambda y, \lambda z) = \frac{\lambda^2}{2}\left[\left(1 - \int_0^t g(s)\,ds\right)a\big(y(t), y(t)\big) + \big(g\Box\partial^2 y\big)(t) + \frac{1}{2}\big\|\Delta z(t)\big\|^2\right] - \frac{\lambda^p}{p}\big\|y(t)\big\|_{p,\Gamma_1}^p.$$

If $\frac{dH(\lambda y,\lambda z)}{d\lambda} = 0$, then we obtain

$$\lambda_1 = \left[\frac{(1-\int_0^t g(s)\,ds)a(y(t),y(t)) + (g\Box\partial^2 y)(t) + \frac{1}{2}\|\Delta z(t)\|^2}{\|y(t)\|_{p,\Gamma_1}^p}\right]^{\frac{1}{p-2}}.$$

It is easy to verify that $\frac{d^2H}{d\lambda^2}|_{\lambda=\lambda_1} < 0$, then from (2.3), (2.4), and (2.8)

$$\begin{aligned}\sup_{\lambda\geq 0} H(\lambda y,\lambda z) &= H(\lambda_1 y,\lambda_1 z)\\ &= \left(\frac{p-2}{2p}\right)\left(\frac{(1-\int_0^t g(s)\,ds)a(y(t),y(t)) + (g\Box\partial^2 y)(t) + \frac{1}{2}\|\Delta z(t)\|^2}{\|y(t)\|_{p,\Gamma_1}^2}\right)^{\frac{p}{p-2}}\\ &\geq \left(\frac{p-2}{2p}\right)\left(\frac{C_1 l\|\Delta y(t)\|^2}{\|y(t)\|_{p,\Gamma_1}^2}\right)^{\frac{p}{p-2}} \geq \left(\frac{p-2}{2p}\right)\left(\frac{C_1 l}{B^2}\right)^{\frac{p}{p-2}}.\end{aligned}$$

By the definition of $e_0$, we conclude that $e_0 > 0$. □

**Lemma 2.2** *Assume that* (H1)–(H2) *hold. Suppose that* $(y_0,y_1)\in W\times L^2(\Omega)$ *and satisfy*

$$I(0) < 0, \qquad E(0) < \epsilon e_0 \quad \text{for any } \epsilon < 1. \tag{2.13}$$

*Then, for some* $T > 0$, *we get* $I(t) < 0$ *and*

$$\begin{aligned}e_0 &< \frac{p-2}{2p}\left[\left(1-\int_0^t g(s)\,ds\right)a\bigl(y(t),y(t)\bigr) + \bigl(g\Box\partial^2 y\bigr)(t) + \frac{1}{2}\bigl\|\Delta z(t)\bigr\|^2\right]\\ &< \frac{p-2}{2p}\bigl\|y(t)\bigr\|_{p,\Gamma_1}^p\end{aligned} \tag{2.14}$$

*for all* $t\in[0,T)$.

*Proof* Using (2.10) and (2.13), we obtain $E(t) < \epsilon e_0$ for all $t\in[0,T)$. We can also have $I(t) < 0$ for all $t\in[0,T)$. It can be showed by contradiction. Suppose that there exists some $t_0 > 0$ such that $I(t_0) = 0$ and $I(t) < 0$ for $0\leq t < t_0$. Then

$$\left(1-\int_0^t g(s)\,ds\right)a\bigl(y(t),y(t)\bigr) + \bigl(g\Box\partial^2 y\bigr)(t) + \frac{1}{2}\bigl\|\Delta z(t)\bigr\|^2 < \bigl\|y(t)\bigr\|_{p,\Gamma_1}^p, \quad 0\leq t < t_0. \tag{2.15}$$

Using Lemma 2.1 and (2.15), we see that

$$\begin{aligned}e_0 &< \frac{p-2}{2p}\left\{\frac{(1-\int_0^t g(s)\,ds)a(y(t),y(t)) + (g\Box\partial^2 y)(t) + \frac{1}{2}\|\Delta z(t)\|^2}{[(1-\int_0^t g(s)\,ds)a(y(t),y(t)) + (g\Box\partial^2 y)(t) + \frac{1}{2}\|\Delta z(t)\|^2]^{\frac{2}{p}}}\right\}^{\frac{p}{p-2}}\\ &= \frac{p-2}{2p}\left[\left(1-\int_0^t g(s)\,ds\right)a\bigl(y(t),y(t)\bigr) + \bigl(g\Box\partial^2 y\bigr)(t) + \frac{1}{2}\bigl\|\Delta z(t)\bigr\|^2\right],\\ &\quad 0\leq t < t_0.\end{aligned} \tag{2.16}$$

Applying (2.15) and (2.16), we obtain

$$\bigl\|y(t)\bigr\|_{p,\Gamma_1}^p > \frac{2pe_0}{p-2} > 0, \quad 0\leq t < t_0.$$

From $t \to \|y(t)\|_{p,\Gamma_1}^p > 0$ is continuous, we have $y(t_0)|_{\Gamma_1} \neq 0$. By (2.12) and $I(t_0) = 0$, we find that

$$e_0 \le \frac{p-2}{2p}\|y(t_0)\|_{p,\Gamma_1}^p = H(t_0).$$

This is contradiction to $H(t_0) \le E(t_0) < e_0$. From Lemma 2.1, we get (2.14). □

We set

$$G(t) = \hat{\epsilon} e_0 - E(t), \tag{2.17}$$

where $\hat{\epsilon} = \max\{0, \epsilon\}$. By (2.10), $G$ is an increasing function. Using (2.9), (2.13), (2.14), and (2.17), we obtain

$$0 < G(0) \le G(t) \le \hat{\epsilon} e_0 + \frac{1}{p}\|y(t)\|_{p,\Gamma_1}^p \le p_0 \|y(t)\|_{p,\Gamma_1}^p, \quad t \in [0, T), \tag{2.18}$$

where $p_0 = \frac{\hat{\epsilon}}{2} + (1 - \hat{\epsilon})\frac{1}{p}$.

**Lemma 2.3** *Let the conditions of Lemma 2.2 hold. Then the solution y of problem (1.1)–(1.7) satisfies*

$$\|y(t)\|_{p,\Gamma_1}^s \le C_3 \|y(t)\|_{p,\Gamma_1}^p, \quad t \in [0, T), \text{ for any } 2 \le s \le p, \tag{2.19}$$

*where* $C_3 > 0$.

*Proof* If $\|y(t)\|_{p,\Gamma_1} \ge 1$, then $\|y(t)\|_{p,\Gamma_1}^s \le \|y(t)\|_{p,\Gamma_1}^p$.
If $\|y(t)\|_{p,\Gamma_1} \le 1$, then

$$\|y(t)\|_{p,\Gamma_1}^s \le \|y(t)\|_{p,\Gamma_1}^2 \le B^2 \|\Delta y(t)\|^2 \le \frac{B^2}{C_1} a\big(y(t), y(t)\big),$$

where we used (2.3) and (2.8). Then there exists a positive constant $C_4 = \max\{1, \frac{B^2}{C_1}\}$ such that

$$\|y(t)\|_{p,\Gamma_1}^s \le C_4 \big(\|y(t)\|_{p,\Gamma_1}^p + a\big(y(t), y(t)\big)\big) \quad \text{for any } 2 \le s \le p. \tag{2.20}$$

By (2.4), (2.9), (2.17), and (2.18),

$$\begin{aligned} &\frac{l}{2} a\big(y(t), y(t)\big) \\ &\quad \le \hat{\epsilon} e_0 - G(t) - \frac{1}{\rho+2}\|y_t(t)\|_{\rho+2}^{\rho+2} - \frac{\alpha}{2}\|\nabla y_t(t)\|^2 \\ &\qquad - \frac{1}{2}(g \Box \partial^2 y)(t) + \frac{1}{p}\|y(t)\|_{p,\Gamma_1}^p - \frac{1}{4}\|\Delta z(t)\|^2 \\ &\quad \le \hat{\epsilon} e_0 + \frac{1}{p}\|y(t)\|_{p,\Gamma_1}^p \le p_0 \|y(t)\|_{p,\Gamma_1}^p. \end{aligned} \tag{2.21}$$

Using (2.20) and (2.21), we get the desired result (2.19). □

## 3 A blow-up of solution

To obtain the blow-up result for solutions with nonpositive initial energy as well as positive initial energy, we use a similar method of [26, 29].

**Theorem 3.1** *Let* (H1)–(H2) *and the conditions of Lemma* 2.2 *hold*, $\epsilon < \frac{p-4}{p-2}$ *and* $p > \max\{4, m\}$. *Moreover, we assume that* $g$ *satisfies*

$$\int_0^\infty g(s)\,ds < \frac{p-2}{p-2+\frac{1}{[(1-\hat{\epsilon})^2(p-2)+2(1-\hat{\epsilon})]}}, \tag{3.1}$$

*where* $\hat{\epsilon} = \max\{0, \epsilon\}$ *and*

$$\xi^{-1}(1) < \left(\frac{p\beta\eta\hat{\epsilon}e_0}{(p-1)|\Gamma_1|}\right)^{\frac{p-1}{p}}, \tag{3.2}$$

*where* $0 < \eta < \min\{2\theta_0, 2\theta_1, 4\theta_2\}$, $0 < \beta < \eta^{\frac{1}{p-1}}$, *for some* $\delta > 0$,

$$\theta_0 = \left(\frac{p}{2}-1\right)(1-\hat{\epsilon}) - \left\{\left(\frac{p}{2}-1\right)(1-\hat{\epsilon}) + \frac{1}{4\delta}\right\}\int_0^t g(s)\,ds > 0, \tag{3.3}$$

$$\theta_1 = \left(\frac{p}{2}-1\right)(1-\hat{\epsilon}) + (1-\delta) > 0, \tag{3.4}$$

$$\theta_2 = \left(\frac{p}{4}-1\right) - \hat{\epsilon}\left(\frac{p}{4}-\frac{1}{2}\right) > 0. \tag{3.5}$$

*Then the solution of system* (1.1)–(1.7) *blows up in finite time*.

*Proof* We suppose that there exists some positive constant $B_0$ such that, for $t > 0$, the solution $y(t)$ of (1.1)–(1.7) satisfies

$$\|y_t(t)\|_{\rho+2}^{\rho+2} + \|\nabla y_t(t)\|^2 + \|\Delta y(t)\|^2 + \|y(t)\|_{p,\Gamma_1}^p \le B_0. \tag{3.6}$$

Let us define

$$F(t) = G^{1-\sigma}(t) + \frac{\varepsilon}{\rho+1}\int_\Omega |y_t(t)|^\rho y_t(t)y(t)\,dx + \alpha\varepsilon\int_\Omega \nabla y_t(t)\nabla y(t)\,dx, \tag{3.7}$$

where $\varepsilon > 0$ shall be taken later and

$$0 < \sigma < \min\left\{\frac{1}{\rho+2}, \frac{p-m}{p(m-1)}\right\}. \tag{3.8}$$

Using (1.1)–(1.6), (2.2), (2.9), and (2.17), we get

$$\begin{aligned}
F'(t) &= (1-\sigma)G^{-\sigma}(t)G'(t) + \frac{\varepsilon}{\rho+1}\|y_t(t)\|_{\rho+2}^{\rho+2} + \alpha\varepsilon\|\nabla y_t(t)\|^2 - \varepsilon\|\Delta z(t)\|^2 \\
&\quad - \varepsilon a\big(y(t), y(t)\big) \\
&\quad + \varepsilon a\big((g*y)(t), y(t)\big) - \varepsilon\big(f\big(y_t(t)\big), y(t)\big)_{\Gamma_1} + \varepsilon\|y(t)\|_{p,\Gamma_1}^p + \varepsilon pE(t) - \varepsilon pE(t) \\
&= (1-\sigma)G^{-\sigma}(t)G'(t) + \frac{\varepsilon}{\rho+1}\|y_t(t)\|_{\rho+2}^{\rho+2} + \alpha\varepsilon\|\nabla y_t(t)\|^2 - \varepsilon\|\Delta z(t)\|^2
\end{aligned}$$

$$\begin{aligned}&-\varepsilon a\big(y(t),y(t)\big)\\&+\varepsilon a\big((g*y)(t),y(t)\big)-\varepsilon\big(f\big(y_t(t)\big),y(t)\big)_{\Gamma_1}+\varepsilon p\big(G(t)-\hat{\epsilon}e_0\big)+\frac{\varepsilon p}{\rho+2}\|y_t(t)\|_{\rho+2}^{\rho+2}\\&+\varepsilon p\bigg(\frac{\alpha}{2}\|\nabla y_t(t)\|^2+\frac{1}{2}\bigg(1-\int_0^t g(s)\,ds\bigg)a\big(y(t),y(t)\big)\\&+\frac{1}{2}\big(g\Box\partial^2 y\big)(t)+\frac{1}{4}\|\Delta z(t)\|^2\bigg).\end{aligned}\tag{3.9}$$

From (2.14), we find that

$$-\hat{\epsilon}e_0>\hat{\epsilon}\bigg(\frac{1}{p}-\frac{1}{2}\bigg)\bigg(\bigg(1-\int_0^t g(s)\,ds\bigg)a\big(y(t),y(t)\big)+\big(g\Box\partial^2 y\big)(t)+\frac{1}{2}\|\Delta z(t)\|^2\bigg).\tag{3.10}$$

Moreover, we give

$$\begin{aligned}a\big((g*y)(t),y(t)\big)&=\int_0^t g(t-s)a\big(y(s)-y(t),y(t)\big)\,ds+\bigg(\int_0^t g(s)\,ds\bigg)a\big(y(t),y(t)\big)\\&\geq\bigg(1-\frac{1}{4\delta}\bigg)\bigg(\int_0^t g(s)\,ds\bigg)a\big(y(t),y(t)\big)-\delta\big(g\Box\partial^2 y\big)(t),\end{aligned}\tag{3.11}$$

for some $\delta>0$. Combining (3.9), (3.10), and (3.11), we deduce that

$$\begin{aligned}F'(t)&\geq(1-\sigma)G^{-\sigma}(t)G'(t)+\varepsilon\bigg(\frac{1}{\rho+1}+\frac{p}{\rho+2}\bigg)\|y_t(t)\|_{\rho+2}^{\rho+2}+\varepsilon\alpha\bigg(1+\frac{p}{2}\bigg)\|\nabla y_t(t)\|^2\\&+\varepsilon\bigg[\bigg(\frac{p}{2}-1\bigg)(1-\hat{\epsilon})-\bigg\{\bigg(\frac{p}{2}-1\bigg)(1-\hat{\epsilon})+\frac{1}{4\delta}\bigg\}\int_0^t g(s)\,ds\bigg]a\big(y(t),y(t)\big)\\&+\varepsilon\bigg[\bigg(\frac{p}{2}-1\bigg)(1-\hat{\epsilon})+(1-\delta)\bigg]\big(g\Box\partial^2 y\big)(t)\\&+\varepsilon\bigg[\bigg(\frac{p}{4}-1\bigg)-\hat{\epsilon}\bigg(\frac{p}{4}-\frac{1}{2}\bigg)\bigg]\|\Delta z(t)\|^2\\&+\varepsilon pG(t)-\varepsilon\big(f\big(y_t(t)\big),y(t)\big)_{\Gamma_1}\end{aligned}\tag{3.12}$$

for some $\delta$ with $0<\delta<1+(\frac{p}{2}-1)(1-\hat{\epsilon})$. By (3.1), (3.3)–(3.5), estimate (3.12) can be rewritten by

$$\begin{aligned}F'(t)&\geq(1-\sigma)G^{-\sigma}(t)G'(t)+\varepsilon\bigg(\frac{1}{\rho+1}+\frac{p}{\rho+2}\bigg)\|y_t(t)\|_{\rho+2}^{\rho+2}+\varepsilon\alpha\bigg(1+\frac{p}{2}\bigg)\|\nabla y_t(t)\|^2\\&+\varepsilon\theta_0 a\big(y(t),y(t)\big)+\varepsilon\theta_1\big(g\Box\partial^2 y\big)(t)+\varepsilon\theta_2\|\Delta z(t)\|^2\\&+\varepsilon pG(t)-\varepsilon\big(f\big(y_t(t)\big),y(t)\big)_{\Gamma_1}.\end{aligned}\tag{3.13}$$

Using a method similar to [30], we now estimate the last term of the right-hand side of (3.13). Setting $\Gamma_{11}=\{x\in\Gamma_1:|y_t(x,t)|\leq1\}$ and $\Gamma_{12}=\{x\in\Gamma_1:|y_t(x,t)|>1\}$, we obtain

$$\big(f\big(y_t(t)\big),y(t)\big)_{\Gamma_1}\leq\int_{\Gamma_{11}}\big|f\big(y_t(x,t)\big)\big|\big|y(x,t)\big|\,d\Gamma+\int_{\Gamma_{12}}\big|f\big(y_t(x,t)\big)\big|\big|y(x,t)\big|\,d\Gamma.\tag{3.14}$$

From (2.5) and Young's inequality, we get

$$\begin{aligned}
&\int_{\Gamma_{11}} \left|f\left(y_t(x,t)\right)\right|\left|y(x,t)\right| \, d\Gamma \\
&\quad \le \left(\int_{\Gamma_{11}} \left|\xi^{-1}(1)\right|^{\frac{p}{p-1}} \, d\Gamma\right)^{\frac{p-1}{p}} \left(\int_{\Gamma_{11}} \left|y(x,t)\right|^p \, d\Gamma\right)^{\frac{1}{p}} \\
&\quad \le \frac{\beta^{p-1}}{p} \left\|y(t)\right\|_{p,\Gamma_1}^p + \frac{(p-1)|\Gamma_1|}{p\beta} \left(\xi^{-1}(1)\right)^{\frac{p}{p-1}}, \quad \beta > 0.
\end{aligned} \tag{3.15}$$

On the other hand, by using (2.6), (2.10), (2.17), and Young's inequality, we have

$$\begin{aligned}
&\int_{\Gamma_{12}} \left|f\left(y_t(x,t)\right)\right|\left|y(x,t)\right| \, d\Gamma \\
&\quad \le c_2 \left(\int_{\Gamma_{12}} \left|y_t(x,t)\right|^m \, d\Gamma\right)^{\frac{m-1}{m}} \left(\int_{\Gamma_{12}} \left|y(x,t)\right|^m \, d\Gamma\right)^{\frac{1}{m}} \\
&\quad \le c_2 \left(\frac{1}{c_1} \int_{\Gamma_{12}} f\left(y_t(x,t)\right) y_t(x,t) \, d\Gamma\right)^{\frac{m-1}{m}} \left(\int_{\Gamma_{12}} \left|y(x,t)\right|^m \, d\Gamma\right)^{\frac{1}{m}} \\
&\quad \le \frac{c_2^m \gamma^m}{m} \left\|y(t)\right\|_{p,\Gamma_1}^m + \frac{m-1}{c_1 m \gamma^{\frac{m}{m-1}}} G'(t), \quad \gamma > 0.
\end{aligned} \tag{3.16}$$

Inserting (3.14)–(3.16) into (3.13), we obtain

$$\begin{aligned}
F'(t) \ge{}& \left[(1-\sigma)G^{-\sigma}(t) - \frac{\varepsilon(m-1)}{c_1 m \gamma^{\frac{m}{m-1}}}\right] G'(t) + \varepsilon\left(\frac{1}{\rho+1} + \frac{p}{\rho+2}\right) \left\|y_t(t)\right\|_{\rho+2}^{\rho+2} \\
&+ \varepsilon\alpha\left(1 + \frac{p}{2}\right) \left\|\nabla y_t(t)\right\|^2 + \varepsilon\theta_0 a\left(y(t), y(t)\right) \\
&+ \varepsilon\theta_1 \left(g \Box \partial^2 y\right)(t) + \varepsilon\theta_2 \left\|\Delta z(t)\right\|^2 + \varepsilon p G(t) \\
&- \frac{\varepsilon\beta^{p-1}}{p} \left\|y(t)\right\|_{p,\Gamma_1}^p - \frac{\varepsilon c_2^m \gamma^m}{m} \left\|y(t)\right\|_{p,\Gamma_1}^m - \frac{\varepsilon(p-1)|\Gamma_1|}{p\beta} \left(\xi^{-1}(1)\right)^{\frac{p}{p-1}}.
\end{aligned} \tag{3.17}$$

We choose $\gamma = (\tau G^{-\sigma}(t))^{-\frac{m-1}{m}}$, $\tau > 0$ will be specified later. Using (2.18), (2.19), and (3.8), we see that

$$\begin{aligned}
-\frac{\varepsilon c_2^m \gamma^m}{m} \left\|y(t)\right\|_{p,\Gamma_1}^m &= -\frac{\varepsilon c_2^m \tau^{1-m}}{m} G^{\sigma(m-1)}(t) \left\|y(t)\right\|_{p,\Gamma_1}^m \\
&\ge -\frac{\varepsilon c_2^m \tau^{1-m}}{m} p_0^{\sigma(m-1)} \left\|y(t)\right\|_{p,\Gamma_1}^{\sigma p(m-1)+m} \ge -\varepsilon C_5 \tau^{1-m} \left\|y(t)\right\|_{p,\Gamma_1}^p,
\end{aligned} \tag{3.18}$$

where $C_5 = \frac{c_2^m p_0^{\sigma(m-1)} C_3}{m}$. Substituting (3.18) into (3.17), we have

$$\begin{aligned}
F'(t) \ge{}& \left[(1-\sigma) - \frac{\varepsilon\tau(m-1)}{c_1 m}\right] G^{-\sigma}(t) G'(t) + \varepsilon\left(\frac{1}{\rho+1} + \frac{p}{\rho+2}\right) \left\|y_t(t)\right\|_{\rho+2}^{\rho+2} \\
&+ \varepsilon\alpha\left(1 + \frac{p}{2}\right) \left\|\nabla y_t(t)\right\|^2 + \varepsilon\theta_0 a\left(y(t), y(t)\right) \\
&+ \varepsilon\theta_1 \left(g \Box \partial^2 y\right)(t) + \varepsilon\theta_2 \left\|\Delta z(t)\right\|^2 + \varepsilon p G(t)
\end{aligned}$$

$$- \varepsilon\left(\frac{\beta^{p-1}}{p} + C_5\tau^{1-m}\right)\|y(t)\|_{p,\Gamma_1}^p - \frac{\varepsilon(p-1)|\Gamma_1|}{p\beta}\left(\xi^{-1}(1)\right)^{\frac{p}{p-1}}. \tag{3.19}$$

Adding and subtracting $\varepsilon\eta G(t)$ on the right-hand side of (3.19) and applying (2.9) and (2.17), we obtain

$$\begin{aligned} F'(t) \geq{}& \left[(1-\sigma) - \frac{\varepsilon\tau(m-1)}{c_1 m}\right]G^{-\sigma}(t)G'(t) + \varepsilon\left(\frac{1}{\rho+1} + \frac{p}{\rho+2} - \frac{\eta}{\rho+2}\right)\|y_t(t)\|_{\rho+2}^{\rho+2} \\ &+ \varepsilon\alpha\left(1 + \frac{p}{2} - \frac{\eta}{2}\right)\|\nabla y_t(t)\|^2 + \varepsilon(p-\eta)G(t) \\ &+ \varepsilon\left\{\theta_0 - \frac{\eta}{2}\left(1 - \int_0^t g(s)\,ds\right)\right\}a\big(y(t),y(t)\big) \\ &+ \varepsilon\left(\theta_1 - \frac{\eta}{2}\right)\left(g\Box\partial^2 y\right)(t) + \varepsilon\left(\theta_2 - \frac{\eta}{4}\right)\|\Delta z(t)\|^2 \\ &+ \varepsilon\left(\frac{\eta}{p} - \frac{\beta^{p-1}}{p} - C_5\tau^{1-m}\right)\|y(t)\|_{p,\Gamma_1}^p \\ &+ \varepsilon\eta e_0\hat{\epsilon} - \frac{\varepsilon(p-1)|\Gamma_1|}{p\beta}\left(\xi^{-1}(1)\right)^{\frac{p}{p-1}}. \end{aligned} \tag{3.20}$$

We fix $\eta$ such that

$$0 < \eta < \min\{2\theta_0, 2\theta_1, 4\theta_2\}, \tag{3.21}$$

then we can choose $\beta > 0$ sufficiently small so that $\eta - \beta^{p-1} > 0$. And then, we select $\tau > 0$ large enough such that $\frac{\eta}{p} - \frac{\beta^{p-1}}{p} - C_5\tau^{1-m} > 0$. Finally, we take $\varepsilon > 0$ with

$$(1-\sigma) - \frac{\varepsilon\tau(m-1)}{c_1 m} > 0, \qquad G^{1-\sigma}(0) + \frac{\varepsilon}{\rho+1}\int_\Omega |y_1|^\rho y_1 y_0\,dx + \alpha\varepsilon\int_\Omega \nabla y_1\nabla y_0\,dx > 0.$$

Condition (3.2) yields

$$\eta e_0\hat{\epsilon} - \frac{(p-1)|\Gamma_1|}{p\beta}\left(\xi^{-1}(1)\right)^{\frac{p}{p-1}} > 0.$$

Therefore, we get from (2.3) and (3.20)

$$F'(t) \geq C\left(\|y_t(t)\|_{\rho+2}^{\rho+2} + \|\nabla y_t(t)\|^2 + \|\Delta y(t)\|^2 + \|y(t)\|_{p,\Gamma_1}^p + G(t)\right), \tag{3.22}$$

where $C > 0$ is a generic constant. Hence we have

$$F(t) \geq F(0) > 0, \quad \forall t \geq 0.$$

By the similar arguments in [31, 32], we see that

$$F^{\frac{1}{1-\sigma}}(t) \leq C\left(\|y_t(t)\|_{\rho+2}^{\rho+2} + \|\nabla y_t(t)\|^2 + \|\Delta y(t)\|^2 + \|y(t)\|_{p,\Gamma_1}^p\right). \tag{3.23}$$

Indeed, using Young's inequality and

$$\left|\int_\Omega |y_t(t)|^\rho y_t(t)y(t)\,dx\right| \leq \|y_t(t)\|_{\rho+2}^{\rho+1}\|y(t)\|_{\rho+2},$$

we obtain

$$\left|\int_\Omega |y_t(t)|^\rho y_t(t)y(t)\,dx\right|^{\frac{1}{1-\sigma}} \le \left(\|y_t(t)\|_{\rho+2}^{\rho+1}\|y(t)\|_{\rho+2}\right)^{\frac{1}{1-\sigma}}$$
$$\le C\left(\|y_t(t)\|_{\rho+2}^{\frac{(\rho+1)\kappa}{1-\sigma}} + \|y(t)\|_{\rho+2}^{\frac{\mu}{1-\sigma}}\right), \tag{3.24}$$

where $\frac{1}{\kappa} + \frac{1}{\mu} = 1$. By taking $\kappa = \frac{(1-\sigma)(\rho+2)}{\rho+1}$ and using (3.8), we get $\kappa > 1$ and $\frac{\mu}{1-\sigma} = \frac{\rho+2}{(1-\sigma)(\rho+2)-(\rho+1)}$. Since $G$ is an increasing function, (2.18) and (3.6), we arrive at

$$\|y(t)\|_{\rho+2}^{\frac{\mu}{1-\sigma}} \le C_0^{\frac{\mu}{1-\sigma}}\|\Delta y(t)\|^{\frac{\mu}{1-\sigma}} \le \frac{(C_0^2B_0)^{\frac{\mu}{2(1-\sigma)}}}{G(0)}G(t) \le C\|y(t)\|_{p,\Gamma_1}^p, \tag{3.25}$$

where $C_0$ is the embedding constant. Similarly, by Young's inequality, we obtain

$$\left|\int_\Omega \nabla y_t(t)\nabla y(t)\,dx\right|^{\frac{1}{1-\sigma}} \le \|\nabla y_t(t)\|^{\frac{1}{1-\sigma}}\|\nabla y(t)\|^{\frac{1}{1-\sigma}}$$
$$\le C\left(\|\nabla y_t(t)\|^2 + \|\nabla y(t)\|^{\frac{2}{1-2\sigma}}\right). \tag{3.26}$$

Like (3.25), we find that

$$\|\nabla y(t)\|^{\frac{2}{1-2\sigma}} \le C_*^{\frac{2}{1-2\sigma}}\|\Delta y(t)\|^{\frac{2}{1-2\sigma}} \le \frac{(C_*^2B_0)^{\frac{1}{1-2\sigma}}}{G(0)}G(t) \le C\|y(t)\|_{p,\Gamma_1}^p, \tag{3.27}$$

where $C_*$ is the embedding constant. From (2.18), (3.7), (3.24)–(3.27), we see that (3.23) holds. Combining (3.22) and (3.23), we deduce that

$$F'(t) \ge CF^{\frac{1}{1-\sigma}}(t) \quad \text{for } t \ge 0. \tag{3.28}$$

By a simple integration of (3.28) over $(0,t)$, we get

$$F^{\frac{\sigma}{1-\sigma}}(t) \ge \frac{1}{F^{-\frac{\sigma}{1-\sigma}}(0) - \frac{C\sigma t}{1-\sigma}} \quad \text{for } t \ge 0.$$

Consequently, $F(t)$ blows up in time $T^* \le \frac{1-\sigma}{C\sigma F^{\frac{\sigma}{1-\sigma}}(0)}$. Furthermore, we have from (3.23)

$$\lim_{t\to T^{*-}}\left(\|y_t(t)\|_{\rho+2}^{\rho+2} + \|\nabla y_t(t)\|^2 + \|\Delta y(t)\|^2 + \|y(t)\|_{p,\Gamma_1}^p\right) = \infty.$$

This leads to a contradiction with (3.6). Therefore the solution of (1.1)–(1.7) blows up in finite time. □

## 4 Conclusions

In this paper, we consider the blow-up of solutions for the quasilinear von Karman equation of memory type. In recent years, there has been published much work concerning the wave equation with nonlinear boundary damping. But as far as we know, there was no blow-up result of solutions to the viscoelastic von Karman equation with nonlinear boundary damping and source terms. Therefore, we will prove a finite time blow-up result of solution with positive initial energy as well as non-positive initial energy. Moreover, we generalize the earlier result under a weaker assumption on the damping term.

**Acknowledgements**
The authors are thankful to the honorable reviewers and editors for their valuable comments and suggestions, which improved the paper.

**Funding**
Research of MJL was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT and Future Planning (2017R1E1A1A03070738). Research of J-RK was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2017R1D1A1B03028291).

**Abbreviations**
Not applicable.

**Availability of data and materials**
Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

**Competing interests**
The authors declare that they have no competing interests.

**Authors' contributions**
The authors declare that the study was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

**Author details**
[1]Department of Mathematics, Pusan National University, Busan, South Korea. [2]Department of Applied Mathematics, Pukyong National University, Busan, South Korea.

## Publisher's Note



Received: 31 January 2019 Accepted: 9 October 2019 Published online: 06 November 2019

**References**

1. Bradley, M.E., Lasiecka, I.: Global decay rates for the solutions to a von Karman plate without geometric conditions. J. Math. Anal. Appl. **181**, 254–276 (1994)
2. Chueshov, I., Lasiecka, I.: Von Karman Evolution Equations: Well-Posedness and Long-Time Dynamics. Springer, New York (2010)
3. Khanmamedov, A.K.: Global attractors for von Karman equations with nonlinear interior dissipation. J. Math. Anal. Appl. **318**, 92–101 (2006)
4. Lasiecka, I.: Finite dimensionality and compactness of attractors for von Karman equations with nonlinear dissipation. NoDEA Nonlinear Differ. Equ. Appl. **6**, 437–472 (1999)
5. Puel, J., Tucsnak, M.: Boundary stabilization for the von Karman equations. SIAM J. Control Optim. **33**, 255–273 (1996)
6. Chueshov, I., Lasiecka, I.: Long-time dynamics of von Karman semi-flows with non-linear boundary/interior damping. J. Differ. Equ. **233**, 42–86 (2007)
7. Favini, A., Horn, M., Lasiecka, I., Tataru, D.: Global existence, uniqueness and regularity of solutions to a von Karman system with nonlinear boundary dissipation. Differ. Integral Equ. **9**(2), 267–294 (1996)
8. Horn, M.A., Lasiecka, I.: Global stabilization of a dynamic von Karman plate with nonlinear boundary feedback. Appl. Math. Optim. **31**, 57–84 (1995)
9. Kang, J.R.: Energy decay rates for von Karman system with memory and boundary feedback. Appl. Math. Comput. **218**, 9085–9094 (2012)
10. Kang, J.R.: A general stability for a von Karman system with memory. Bound. Value Probl. **2015**, 204 (2015)
11. Raposo, C.A., Santos, M.L.: General decay to a von Karman systems with memory. Nonlinear Anal. **74**, 937–945 (2011)
12. Rivera, J.E.M., Menzala, G.P.: Decay rates of solutions of a von Karman system for viscoelastic plates with memory. Q. Appl. Math. **LVII**(1), 181–200 (1999)
13. Rivera, J.E.M., Oquendo, H.P., Santos, M.L.: Asymptotic behavior to a von Karman plate with boundary memory conditions. Nonlinear Anal. **62**, 1183–1205 (2005)
14. Santos, M.L., Soufyane, A.: General decay to a von Karman plate system with memory boundary conditions. Differ. Integral Equ. **24**(1–2), 69–81 (2011)
15. Kang, J.R.: General stability for a von Karman plate system with memory boundary conditions. Bound. Value Probl. **2015**, 167 (2015)
16. Park, J.Y., Park, S.H., Kang, Y.H.: General decay for a von Karman equation of memory type with acoustic boundary conditions. Z. Angew. Math. Phys. **63**, 813–823 (2012)
17. Kang, J.R.: Asymptotic behavior to a von Karman equations of memory type with acoustic boundary conditions. Z. Angew. Math. Phys. **67**, 48 (2016)

18. Park, S.H.: Energy decay for a von Karman equation with time-varying delay. Appl. Math. Lett. **55**, 10–17 (2016)
19. Kim, D.W., Park, J.Y., Kang, Y.H.: Energy decay rate for a von Karman system with a boundary nonlinear delay term. Comput. Math. Appl. **75**, 3269–3282 (2018)
20. Ball, J.: Remarks on blow up and nonexistence theorems for nonlinear evolutions equations. Quart. J. Math. Oxford **28**, 473–486 (1977)
21. Levine, H.A.: Instability and nonexistence of global solutions of nonlinear wave equation of the form $Pu_{tt} = Au + F(u)$. Trans. Am. Math. Soc. **192**, 1–21 (1974)
22. Levine, H.A.: Some additional remarks on the nonexistence of global solutions to nonlinear wave equation. SIAM J. Math. Anal. **5**, 138–146 (1974)
23. Georgiev, V., Todorova, G.: Existence of solutions of the wave equation with nonlinear damping and source terms. J. Differ. Equ. **109**, 295–308 (1994)
24. Messaoudi, S.A.: Blow up in a nonlinearly damped wave equation. Math. Nachr. **231**, 1–7 (2001)
25. Messaoudi, S.A.: Blow up and global existence in a nonlinear viscoelastic wave equation. Math. Nachr. **260**, 58–66 (2003)
26. Messaoudi, S.A.: Blow-up of positive-initial-energy solutions of a nonlinear viscoelastic hyperbolic equation. J. Math. Anal. Appl. **320**, 902–915 (2006)
27. Song, H.: Blow up of arbitrarily positive initial energy solutions for a viscoelastic wave equation. Nonlinear Anal., Real World Appl. **26**, 306–314 (2015)
28. Park, S.H., Lee, M.J., Kang, J.R.: Blow-up results for viscoelastic wave equations with weak damping. Appl. Math. Lett. **80**, 20–26 (2018)
29. Liu, W.J., Yu, J.: On decay and blow-up of the solution for a viscoelastic wave equation with boundary damping and source terms. Nonlinear Anal. **74**, 2175–2190 (2011)
30. Ha, T.G.: Blow-up for wave equation with weak boundary damping and source terms. Appl. Math. Lett. **49**, 166–172 (2015)
31. Hao, J., Wei, H.: Blow-up and global existence for solution of quasilinear viscoelastic wave equation with strong damping and source term. Bound. Value Probl. **2017**, 65 (2017)
32. Liu, W.J.: General decay and blow-up of solution for a quasilinear viscoelastic problem with nonlinear source. Nonlinear Anal. **73**, 1890–1904 (2010)
33. Liu, W.J., Sun, Y., Li, G.: On decay and blow-up of solutions for a singular nonlocal viscoelastic problem with a nonlinear source term. Topol. Methods Nonlinear Anal. **49**(1), 299–323 (2017)
34. Liu, W.J., Zhuang, H.F.: Global existence, asymptotic behavior and blow-up of solutions for a suspension bridge equation with nonlinear damping and source terms. Nonlinear Differ. Equ. Appl. **24**, 67 (2017)
35. Messaoudi, S.A., Bonfoh, A., Mukiawa, S., Enyi, C.: The global attractor for a suspension bridge with memory and partially hinged boundary conditions. Z. Angew. Math. Mech. **97**(2), 159–172 (2017)
36. Song, H.: Global nonexistence of positive initial energy solutions for a viscoelastic wave equation. Nonlinear Anal. **125**, 260–269 (2015)
37. Song, H., Zhong, C.: Blow-up of solutions of a nonlinear viscoelastic wave equation. Nonlinear Anal., Real World Appl. **11**, 3877–3883 (2010)
38. Vitillaro, E.: Global existence for the wave equation with nonlinear boundary damping and source terms. J. Differ. Equ. **186**, 259–298 (2002)
39. Chueshov, I., Lasiecka, J.: Global attractors for von Karman evolutions with nonlinear boundary dissipation. J. Differ. Equ. **198**, 196–231 (2004)
40. Ha, T.G.: Asymptotic stability of the semilinear wave equation with boundary damping and source term. C.R. Acad. Sci. Paris **352**, 213–218 (2014)