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Shape optimization of turbine blade cooling system using topological sensitivity analysis method

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Abstract

The topological sensitivity analysis method gives the variation of a criterion with respect to the creation of a small hole in the domain. In this paper, we use this method to solve an inverse problem related to the turbine blade cooling. The aim is to optimize the hole characteristics created in the blade vane in order to improve the behavior of the cooling system. A topological optimization algorithm is proposed and some numerical results, showing the efficiency of our approach, are presented.

Keywords: Geometric inverse problem; Topological sensitivity analysis; Heat equation

1 Introduction

Modern gas turbine engines are designed to operate at high temperature (1200–1400°C) to improve thermal efficiency and power output. The temperature variation within the blade material (which causes thermal stresses) must be limited in order to ensure a reasonable lifetime. Therefore, there is a need for an efficient cooling system. Discrete jet film is one of the techniques most used for protecting the blades. This process is achieved by injecting cooler air through discrete holes created in the blade vane (see Fig. 1).

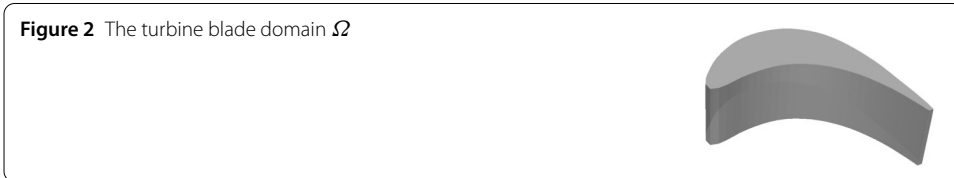
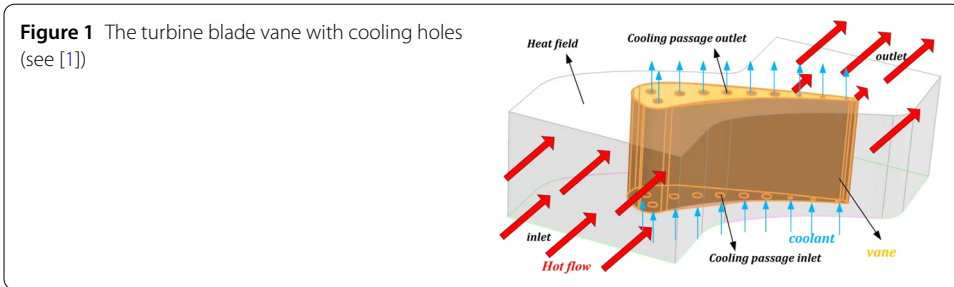
In order to improve the behavior of this cooling system, a considerable effort has been devoted to optimize size, location, and shape of the created holes. Various approaches and techniques have been developed and exploited to address this issue [1–3].

In this paper, we suggest a mathematical approach based on the topological sensitivity analysis method [4–11]. The main idea is to compute the asymptotic topological expansion with respect to the insertion of a small cooling hole.

This technique has been successfully used in different applications: identification of gas bubbles created during the mould filling process [12], optimization of injectors locations in water reservoirs [13], geometric control problem for fluid flow [14], etc.

However, most contributions have been limited to the steady state regime. In this paper, we exploit this idea for solving a time-dependent topological optimization problem associated with a parabolic partial differential equation.

The paper is organized as follows. We begin by the problem formulation in Sect. 2. The topological sensitivity analysis method is introduced in Sect. 3. The variation of the cost function with respect to a small topological perturbation of the domain is computed and



an asymptotic expansion is presented. Based on the obtained theoretical results, we propose in Sect. 4 a simple and efficient reconstruction procedure that we use for some numerical investigations.

2 Formulation of the problem

Let $\Omega \subset \mathbb{R}^3$ be the domain describing the turbine vane (see Fig. 2). We denote by $\{\mathcal{H}_k^*, k = 1, \dots, m\}$ the cooling holes to be created in Ω . The size, location, and shape of the unknown holes \mathcal{H}_k^* can be formulated as solution to the following topology optimization problem:

$$(P) \quad \min_{\mathcal{H} \subset \Omega} j(\Omega \setminus \overline{\mathcal{H}}).$$

Here j is a shape function measuring the temperature gradient, defined by

$$j(\Omega \setminus \overline{\mathcal{H}}) = \int_0^T \int_{\Omega \setminus \overline{\mathcal{H}}} |\nabla \theta_{\mathcal{H}}|^2 dx dt,$$

where $\theta_{\mathcal{H}}$ is the temperature field solution to the following heat transfer problem:

$$\begin{cases} \frac{\partial \theta_{\mathcal{H}}}{\partial t} - \Delta \theta_{\mathcal{H}} = Q & \text{in } \Omega \setminus \overline{\mathcal{H}} \times (0, T), \\ \nabla \theta_{\mathcal{H}} \cdot n = g_N & \text{on } \partial \Omega \times (0, T), \\ \theta_{\mathcal{H}} = 0 & \text{on } \partial \mathcal{H} \times (0, T), \\ \theta_{\mathcal{H}}(\cdot, 0) = \varphi & \text{in } \Omega \setminus \overline{\mathcal{H}}, \end{cases}$$

with Q being a given source term, g_N a heat flux, and φ the initial temperature distribution.

To solve this optimization problem, we propose in this work a fast and accurate algorithm based on the topological sensitivity analysis method. It consists in studying the variation of function j with respect to the creation of a small hole $\mathcal{H}_{z,\varepsilon} = z + \varepsilon \mathcal{H}$ ($\varepsilon \in]0, 1[$ and $\mathcal{H} \subset \mathbb{R}^3$) inside the domain Ω . The concept of topological sensitivity allows finding the place where the creation of a small cooling hole would bring the best possible improvement to the performance of the turbine.

3 Topological sensitivity analysis

We present in this section a topological sensitivity expansion of the design function j with respect to the insertion of a small cooling hole $\mathcal{H}_{z,\varepsilon}$ inside the domain Ω . We consider

- θ_0 to be the solution to the heat transfer problem in the nonperturbed domain Ω

$$\begin{cases} \frac{\partial \theta_0}{\partial t} - \Delta \theta_0 = Q & \text{in } \Omega \times (0, T), \\ \nabla \theta_0 \cdot n = g_N & \text{on } \partial \Omega \times (0, T), \\ \theta_0(\cdot, 0) = \varphi & \text{in } \Omega. \end{cases} \tag{1}$$

- v_0 to be the solution to the associated adjoint problem

$$\begin{cases} -\frac{\partial v_0}{\partial t} - \Delta v_0 = -DJ_0(\theta_0) & \text{in } \Omega \times (0, T), \\ \nabla v_0 \cdot n = 0 & \text{on } \partial \Omega \times (0, T), \\ v_0(\cdot, T) = \varphi & \text{in } \Omega, \end{cases} \tag{2}$$

where J_0 is a cost function defined on $H^1(\Omega)$ by

$$J_0(\theta) = \int_{\Omega} |\nabla \theta|^2 dx, \quad \forall \theta \in H^1(\Omega).$$

The shape function j satisfies the following theorem (see [15]).

Theorem 3.1 *Let $\mathcal{H}_{z,\varepsilon} = z + \varepsilon \mathcal{H}$ be a small geometric perturbation inserted in the background domain Ω . The shape function j admits the following asymptotic expansion:*

$$j(\Omega \setminus \overline{\mathcal{H}_{z,\varepsilon}}) = j(\Omega) + \varepsilon \delta j(z) + o(\varepsilon). \tag{3}$$

Here δj is the topological sensitivity function defined in Ω by

$$\delta j(x) = \int_0^T \theta_0(x, t) [v_0(x, t) + \theta_0(x, t)] dt \left(- \int_{\partial \mathcal{H}} q(y) ds(y) \right), \quad \forall x \in \Omega, \tag{4}$$

where $q \in H^{-1/2}(\partial \mathcal{H})$ solves the following boundary integral equation:

$$\int_{\partial \mathcal{H}} \Gamma(y-x) q(x) ds(x) = -1, \quad \forall y \in \partial \mathcal{H}. \tag{5}$$

In the particular case where $\mathcal{H} = B(0, 1)$, we have

Corollary 3.2 *If \mathcal{H} is the unit ball centered at the origin, then*

$$\int_{\partial \mathcal{H}} q(y) ds(y) = -4\pi,$$

and we have

$$j(\Omega \setminus \overline{\mathcal{H}_{z,\varepsilon}}) - j(\Omega) = \varepsilon \delta j(z) + o(\varepsilon),$$

where

$$\delta j(x) = 4\pi \int_0^T \theta_0(x, t) [v_0(x, t) + \theta_0(x, t)] dt, \quad \forall x \in \Omega, \forall x \in \Omega. \quad (6)$$

4 Numerical results

In order to improve the behavior of the cooling system, we need to optimize the location and shape of the created cooling holes. The unknown cooling holes $\{\mathcal{H}_k^*, k = 1, \dots, m\}$ are likely to be created at zones where the topological sensitivity function δj is most negative. More precisely, each hole \mathcal{H}_k^* will be approximated by a level set curve of the scalar function δj . The main steps of our procedure are the following:

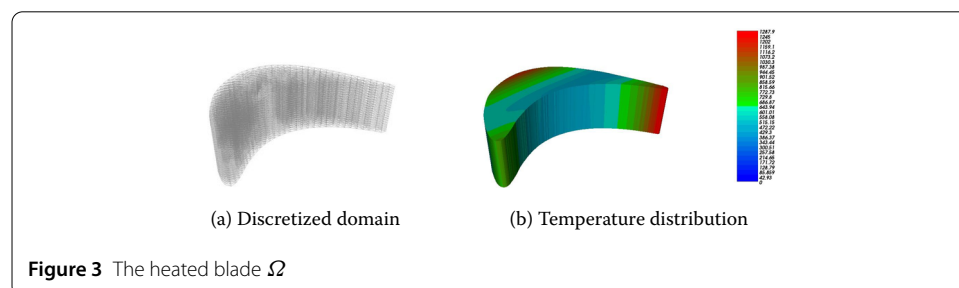
One-iteration algorithm:

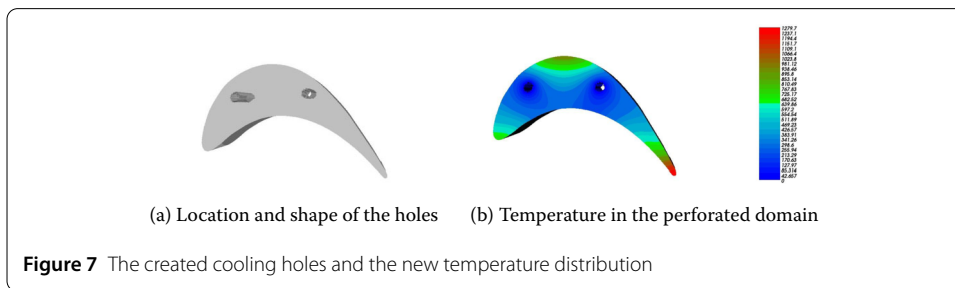
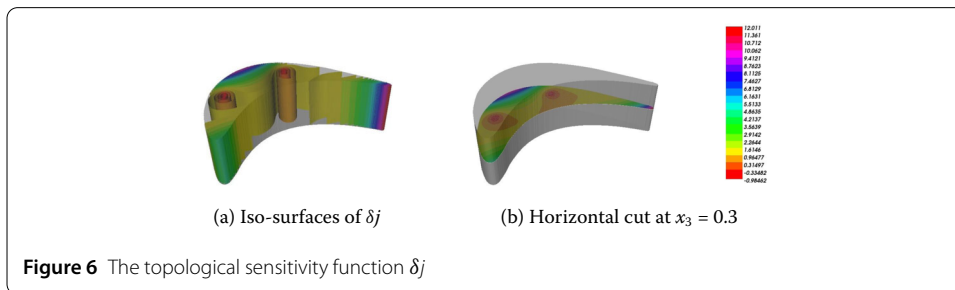
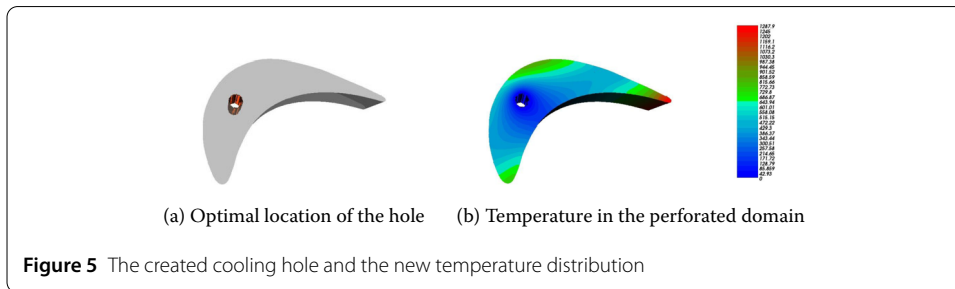
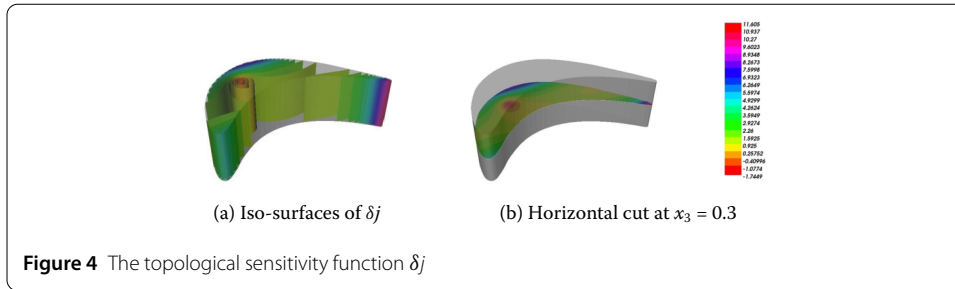
- Solve problem (1) and its associated adjoint problem (2),
- Compute the topological sensitivity function $\delta j(x)$, $\forall x \in \Omega$ defined by (6),
- Determine $\zeta^* \in [0, 1]$ such that $j(\Omega \setminus \overline{\mathcal{H}_{\zeta^*}}) \leq j(\Omega \setminus \overline{\mathcal{H}_{\zeta}})$, $\forall \zeta \in [0, 1]$, where $\mathcal{H}_{\zeta} = \{x \in \Omega; \delta j(x) < \zeta \delta_{\min}\}$, with δ_{\min} is the minimal value of δj in Ω .

Next, we will present some numerical simulations using the proposed algorithm. The two problems are discretized using triangular mesh and \mathbb{P}^1 finite element method [16] for the spacial variable. The temporal discretization is based on a finite difference approximation method where the time computational is given by $T = 1$. The numerical simulations are done using the free software FreeFem++ [17]. The initial computational domain is shown in Fig. 2. The temperature distribution in the heated blade domain Ω is depicted in Fig. 3. Next, we will present some numerical results. The first example is concerned with the creation of one cooling hole. Then, we present the case of two holes, and finally the case of multiple cooling holes.

4.1 Creating one cooling hole

The aim is to determine the best location to create a cooling hole that will reduce the temperature gradient as much as possible. The results of this test are summarized in Figs. 4 and 5. The iso-surfaces of the topological sensitivity function δj and an horizontal cut of δj at $x_3 = 0.3$ are plotted in Fig. 4. The created cooling hole and the new temperature distribution in the perforated domain are illustrated in Fig. 5. As one can observe, the hole is inserted at the zone where the topological sensitivity function is the most negative. We can see that the temperature distribution is reduced comparing with the initial temperature presented in Fig. 3.





4.2 Creating two cooling holes

In this test, we apply our proposed algorithm for detecting the best location of two cooling holes which minimize the temperature gradient as much as possible. The obtained numerical results are presented in Figs. 6 and 7. The iso-surfaces of the topological sensitivity function δ_j are plotted in Fig. 6. The best location of the two cooling holes and the new temperature distribution in the perforated domain are shown in Fig. 7.

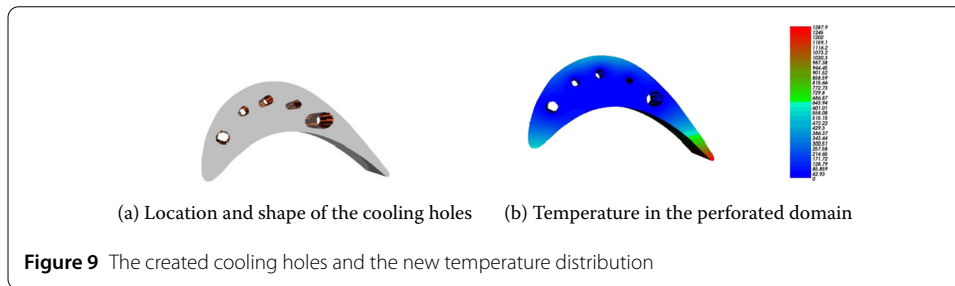
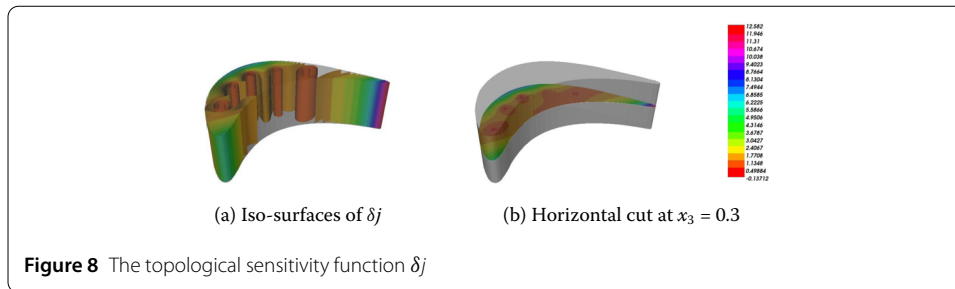


Table 1 The percentage of reduction after inserting the holes

Test cases	Creating one hole	Creating two holes	Creating multiple holes
$j(\Omega)$	$1230 \cdot 10^3$	$1230 \cdot 10^3$	$1230 \cdot 10^3$
$j(\Omega \setminus \mathcal{H})$	$753 \cdot 10^3$	$639 \cdot 10^3$	$438 \cdot 10^3$
Percentage of reduction	38%	48%	64%

4.3 Creating multiple cooling holes

This test is devoted to the creation of multiple cooling holes. In Fig. 8, we present the iso-surfaces of the topological sensitivity function. As one can observe, the function δj admits five negative local minima $\{z_k, 1 \leq k \leq 5\}$ inside the domain Ω . We will create a hole \mathcal{H}_k around each point z_k . The shape and size of the holes $\mathcal{H}_k, 1 \leq k \leq 5$ are defined by a level set curve of the function δj

$$\mathcal{H}_k = \{x \in \Omega; \delta j(x) \leq c_k\},$$

where c_k is chosen in such way that the shape function j decreases as much as possible.

In Fig. 9, we present the obtained location of the cooling holes and the new temperature distribution in the perforated domain.

4.4 Optimization quality

Finally, in order to show the effects of the created holes on the behavior of the cooling system, we summarize in Table 1 the percentage of reduction of the cost function obtained after inserting the holes in the studied cases. This shows the efficiency of the used approach.

5 Conclusion

The numerical study of the optimal hole creation problem in a the turbine blade cooling system improving the behavior of the cooling system has been studied. The used technique consists in studying the asymptotic expansion of the cost function with respect to the

variation of the domain. A fast and accurate algorithm is proposed for creating cooling holes in the blade vane. The presented numerical simulations show the efficiency of the suggested approach.

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Authors' contributions

The authors declare that the study was realized in collaboration with equal responsibility. All authors read and approved the final manuscript.

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