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On the fractional partial integro-differential equations of mixed type with non-instantaneous impulses

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Abstract

In this paper, we consider the initial boundary value problem for a class of nonlinear fractional partial integro-differential equations of mixed type with non-instantaneous impulses in Banach spaces. Sufficient conditions of existence and uniqueness of PC-mild solutions for the equations are obtained via general Banach contraction mapping principle, Krasnoselskii's fixed point theorem, and α-order solution operator.

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1 Introduction

In this paper, we study the following initial boundary value problem of nonlinear fractional partial integro-differential equations of mixed type with non-instantaneous impulses:

$$\begin{cases} \frac{\partial}{\partial t}u(x,t)) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)} \frac{\partial^2}{\partial x^2}u(x,s)\,ds + f(t,u(x,t),\mathcal{G}u(x,t),\mathcal{S}u(x,t)), \\ t \in (s_k,t_{k+1}], \quad k = 0,1,\ldots,m, \\ u(0,t) = u(\pi,t) = 0, \quad t \in [0,b], \\ u(x,t) = l_k(t,u(x,t)), \quad t \in (t_k,s_k], k = 1,2,\ldots,m, \\ u(x,0) = \varphi(x), \quad x \in [0,\pi], \end{cases} \tag{1.1}$$

where $\alpha \in (1,2)$, $f : [0,b] \times \mathbb{R}^3 \to \mathbb{R}$, $l_k : [0,b] \times \mathbb{R} \to \mathbb{R}$, $k = 1,2,\ldots,m$, $\varphi \in L^2([0,\pi])$, $\mathcal{G}$ and $\mathcal{S}$ are defined by

$$\mathcal{G}u(x,t) = \int_0^t \mathcal{K}\bigl(t,s,u(x,s)\bigr)\,ds, \qquad \mathcal{S}u(x,t) = \int_0^b \mathcal{H}\bigl(t,s,u(x,s)\bigr)\,ds, \tag{1.2}$$

$\mathcal{K} : D \times \mathbb{R} \to \mathbb{R}_+$ and $\mathcal{H} : D_0 \times \mathbb{R} \to \mathbb{R}_+$ are continuous and nonlinear functions, $D = \{(t,s) \in \mathbb{R}^2 : 0 \le s \le t \le b\}$, $D = \{(t,s) \in \mathbb{R}^2 : 0 \le t,s \le b\}$, $\mathbb{R}_+ = [0,+\infty)$. The pre-fixed numbers s_i

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and t_i satisfy $0 = s_0 < t_1 \leq s_1 < t_2 \leq \cdots < t_m \leq s_m < t_{m+1} = b$. The operator $\mathcal{G}$ is an integral with variable upper limit, the operator $\mathcal{H}$ is an ordinary definite integral. Therefore, problem (1.1) is called the mixed type integro-differential equations.

The theory of differential equations with instantaneous impulses often describes some processes which have a sudden change in their states at certain times, especially in biology, dynamics, physics, engineering etc. In recent years, a lot of researchers have obtained numerous good results about the fractional differential equations, for example, see [1–17] and the references therein. In [1, 5], the authors studied the stability for impulsive systems; in [2–4, 6, 13, 14], the authors studied the existence results for impulsive differential equations. Yan [7], Chen, Zhang, and Li [8, 11] studied the approximate controllability of the fractional evolution equations.

Meanwhile, fractional differential equations with non-instantaneous impulsive effects have been applied widely as mathematical models to consider many phenomena in biology, dynamics, physics, control model, etc., see [18–24] and the references therein. In [18], Hernandez and O'Regan firstly studied the integer differential equations with non-instantaneous impulses. In [19, 20], Chen, Zhang, and Li studied the non-autonomous evolution equations with non-instantaneous impulses and obtained the main results of the existence. In [21–24], the authors studied the controllability for the fractional differential systems with non-instantaneous impulses. In [25–27], the authors studied the initial boundary value problem for time fractional partial differential equations with delay and discussed the existence and uniqueness of the mild solutions. In [28, 29], the authors also studied the differential equations of mixed type. Guo [28] studied the existence and uniqueness of the following integer nonlinear integro-differential equations of mixed type in a Banach space E:

$$\begin{cases} u'(t) = f(t, u(t), \mathcal{G}u(t), \mathcal{S}u(t)), & t \in (0, a], \\ u(0) = u_0, \end{cases} \tag{1.3}$$

where

$$\mathcal{G}u(x,t) = \int_0^t K(t,s)u(x,s)\,ds, \qquad \mathcal{S}u(x,t) = \int_0^a H(t,s)u(x,s)\,ds, \tag{1.4}$$

the kernels K and H are linear functions. Chen, Zhang, and Li [19] studied the existence of the following fractional non-autonomous integro-differential evolution equations of mixed type:

$$\begin{cases} {}^cD_t^\alpha(t) + A(t)u(t) = f(t, u(t), \mathcal{G}u(t), \mathcal{S}u(t)), & t \in (0, a], \\ u(0) = A^{-1}(0)u_0, \end{cases} \tag{1.5}$$

where the operators $\mathcal{G}$ and $\mathcal{S}$ are the same as in (1.4), and the kernels K and H are also linear functions.

To the best of our knowledge, we have not found the relevant results that study the initial boundary value problem for the fractional partial integro-differential equations of mixed type with non-instantaneous impulses. Therefore, motivated by the above-mentioned papers, we study the existence of PC-mild solutions for problem (1.1). In this paper, the kernels $\mathcal{K}$ and $\mathcal{H}$ of the operators $\mathcal{G}$ and $\mathcal{S}$ are nonlinear functions. The nonlinear term f

satisfies the Lipschitz condition, where the Lipschitz coefficients are Lebesgue integrable functions. In the proof of the main results by the general Banach contraction mapping principle, we do not need extra conditions to ensure the contraction coefficients less than one. Our main results of this paper generalize and improve some corresponding results.

2 Preliminaries

Let $E = L^2([0,\pi])$ be a Banach space, $J = [0,b]$, $C(J,E) = \{u : J \to E \text{ is continuous}\}$, $PC(J,E) = \{u : J \to E : u \in C((s_k,t_{k+1}],E)$, and there exist $u(t_k^+)$ and $u(t_k^-)$ with $u(t_k^-) = u(t_k), k = 1,2,\ldots,m\}$ with the PC-norm $\|u\|_{PC} = \sup\{\|u(t)\| : t \in J\}$. $A : D(A) \subset E \to E$ defined by $Au = \frac{\partial^2}{\partial x^2}u$ with the domain $D(A) = \{u \in E : u'' \in E, u(0) = u(\pi) = 0\}$, then A is a sectorial operator of type μ. Let $u(x,t) = u(\cdot,t)$,

$$f\big(t,u(x,t),\mathcal{G}u(x,t),\mathcal{S}u(x,t)\big) = f\big(t,u(\cdot,t),\mathcal{G}u(\cdot,t),\mathcal{S}u(\cdot,t)\big), \quad t \in J,$$
$$u_0 = \varphi(\cdot),$$

then problem (1.1) can be rewritten as the following abstract form (2.1):

$$\begin{cases} \frac{d}{dt}u(t)) = \int_0^t \frac{(t-s)^{\alpha-2}}{\Gamma(\alpha-1)}Au(s)\,ds + f(t,u(t),\mathcal{G}u(t),\mathcal{S}u(t)), \\ t \in (s_k,t_{k+1}], \quad k = 0,1,\ldots,m, \\ u(t) = l_k(t,u(t)), \quad t \in (t_k,s_k], k = 1,2,\ldots,m, \\ u(0) = u_0. \end{cases} \tag{2.1}$$

If there exist constant $0 < \theta < \pi/2$, $\widetilde{M} > 0$, $\mu \in \mathbb{R}$ such that its resolvent exists outside the sector

$$\mu + \mathcal{T}_\theta := \big\{\mu + s : \lambda \in \mathbb{C}, \big|arg(-\lambda)\big| < \theta\big\}, \qquad \big\|(\lambda - A)^{-1}\big\| \le \frac{\widetilde{M}}{|\lambda - \mu|}, \quad \lambda \notin \mu + \mathcal{T}_\theta,$$

then the operator A is called sectorial operator of type μ, where the linear operator A in problem (2.1) is sectorial of type μ with $0 < \theta < \pi(1 - \alpha/2)$.

Definition 2.1 ([30]) Let A be a closed and linear operator with domain $D(A)$ defined on a Banach space E. If there exist a real number μ and a strongly continuous function $\mathcal{T}_\alpha : \mathbb{R}_+ \to \mathcal{L}(E)$ such that

$$\big\{\lambda^\alpha : Re\lambda > \mu\big\} \subset \rho(A)$$

and

$$\lambda^{\alpha-1}\big(\lambda^\alpha - A\big)^{-1}u = \int_0^\infty e^{-\lambda t}\mathcal{T}_\alpha(t)u\,dt, \quad Re(\lambda) > \mu, u \in E,$$

where $\mathcal{L}(E)$ means the space of bounded linear operators from E to E, then $\mathcal{T}_\alpha(t)$ is called the α-order solution operator generated by A.

Definition 2.2 A function $u \in PC(J,E)$ is called a PC-mild solution of Eqs. (2.1), if $u(0) = u_0$, and

$$u(t) = \begin{cases} \mathcal{T}_\alpha(t)u_0 + \int_0^t \mathcal{T}_\alpha(t-s)f(s,u(s),\mathcal{G}u(s),\mathcal{S}u(s))\,ds, & t \in [0,t_1], \\ l_k(t,u(t)), & t \in (t_k,s_k], k = 1,2,\ldots,m, \\ \mathcal{T}_\alpha(t-s_k)l_k(s_k,u(s_k)) + \int_{s_k}^t \mathcal{T}_\alpha(t-s)f(s,u(s),\mathcal{G}u(s),\mathcal{S}u(s))\,ds, \\ t \in (s_k,t_{k+1}], \quad k = 1,2,\ldots,m. \end{cases} \tag{2.2}$$

Lemma 2.1 ([31, 32]) *Let* $0 < \varrho < 1$, $\gamma > 0$,

$$S = \varrho^n + C_n^1 \varrho^{n-1}\gamma + \frac{C_n^2 \varrho^{n-2}\gamma^2}{2!} + \cdots + \frac{\gamma^n}{n!}, \quad n \in \mathbb{N}.$$

Then, for all constant $0 < \xi < 1$ *and all real number* $s > 1$, *we get*

$$S \le O\left(\frac{\xi^n}{\sqrt{n}}\right) + o\left(\frac{1}{n^s}\right) = o\left(\frac{1}{n^s}\right), \quad n \to +\infty.$$

Lemma 2.2 (Krasnoselskii's fixed point theorem) *Let D be a bounded closed and convex subset of a Banach space E, and let* Φ_1, Φ_2 *be maps of D into E such that* $\Phi_1 x + \Phi_2 y \in D$ *for all* $x,y \in D$. *If* Φ_1 *is a contraction and* Φ_2 *is completely continuous, then the operator* $\Phi_1 + \Phi_2$ *has a fixed point on D.*

3 Main results

We assume that there exists a constant $M > 0$ such that $\|\mathcal{T}_\alpha\| \le M$ for all $t \in J$. Define an operator $\Phi : PC(J;E) \to PC(J;E)$ by

$$(\Phi u)(t) = (\Phi_1 u)(t) + (\Phi_2 u)(t), \tag{3.1}$$

where

$$(\Phi_1 u)(t) = \begin{cases} \mathcal{T}_\alpha(t)u_0, & t \in [0,t_1], \\ l_k(t,u(t)), & t \in (t_k,s_k], k = 1,2,\ldots,m, \\ \mathcal{T}_\alpha(t-s_k)l_k(s_k,u(s_k)), & t \in (s_k,t_{k+1}], k = 1,2,\ldots,m, \end{cases} \tag{3.2}$$

$$(\Phi_2 u)(t) = \begin{cases} \int_0^t \mathcal{T}_\alpha(t-s)f(s,u(s),\mathcal{G}u(s),\mathcal{S}u(s))\,ds, & t \in [0,t_1], \\ 0, & t \in (t_k,s_k], k = 1,2,\ldots,m, \\ \int_{s_k}^t \mathcal{T}_\alpha(t-s)f(s,u(s),\mathcal{G}u(s),\mathcal{S}u(s))\,ds, & t \in (s_k,t_{k+1}], k = 1,2,\ldots,m. \end{cases} \tag{3.3}$$

Firstly, we give the following hypotheses:

(H_1) The function $f : J \times E^3 \to E$ is continuous and there exist nonnegative Lebesgue integrable functions $l_i \in L^1(J,\mathbb{R}^+)$ $(i = 1,2,3)$ such that, for all $t \in J$, $u_i, v_i \in E$ $(i = 1,2,3)$, we have

$$\|f(t,u_1,u_2,u_3) - f(t,v_1,v_2,v_3)\| \le \sum_{i=1}^3 l_i(t)\|u_i - v_i\|.$$

(H_1') The function $f : J \times E^3 \to E$ is continuous, for a constant $r > 0$, there exist a positive constant $\Im$, a Lebesgue integrable function $\psi \in L^1(J, \mathbb{R}^+)$, and a continuous nondecreasing function $\Theta : \mathbb{R}^+ \to (0, +\infty)$ such that, for any $t \in J$ and $u_i \in E$ ($i = 1, 2, 3$), we have

$$\|u_i\| \le r, \qquad \bigl\|f(t, u_1, u_2, u_3)\bigr\| \le \psi(t)\Theta(r), \qquad \liminf_{r\to+\infty} \frac{\Theta(r)}{r} := \Im < +\infty.$$

(H_2) The functions $l_k : J \times E \to E$ are continuous and there exist nonnegative constants l_{l_k} such that, for all $t \in J$, $u, v \in E$, we have

$$\bigl\|l_k(t, u) - l_k(t, u)\bigr\| \le l_{l_k}\|u - v\|, \quad k = 1, 2, \ldots, m.$$

(H_3) The functions $\mathcal{K} : J \times J \times E \to E$ and $\mathcal{H} : J \times J \times E \to E$ are continuous, and there exist nonnegative constants $l_{\mathcal{K}}$, $l_{\mathcal{H}}$ such that, for all $t, s \in J$, $u, v \in E$, we have

$$\bigl\|\mathcal{K}(t, s, u) - \mathcal{K}(t, s, v)\bigr\| \le l_{\mathcal{K}}\|u - v\|, \qquad \bigl\|\mathcal{H}(t, s, u) - \mathcal{H}(t, s, v)\bigr\| \le l_{\mathcal{H}}\|u - v\|.$$

Theorem 3.1 *If hypotheses* (H_1)–(H_3) *hold and* $0 \le \tau < 1$ ($\tau = \max\{l_{l_k}, Ml_{l_k}\}$), *then problem* (2.1) *has a unique PC-mild solution* $u^* \in PC(J, E)$, *which means that problem* (1.1) *has a unique PC-mild solution.*

Proof For any $u, v \in PC(J, E)$, by (3.2) we have

$$\bigl\|(\Phi_1 u)(t) - (\Phi_1 v)(t)\bigr\| \le \begin{cases} 0, & t \in [0, t_1], \\ \tau\|u - v\|_{PC}, & t \in (t_k, s_k], k = 1, 2, \ldots, m, \\ \tau\|u - v\|_{PC}, & t \in (s_k, t_{k+1}], k = 1, 2, \ldots, m, \end{cases} \tag{3.4}$$

which means

$$\bigl\|(\Phi_1 u)(t) - (\Phi_1 v)(t)\bigr\| \le \tau\|u - v\|_{PC},$$

where $t \in [0, t_1] \cup (t_k, s_k] \cup (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$. Then we obtain

$$\bigl\|\bigl(\Phi_1^2 u\bigr)(t) - \bigl(\Phi_1^2 v\bigr)(t)\bigr\| \le \tau^2\|u - v\|_{PC},$$

where $t \in [0, t_1] \cup (t_k, s_k] \cup (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$. It is clear that we have

$$\bigl\|\bigl(\Phi_1^n u\bigr)(t) - \bigl(\Phi_1^n v\bigr)(t)\bigr\| \le \tau^n\|u - v\|_{PC}, \tag{3.5}$$

where $t \in [0, t_1] \cup (t_k, s_k] \cup (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$.

For any real number $0 < \varepsilon < 1$, there exists a continuous function $\phi(s)$ such that $\int_0^b |l(s) - \phi(s)|\,ds < \varepsilon$, where $l(s) = M(l_1(s) + bl_{\mathcal{K}}l_2(s) + bl_{\mathcal{H}}l_3(s))$ is a Lebesgue integrable function. For any $t \in [0, t_1]$, $u, v \in PC(J;E)$, by hypotheses H_1, H_3 and formula (3.3), we have

$$\begin{aligned}
&\bigl\|(\Phi_2 u)(t) - (\Phi_2 v)(t)\bigr\| \\
&\quad\le \int_0^t \mathcal{T}_\alpha(t-s)\|f(s,u(s),\mathcal{S}u\bigl((s)V\bigr) - f(s,v(s),\mathcal{G}v(s),\mathcal{S}v\bigl((s)\bigr)\|\,ds \\
&\quad\le \int_0^t M\bigl(l_1(s) + bl_{\mathcal{K}}l_2(s) + bl_{\mathcal{H}}l_3(s)\bigr)\bigl\|u(s) - v(s)\bigr\|\,ds \\
&\quad\le \int_0^t l(s)\,ds\|u - v\|_{PC} \\
&\quad\le \biggl(\int_0^t \bigl|l(s) - \phi(s)\bigr|\,ds + \int_0^t \bigl|\phi(s)\bigr|\,ds\biggr)\|u - v\|_{PC} \\
&\quad\le (\varepsilon + \nu t)\|u - v\|_{PC} \\
&\quad= \biggl(C_1^0\varepsilon^1 + C_1^1\frac{(\nu t)^1}{1!}\biggr)\|u - v\|_{PC},
\end{aligned} \tag{3.6}$$

where $\max_{t\in J}|\phi(t)| = \nu$. Assume that, for any natural number k, we have

$$\bigl\|\bigl(\Phi_2^k u\bigr)(t) - \bigl(\Phi_2^k v\bigr)(t)\bigr\| \le \biggl(C_k^0\varepsilon^k + C_k^1\varepsilon^{k-1}\frac{(\nu t)^1}{1!} + \cdots + C_k^k\varepsilon^{k-k}\frac{(\nu t)^k}{k!}\biggr)\|u - v\|_{PC}. \tag{3.7}$$

By the formula $C_{k+1}^m = C_k^m + C_k^{m-1}$ and (3.7), we get

$$\begin{aligned}
&\bigl\|\bigl(\Phi_2^{k+1} u\bigr)(t) - \bigl(\Phi_2^{k+1} v\bigr)(t)\bigr\| \\
&\quad\le \int_0^t M\bigl\|f\bigl(s,\bigl(\Phi_2^k u\bigr)(s),\mathcal{G}\bigl(\Phi_2^k u\bigr)(s),\mathcal{S}\bigl(\Phi_2^k u\bigr)(s)\bigr) \\
&\qquad - f\bigl(s,\bigl(\Phi_2^k v\bigr)(s),\mathcal{G}\bigl(\Phi_2^k v\bigr)(s),\mathcal{S}\bigl(\Phi_2^k v\bigr)(s)\bigr)\bigr\|\,ds \\
&\quad\le \int_0^t M\bigl(l_1(s) + bl_{\mathcal{K}}l_2(s) + bl_{\mathcal{H}}l_3(s)\bigr)\bigl\|\bigl(\Phi_2^k u\bigr)(s) - \bigl(\Phi_2^k v\bigr)(s)\bigr\|\,ds \\
&\quad= \int_0^t l(s)\bigl\|\bigl(\Phi_2^k u\bigr)(s) - \bigl(\Phi_2^k v\bigr)(s)\bigr\|\,ds \\
&\quad\le \biggl(\int_0^t \bigl|l(s) - \phi(s)\bigr|\biggl(C_k^0\varepsilon^k + C_k^1\varepsilon^{k-1}\frac{(\nu s)^1}{1!} + \cdots + C_k^k\varepsilon^{k-k}\frac{(\nu s)^k}{k!}\biggr)\,ds\biggr)\|u - v\|_{PC} \\
&\qquad + \biggl(\int_0^t \bigl|\phi(s)\bigr|\biggl(C_k^0\varepsilon^k + C_k^1\varepsilon^{k-1}\frac{(\nu s)^1}{1!} + \cdots + C_k^k\varepsilon^{k-k}\frac{(\nu s)^k}{k!}\biggr)\,ds\biggr)\|u - v\|_{PC} \\
&\quad\le \varepsilon\biggl(C_k^0\varepsilon^k + C_k^1\varepsilon^{k-1}\frac{(\nu t)^1}{1!} + \cdots + C_k^k\varepsilon^{k-k}\frac{(\nu t)^k}{k!}\biggr)\|u - v\|_{PC} \\
&\qquad + \nu\int_0^t \biggl(C_k^0\varepsilon^k + C_k^1\varepsilon^{k-1}\frac{(\nu s)^1}{1!} + \cdots + C_k^k\varepsilon^{k-k}\frac{(\nu s)^k}{k!}\biggr)\,ds\|u - v\|_{PC} \\
&\quad\le \biggl(C_{k+1}^0\varepsilon^{k+1} + C_{k+1}^1\varepsilon^k\frac{(\nu t)^1}{1!} + \cdots + C_{k+1}^{k+1}\varepsilon^{(k+1)-(k+1)}\frac{(\nu t)^{k+1}}{(k+1)!}\biggr)\|u - v\|_{PC}.
\end{aligned}$$

By mathematical methods of induction, for any natural number n, we get

$$\|\Phi_2^n u - \Phi_2^n v\|_{PC} \leq \left(C_n^0 \varepsilon^n + C_n^1 \varepsilon^{n-1} \frac{\varsigma^1}{1!} + \cdots + C_n^n \varepsilon^{n-n} \frac{\varsigma^n}{n!}\right) \|u - v\|_{PC}, \tag{3.8}$$

where $\varsigma = \nu b$. By Lemma 2.1, we have

$$\begin{aligned} \|\Phi_2^n u - \Phi_2^n v\|_{PC} &\leq \left[O\left(\frac{\eta^n}{\sqrt{n}}\right) + o\left(\frac{1}{h^\lambda}\right)\right] \|u - v\|_{PC} \\ &= o\left(\frac{1}{n^\lambda}\right) \|u - v\|_{PC}, \quad (n \to +\infty), \end{aligned} \tag{3.9}$$

where $0 < \eta < 1$, $\lambda > 1$. It is easy to see that the above Eq. (3.9) holds for $t \in (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$. By (3.5) and (3.9), we obtain

$$\|\Phi^n u - \Phi^n v\|_{PC} \leq \left(\tau^n + o\left(\frac{1}{n^\lambda}\right)\right) \|u - v\|_{PC}, \quad \forall n > n_0.$$

Thus, for any fixed constant $\lambda > 1$, we can find a positive integer n_0 such that, for any $n > n_0$, we get $0 < \tau^n + \frac{1}{n^\lambda} < 1$. Therefore, for any $u, v \in PC(J, E)$, we have

$$\|\Phi^n u - \Phi^n v\|_{PC} \leq \left(\tau^n + \frac{1}{n^\lambda}\right) \|u - v\|_{PC} \leq \|u - v\|_{PC}, \quad \forall n > n_0.$$

By the general Banach contraction mapping principle, we get that the operator Φ defined by (3.1) has a unique fixed point $u^* \in PC(J, E)$, which means that problem (1.1) has a unique PC-mild solution. □

Remark 3.1 In Theorem 3.1, we prove the existence and uniqueness of the PC-mild solutions for problem (1.1) using the general Banach contraction mapping principle. Note that we do not need extra conditions to ensure the contraction constant $0 < k < 1$ for the operator Φ_2. Therefore, Theorem 3.1 improves some results that have been studied by the Banach contraction mapping principle.

Theorem 3.2 *Assume that the solution operator $\mathcal{T}_\alpha(t)$ $(t \in J)$ generated by A is compact and functions l_k $(k = 1, 2, \ldots, m)$ are bounded. If hypotheses (H_1'), (H_2), and (H_3) hold, then problem (2.1) has at least one PC-mild solution $u^* \in PC(J, E)$, which means that problem (1.1) has at least one PC-mild solution provided that*

$$\varpi(\Im\Delta + L) < 1, \tag{3.10}$$

where

$$\varpi = \max\{1, M\}, \qquad L = \max_{k=1,2,\ldots,m} l_{l_k}, \qquad \Delta = \max_{k=0,1,\ldots m} \|\psi\|_{L^1([s_k, t_{k+1}], \mathbb{R}^+)}.$$

Proof Step 1. We prove that there exists a positive constant R such that the operator $\Phi(B_R) \subset B_R$. If the judgment is not right, then for any positive constant r, there would exist $u_r \in B_r$ and $t_r \in J$ such that $\|(\Phi u_r)\| > r$.

If $t_r \in [0, t_1]$, then by (3.1) and (H_1'), we have

$$\begin{aligned} \|(\Phi u_r)(t_r)\| &\le \|\mathcal{T}_\alpha(t_r)u_0\| + \left\| \int_0^{t_r} \mathcal{T}_\alpha(t_r - s) f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\, ds \right\| \\ &\le M\|u_0\| + M\int_o^{t_r} \Theta(r)\psi(s)\, ds \\ &\le M(\|u_0\| + \Theta(r)\|\psi\|_{L^1([0,t)1],\mathbb{R}^+)}. \end{aligned} \tag{3.11}$$

If $t_r \in (t_k, s_k]$, $k = 1, 2, \dots, m$, then by (3.1) and (H_2), we have

$$\|(\Phi u_r)(t_r)\| \le \|l_k(t_r, u_r(t_r))\| \le l_{l_k}\|u_r(t_r)\| + \|l_k(t_r, \theta)\| \le Lr + M^*, \tag{3.12}$$

where $M^* = \max_{k=1,2,\dots,m} \sup_{t\in J} \|l_k(t_r, \theta)\|$.

If $t_r \in (s_k, t_{k+1}]$, $k = 1, 2, \dots, m$, then by (3.1), (H_1'), and (H_2), we have

$$\begin{aligned} \|(\Phi u_r)(t_r)\| &\le \|\mathcal{T}_\alpha(t_r - s_k) l_k\big(s_k, u_r(s_k)\big)\| \\ &\quad + \left\| \int_{s_k}^{t_r} \mathcal{T}_\alpha(t_r - s) f\big(s, u_r(s), \mathcal{G}u_r(s), \mathcal{S}u_r(s)\big)\, ds \right\| \\ &\le M\big(l_{l_k}\|u_r(s_k)\| + \|l_k(s_k, \theta)\|\big) + M\int_{s_k}^{t_r} \Theta(r)\psi(s)\, ds \\ &\le M(Lr + M^* + \Theta(r)\|\psi\|_{L^1([0,t)1],\mathbb{R}^+)}. \end{aligned} \tag{3.13}$$

We know that the inequality $\|(\Phi u_r)(t_r)\| > r$ holds, by (3.10)–(3.13), we get

$$r < \|(\Phi u_r)(t_r)\| \le \varpi\big(\|u_0\| + \Theta(r)\Delta + Lr + M^*\big), \tag{3.14}$$

then

$$1 < \frac{\|(\Phi u_r)(t_r)\|}{r} \le \frac{\varpi(\|u_0\| + \Theta(r)\Delta + Lr + M^*)}{r}. \tag{3.15}$$

Let $r \to +\infty$, we have $\varpi(\Im\Delta + L) \ge 1$, which contradicts (3.10). Thus, we have that the operator $\Phi(B_R) \subset B_R$.

Step 2. We prove that $\Phi_1 : B_R \to B_R$ is a contraction map. For $t \in [0, t_1]$ and $u, v \in B_R$, by (3.2) we have

$$\|(\Phi_1 u)(t) - (\Phi_1 v)(t)\| = 0. \tag{3.16}$$

For $t \in (t_k, s_k]$, $k = 1, 2, \dots, m$, and $u, v \in B_R$, by (3.2) and (H_2), we have

$$\|(\Phi_1 u)(t) - (\Phi_1 v)(t)\| \le l_{l_k}\|u(t) - v(t)\| \le L\|u - v\|_{PC}. \tag{3.17}$$

For $t \in (s_k, t_{k+1}]$, $k = 1, 2, \dots, m$, and $u, v \in B_R$, by (3.2) and (H_2), we have

$$\|(\Phi_1 u)(t) - (\Phi_1 v)(t)\| \le \varpi l_{l_k}\|u(t) - v(t)\| \le \varpi L\|u - v\|_{PC}. \tag{3.18}$$

By (3.16)–(3.18), for any $u, v \in B_R$, we have

$$\|\Phi_1 u - \Phi_1 v\|_{PC} \leq \varpi L \|u - v\|_{PC}. \tag{3.19}$$

From (3.10) we know that the operator $\Phi_1 : B_R \to B_R$ is a contraction map.

Step 3. We prove that Φ_2 is a continuous operator in B_R. Let $\{u_n\}_0^\infty \subset B_R$, and $u_n \to u \in B_R$. By (H_1') and (H_3), we have

$$f\big(s, u_n(s), \mathcal{G}u_n(s), \mathcal{S}u_n(s)\big) \to f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big), \quad n \to \infty, s \in J. \tag{3.20}$$

For any $s \in J$, by (H_1), we get

$$\big\|f\big(s, u_n(s), \mathcal{G}u_n(s), \mathcal{S}u_n(s)\big) - f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\big\| \leq 2\psi_R(s), \quad n \to \infty, s \in J, \tag{3.21}$$

where the function $2\psi_R(s)$ is Lebesgue integrable for $s \in [0, t_1]$ and $t \in (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$. Using the Lebesgue dominated convergence theorem and (3.3), (3.20), (3.21), for $t \in [0, t_1]$, we have

$$\begin{aligned} &\big\|(\Phi_2 u_n)(t) - (\Phi_2 u)(t)\big\| \\ &\quad \leq M \int_0^t \big\|f\big(s, u_n(s), \mathcal{G}u_n(s), \mathcal{S}u_n(s)\big) - f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\big\|\, ds \\ &\quad \to 0 \quad \text{as } n \to \infty. \end{aligned} \tag{3.22}$$

For $t \in (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$, we have

$$\begin{aligned} &\big\|(\Phi_2 u_n)(t) - (\Phi_2 u)(t)\big\| \\ &\quad \leq M \int_{s_k}^t \big\|f\big(s, u_n(s), \mathcal{G}u_n(s), \mathcal{S}u_n(s)\big) - f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\big\|\, ds \\ &\quad \to 0 \quad \text{as } n \to \infty. \end{aligned} \tag{3.23}$$

From (3.22) and (3.23), for $s \in [0, t_1]$ and $t \in (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$, we have

$$\|\Phi_2 u_n - \Phi_2 u\|_{PC} \to 0 \quad \text{as } n \to \infty.$$

Thus, Φ_2 is a continuous operator in B_R.

Step 4. We prove that the operator $\Phi_2 : B_R \to B_R$ is compact. Firstly, we prove that $\{(\Phi_2 u)(t) : u \in B_R\}$ is relatively compact in E for any $t \in [0, t_1]$ and $t \in (s_k, t_{k+1}]$, $k = 1, 2, \ldots, m$. For any fixed t $(0 < t \leq t_1)$ and $0 < \epsilon < t$, let $u \in B_R$ and define

$$(\Phi_{2,\epsilon} u)(t) = \int_0^{t-\epsilon} \mathcal{T}_\alpha(t - s) f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\, ds.$$

Due to the compactness of $\mathcal{T}_\alpha(t)$, the set $\{(Q_{2,\epsilon}u)(t) : u \in B_R\}$ is relatively compact in E for all ϵ $(0 < \epsilon < t)$. For any $u \in B_R$, we get

$$\begin{aligned}\|(\Phi_2 u)(t) - (\Phi_{2,\epsilon}u)(t)\| &\le \left\| \int_{t-\epsilon}^{t} \mathcal{T}_\alpha(t-s) f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\,ds \right\| \\ &\le M\Phi(R)\int_{t-\epsilon}^{t} \psi_R(s)\,ds \to 0 \quad \text{as } \epsilon \to 0,\end{aligned}$$

which means that the set $\{(\Phi_2 u)(t) : u \in B_R\}$ is totally bounded. Therefore, the set $\{(\Phi_2 u)(t) : u \in B_R\}$ is relatively compact in E. Similar to the proof for $t \in [0, t_1]$, we can prove that $\Phi_2(B_R)(t) \subset E$, $t \in (s_k, t_{k+1}]$, $k = 1, 2, \dots, m$, is precompact.

Secondly, we prove that $\Phi_2(B_R)$ is equicontinuous. Case 1. For $[0, t_1]$, let $0 \le \tau_1 < \tau_2 \le t_1$, $u \in B_R$,

$$\begin{aligned}\|(\Phi_2 u)(\tau_2) - (\Phi_2 u)(\tau_1)\| &\le \left\| \int_0^{\tau_1} \big(\mathcal{T}_\alpha(\tau_2 - s) - \mathcal{T}_\alpha(\tau_1 - s)\big) f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\,ds \right\| \\ &\quad + \left\| \int_{\tau_1}^{\tau_2} \mathcal{T}_\alpha(\tau_2 - s) f\big(s, u(s), \mathcal{G}u(s), \mathcal{S}u(s)\big)\,ds \right\| \\ &\le \sup_{s\in[0,t_1]} \big\|\mathcal{T}_\alpha(\tau_2 - s) - \mathcal{T}_\alpha(\tau_1 - s)\big\| \Theta(R) \int_0^{\tau_1} \psi_R(s)\,ds \\ &\quad + 2M\Theta(R)\int_{\tau_1}^{\tau_2} \psi_R(s)\,ds.\end{aligned}$$

The operator $\mathcal{T}_\alpha(t)$ is compact, which means that the operator $\mathcal{T}_\alpha(t)$ is continuous in the sense of uniform operator topology. Thus, $\|(\Phi_2 u)(\tau_2) - (\Phi_2 u)(\tau_1)\| \to 0$ as $\tau_2 \to \tau_1$.

Case 2. For $(s_k, t_{k+1}]$ $(k = 1, 2, \dots, m)$, let $s_k \le \tau_1 < \tau_2 \le t_{k+1}$, $u \in B_R$. Similar to the proof for Case 1, we have $\|(\Phi_2 u)(\tau_2) - (\Phi_2 u)(\tau_1)\| \to 0$ as $\tau_2 \to \tau_1$. Thus, $\Phi_2(B_R)$ is equicontinuous. By the Arzelá–Ascoli theorem, we get that $\Phi_2 : B_R \to B_R$ is completely continuous. Therefore, by Lemma 2.2 we get that the operator Φ has a fixed point u^* in B_R, which is a PC- mild solution of problem (2.1). It implies that problem (1.1) has a PC-mild solution on the interval $[0, b]$. □

4 Conclusion

In this paper, we turn the initial boundary value problem for the fractional partial integro-differential equations of mixed type with non-instantaneous impulses into the abstract form. The kernels $\mathcal{K}$ and $\mathcal{H}$ of the operators $\mathcal{G}$ and $\mathcal{S}$ are nonlinear functions. The nonlinear term f satisfies the Lipschitz condition, where the Lipschitz coefficients are Lebesgue integrable functions. The main results are obtained via general Banach contraction mapping principle, Krasnoselskii's fixed point theorem, and α-order solution operator. In the proof of the main results by the general Banach contraction mapping principle, we do not need extra conditions to ensure the contraction coefficients less than one. Our main results of this paper generalize and improve some corresponding results.

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