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Dynamical behavior of perturbed Gerdjikov–Ivanov equation through different techniques

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Abstract

The objective of this work is to investigate the perturbed Gerdjikov–Ivanov (GI) equation along spatio-temporal dispersion which explains the dynamics of soliton dispersion and evolution of propagation distance in optical fibers, photonic crystal fibers (PCF), and metamaterials. The algorithms, namely hyperbolic extended function method and generalized Kudryashov's method, are constructed to obtain the new soliton solutions. The dark, bright, periodic, and singular solitons are derived of the considered equation with the appropriate choice of parameters. These results are novel, confirm the stability of optical solitons, and have not been studied earlier. The explanation of evaluated results is given by sketching the various graphs in 3D, contour and 2D plots by using Maple 18. Graphical simulations divulge that varying the wave velocity affects the dynamical behaviors of the model. In summary, this research adds to our knowledge on how the perturbed GI equation with spatio-temporal dispersion behaves. The obtained soliton solutions and the methods offer computational tools for further analysis in this field. This work represents an advancement in our understanding of soliton dynamics and their applications in photonic systems.

Keywords: Perturbed Gerdjikov–Ivanov (GI) equation; Spatio-temporal dispersion; Extended hyperbolic function method (EHFM); Generalized Kudryashov's method

1 Introduction

The experimental and theoretical studies show that solitons play a significant role in different fields such as fluid dynamics [1], nonlinear optics [2, 3], quantum electronics [4, 5], and plasma physics [6–8]. Nowadays, optical solitons have pervasive significance in the field of social media, transoceanic transmission, and in transcontinental services. Several equations such as Schrödinger–Hirota (SH) model, Chen–Lee–Liu (CLL) equation, nonlinear Schrödinger's equation (NLSE), Biswas–Arshed equation (BAE), and Manakov model have been employed in investigating the behavior of solitons in optical fibers, metamaterials, and couplers [9–13]. Optical solitons are fabricated by evaluating the dispersive and nonlinear terms in NLSE, and to study this, many mathematical approaches were constructed such as extended sinh-Gordon equation expansion method [14], extended trial equation method [15], Hirota method [16], Kudryashov's method [17], mapping method [18], generalized tanh method [19], extended direct algebraic method [20, 21], Nucci's reduction method [22], new auxiliary equation scheme [23, 24], and ϕ^6-model expansion [25].

In this work, we considered the perturbed Gerdjikov–Ivanov (GI) equation [26–28] which is the most familiar type of NLSE and has accrued attention since its evolution. The perturbed GI equation is one of the important dynamic models which describes the propagation of the ultrashort signal in photonic crystal fibers, fiber optics, and also demonstrates a significant role in nonlinear fiber optics. The GI model is distinct from NLSE because it is studied with a quintic nonlinearity [18] while NLSE carries a cubic nonlinearity [29, 30]. The dimensionless GI equation is described as [27]

$$\iota y_t + a y_{xx} + b|y|^4 y + \iota c y^2 y_x^* = 0, \tag{1}$$

where $y(x,t)$ is a complex-valued function showing the wave profile, the independent variables x and t indicate the distance along fiber and temporal variables, respectively, y^* stands for the complex conjugate of y, while a, b, and c are real-valued constants representing velocity dispersion, quintic nonlinearity, and nonlinear dispersion, respectively.

By adding the perturbation terms, Eq. (1) extends to the form [26]

$$\iota y_t + a y_{xx} + b|y|^4 y + \iota c y^2 y_x^* = \iota\left[\beta y_x + \lambda\left(|y|^2 y\right)_x + u\left(|y|^2\right)_x y\right], \tag{2}$$

where β, λ, and u are the coefficients of inter-model dispersion, self-steeping and dispersion of high order, respectively.

In the literature, many techniques are examined to probe the miscellaneous kinds of solutions of GI equation, e.g., the semi-inverse variational method has been investigated to extract the bright solitons of perturbed GI equation [31], the sine-Gordon method has been used to investigate the distinct types of solitons [32], singular and bright solitons have been observed by extended trial equation [33] and in [34, 35]. Recently, some new solutions were explored by applying algebro-geometric method, the Darboux transformation, tangent expansion method, and extended auxiliary equation technique [36–39].

This paper obtains the soliton solution for perturbed GI equation via extended hyperbolic function method [40–42] and generalized Kudryashov method [43–45]. The EHFM utilizes two types of ordinary differential equation (ODE), and being able to produce multiple types of solution is one of its strengths [46–49]. This versatility can be particularly useful in exploring the behavior of complex physical systems described by NLPDEs. The generalized Kudryashov's method is a valuable, simple, and compatible tool for researchers dealing with nonlinear equations; it has the ability to obtain a variety of solutions through parameter variation and has been applied on many equations [50–52]. Our presented approaches have the capability to provide different solutions, in particular periodic, singular, dark and bright soliton solutions. Meanwhile, other methods, like the unified approach, generate singular and periodic solutions, whereas the sine–cosine method provides periodic solitons [53]. The auxiliary equation method delivers dark and periodic soliton solutions [54]. Our knowledge and analysis of the literature confirms that the perturbed GI equation has not been solved yet via these two approaches.

The remaining paper is structured as follows. The algorithms of the extended hyperbolic function method and generalized Kudryashov's method are given in Sects. 2 and 3, respectively. Section 4 elaborates the formulation of equation. Sections 5 and 6 explain the solutions of perturbed GI using EHFM and generalized Kudryshaov's method, respectively. Section 7 provides the results and a discussion. In the last section, concluding remarks are given.

2 Extended hyperbolic function method

In this section, stepwise details of EHFM are given [40–42].

Consider the NLPDE

$$G(y, y_t, y_x, y_{tt}, y_{tx}, \dots) = 0, \tag{3}$$

where G is a polynomial in y and its derivatives.

In the first step, the governing equation is reduced using a transformation given by

$$y(x,t) = Y(\eta)e^{\iota\psi(x,t)}, \qquad \eta = x - \nu t, \qquad \psi = -qx + \varpi t + \vartheta, \tag{4}$$

where ν is any constant. This transformation changes Eq. (3) into a nonlinear ordinary differential equation (ODE) of the form

$$H\big(Y, Y', Y'', \dots\big) = 0, \tag{5}$$

where the primes indicate the derivatives with respect to η.

Suppose that the solution of Eq. (5) can be written as

$$Y(\eta) = \sum_{j=0}^{M} a_j \chi^j(\eta), \tag{6}$$

where $a_j \neq 0$, $\chi(\eta)$ is a real function, and M is found by using a homogeneous balancing rule.

Now $\chi(\eta)$ satisfies two different types of ODE.

Type 1:

$$\chi'(\eta) = \chi\sqrt{g + h\chi^2}, \quad g, h \in \mathbb{R}. \tag{7}$$

The above equation provides the following solutions:

Case 1: If $g > 0$, $h > 0$, then

$$\chi_1(\eta) = -\sqrt{\frac{g}{h}}\operatorname{csch}\sqrt{g}(\eta).$$

Case 2: If $g < 0$, $h > 0$, then

$$\chi_2(\eta) = \sqrt{\frac{-g}{h}}\sec\sqrt{-g}(\eta).$$

Case 3: If $g > 0$, $h < 0$, then

$$\chi_3(\eta) = \sqrt{\frac{g}{-h}}\operatorname{sech}\sqrt{g}(\eta).$$

Case 4: If $g < 0$, $h > 0$, then

$$\chi_4(\eta) = \sqrt{\frac{-g}{h}}\csc\sqrt{-g}(\eta).$$

Case 5: If $g < 0$, $h > 0$, then

$$\chi_5(\eta) = \cos\sqrt{-g}(\eta) + \iota\sin\sqrt{-g}.$$

Case 6: If $g = 0$, $h > 0$, then

$$\chi_6(\eta) = \frac{1}{\sqrt{h}(\eta)}.$$

Case 7: If $g = 0$, $h < 0$, then

$$\chi_7(\eta) = \frac{1}{\sqrt{-h}(\eta)}.$$

Type 2:

$$\chi'(\eta) = g + h\chi^2, \quad g, h \in \mathbb{R}. \tag{8}$$

Case 1: If $gh > 0$, then

$$\chi_8(\eta) = \operatorname{sgn}(g)\sqrt{\frac{g}{h}}\tan\bigl(\sqrt{gh}(\eta)\bigr).$$

Case 2: If $gh > 0$, then

$$\chi_9(\eta) = -\operatorname{sgn}(g)\sqrt{\frac{g}{h}}\cot\bigl(\sqrt{gh}(\eta)\bigr).$$

Case 3: If $gh < 0$, then

$$\chi_{10}(\eta) = sgn(g)\sqrt{\frac{-g}{h}}\tanh\bigl(\sqrt{-gh}(\eta)\bigr).$$

Case 4: If $gh < 0$,

$$\chi_{11}(\eta) = \operatorname{sgn}(g)\sqrt{\frac{-g}{h}}\coth\bigl(\sqrt{-gh}(\eta)\bigr).$$

Case 5: If $g = 0$, $h > 0$, then

$$\chi_{12}(\eta) = -\frac{1}{h(\eta)}.$$

Case 6: If $g < 0$, $h = 0$, then

$$\chi_{13}(\eta) = g(\eta).$$

At the end, putting Eq. (6) into Eq. (5) and by using Eqs. (7) and (8), the system of algebraic equations is obtained. By solving this system with the aid of Mathematica, the values of constants are retrieved.

3 Generalized Kudryashov's method

In this section, the main steps of generalized Kudryashov's method [43–45] are given.

In the first step, consider the NLPDE and wave transformation which converts the NLPDE into an ODE as described in Eqs. (3)–(5). According to the generalized Kudryashov's method, the solution of Eq. (5) is described as

$$Y(\eta) = \frac{\sum_{i=0}^{N} a_0 p^i(\eta)}{\sum_{j=0}^{M} b_0 p^j(\eta)}, \tag{9}$$

where $a_i, b_j \neq 0$, N and M are found via a homogeneous balancing rule, and $p(\eta)$ satisfies the following ODE:

$$p'(\eta) = p^2(\eta) - p(\eta), \tag{10}$$

where

$$p(\eta) = \frac{1}{1 + De^{\eta}}, \tag{11}$$

with D being a constant of integration.

In the last step, substitution of Eqs. (9) and (10) into Eq. (5) gives a polynomial in $p(\eta)$. After this, the coefficients next to all powers of $p(\eta)$ are equated to zero and, by solving the acquired system, the values of the required constants are attained.

4 Mathematical preliminaries

For solving Eq. (2), consider the following transformation:

$$y(x,t) = Y(\eta)e^{\iota\psi(x,t)}, \qquad \eta = x - \nu t, \qquad \psi = -qx + \varpi t + \vartheta, \tag{12}$$

where $y(x,t)$, q, ϖ, ν, and ϑ represent the phase component, frequency, wave number, velocity, and phase component of soliton, respectively.

By plugging Eq. (12) into Eq. (2) and then decomposing it into imaginary and real parts, the imaginary part yields

$$\nu = -2aq - \beta, \qquad c = -3\lambda + 2u,$$

while the real part gives

$$aY'' - \left(\varpi + aq^2 + \beta q\right)Y - (\lambda + c)qY^3 + bY^5 = 0. \tag{13}$$

The following transformation:

$$Y = Z^{\frac{1}{2}}, \tag{14}$$

reduces Eq. (13) to the following ODE:

$$a\left(-\left(Z'\right)^2 + 2ZZ''\right) - 4\left(\varpi + aq^2 + \beta q\right)Z^2 - 4(\lambda + c)qZ^3 + 4bZ^4 = 0. \tag{15}$$

5 Application of EHFM

In this section, EHFM is applied to extract the solution of the perturbed GI equation.

Type 1. By using the balancing rule, we get $M = 1$ in Eq. (15). By inserting Eqs. (6) and (7) into Eq. (15) and collecting the factors in front of powers of $\chi(\eta)$, a system of equations is obtained. By solving this system, the following values of constants are obtained:

$$a_0 = \frac{3(cq + \lambda q)}{8b}, \qquad g = \frac{3q^2(c+\lambda)^2}{16ab}, \qquad a_1 = \sqrt{-\frac{3ah}{4b}},$$
$$\varpi = -\frac{q(16ab + 16\beta bq + 15c^2q + 30cq\lambda + 15q\lambda^2)}{16b}.$$

Now by using these values in the solutions of Eq. (7), we obtain

Case 1: If $g > 0$, $h > 0$, then

$$y_1 = \left(\frac{3(cq+\lambda q)}{8b} - \frac{3}{8}\sqrt{-\frac{ah}{b}}\sqrt{\frac{q^2(c+\lambda)^2}{abh}}\operatorname{csch}\sqrt{\frac{3q^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 2: If $g < 0$, $h > 0$, then

$$y_2 = \left(\frac{3(cq+\lambda q)}{8b} + \frac{3}{8}\sqrt{-\frac{ah}{b}}\sqrt{-\frac{q^2(c+\lambda)^2}{abh}}\sec\sqrt{-\frac{3q^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 3: If $g > 0$, $h < 0$, then

$$y_3 = \left(\frac{3(cq+\lambda q)}{8b} + \frac{3}{8}\sqrt{-\frac{ah}{b}}\sqrt{-\frac{q^2(c+\lambda)^2}{abh}}\operatorname{sech}\sqrt{\frac{3q^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 4: If $g < 0$, $h < 0$, then

$$y_4 = \left(\frac{3(cq+\lambda q)}{8b} + \frac{3}{8}\sqrt{-\frac{ah}{b}}\sqrt{-\frac{q^2(c+\lambda)^2}{abh}}\csc\sqrt{\frac{-3q^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 5: If $g = 0$, $h > 0$, then

$$y_5 = \left(\frac{3(cq+\lambda q)}{8b} + \sqrt{-\frac{3ah}{4b}}\frac{1}{\sqrt{h}\eta}\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 6: If $g = 0$, $h < 0$, then

$$y_6 = \left(\frac{3(cq+\lambda q)}{8b} + \sqrt{-\frac{3ah}{4b}}\frac{1}{\sqrt{-h}\eta}\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Type 2. Similarly, by putting Eqs. (6) and (8) into Eq. (15), the system of equations is obtained. By solving this system, the following values of constants are obtained:

$$a_0 = \frac{3(cq+\lambda q)}{8b}, \qquad g = \frac{3q^2(c+\lambda)^2}{16ab}, \qquad a_1 = \sqrt{-\frac{3ah}{4b}},$$
$$\beta = \frac{-16abq^2 - 16b\varpi - 3c^2q^2 - 6c\lambda q^2 - 3\lambda^2q^2}{16bq}.$$

Now by inserting these values into the solutions of Eq. (7), we acquire

Case 1: If $gh > 0$, then

$$y_7 = \left(\frac{3(cq+\lambda q)}{8b} + \frac{3}{8}\sqrt{-\frac{ah}{b}}\operatorname{sgn}\left(\frac{3q^2(c+\lambda)^2}{16ab}\right)\sqrt{\frac{q^2(c+\lambda)^2}{abh}}\tan\sqrt{\frac{3hq^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 2: If $gh > 0$, then

$$y_8 = \left(\frac{3(cq+\lambda q)}{8b} - \frac{3}{8}\sqrt{-\frac{ah}{b}}\operatorname{sgn}\left(\frac{3q^2(c+\lambda)^2}{16ab}\right)\sqrt{\frac{q^2(c+\lambda)^2}{abh}}\cot\sqrt{\frac{3hq^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 3: If $gh < 0$, then

$$y_9 = \left(\frac{3(cq+\lambda q)}{8b} + \frac{3}{8}\sqrt{-\frac{ah}{b}}\operatorname{sgn}\left(\frac{3q^2(c+\lambda)^2}{16ab}\right)\sqrt{-\frac{q^2(c+\lambda)^2}{abh}}\tanh\sqrt{-\frac{3hq^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 4: If $gh < 0$, then

$$y_{10} = \left(\frac{3(cq+\lambda q)}{8b} + \frac{3}{8}\sqrt{-\frac{ah}{b}}\operatorname{sgn}\left(\frac{3q^2(c+\lambda)^2}{16ab}\right)\sqrt{\frac{q^2(c+\lambda)^2}{abh}}\coth\sqrt{\frac{3hq^2(c+\lambda)^2}{16ab}}\eta\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

Case 5: If $g = 0$, $h < 0$, then

$$y_{11} = \left(\frac{3(cq+\lambda q)}{8b} + \sqrt{-\frac{3ah}{4b}}\frac{1}{-h\eta}\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}.$$

6 Application of generalized Kudryashov's method

According to the conditions of generalized Kudryashov's method, the balancing rule in Eq. (15) is used, which creates the relation $M + 1 = N$, in which $M = 1$ delivers $N = 2$. Hence, Eq. (9) is written as

$$Z(\eta) = \frac{a_0 + a_1 p(\eta) + a_2 p(\eta)^2}{b_0 + b_1 p(\eta)}, \tag{16}$$

where $a_2, b_1 \neq 0$ and the remaining constants (a_0, a_1, and b_0) are to be found. Inserting Eqs. (16) and (10) into Eq. (15), the system of algebraic equations is obtained which produces the following constants:

$$a_2 = a_2, \qquad a_1 = a_1, \qquad b_0 = \frac{a_1 b_1}{a_2}, \qquad b_1 = b_1, \qquad a_0 = 0, \qquad \lambda = \frac{4a_2 b - 3b_1 cq}{3b_1 q}.$$

Inserting the above values, together with Eq. (11), into Eq. (16), we get

$$Z_{1,1} = \frac{a_2}{b_1(De^{\eta} + 1)}. \tag{17}$$

In hyperbolic form, this solution can be written as

$$Z_{1,1} = \frac{a_2}{b_1}\left(\frac{1}{(D+1)\cosh(\frac{\eta}{2}) + (D-1)\sinh(\frac{\eta}{2})}\right). \tag{18}$$

Now, by using Eqs. (14) and (12), we acquire

$$y_{1,1} = \left(\frac{a_2}{b_1}\left(\frac{1}{(D+1)\cosh(\frac{\eta}{2}) + (D-1)\sinh(\frac{\eta}{2})}\right)\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}. \tag{19}$$

Since D is the constant of integration, we can assign different values of D. If we designate $D = 1$, the above equation is written as

$$y_{1,2} = \left(\frac{a_2}{2b_1}\left(\operatorname{sech}\left(\frac{\eta}{2}\right)\right)\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}. \tag{20}$$

If $D = -1$ then Eq. (18) becomes

$$y_{1,3} = \left(\frac{a_2}{-2b_1}\left(\operatorname{csch}\left(\frac{\eta}{2}\right)\right)\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}. \tag{21}$$

If $D = 2$, then our solution becomes

$$y_{1,4} = \left(\frac{a_2}{b_1}\left(\frac{1}{3\cosh(\frac{\eta}{2}) + \sinh(\frac{\eta}{2})}\right)\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}. \tag{22}$$

If we put $D = 1$ in Eq. (17), the following solution is obtained:

$$y_{1,5} = \left(\frac{a_2}{-2b_1}\left(1 - \tanh\left(\frac{\eta}{2}\right)\right)\right)^{\frac{1}{2}} e^{\iota\psi(x,t)}. \tag{23}$$

7 Results and discussion

The soliton solutions obtained from the perturbed GI equation have many applications in the optical communication field, e.g., in optical fiber networks, where information is transmitted as light pulses and solitons play a significant role. These soliton solutions have the ability to maintain their shape and energy over long distances without distortion and transmit high speed data through an optical fiber, enhancing the efficiency of optical communication systems.

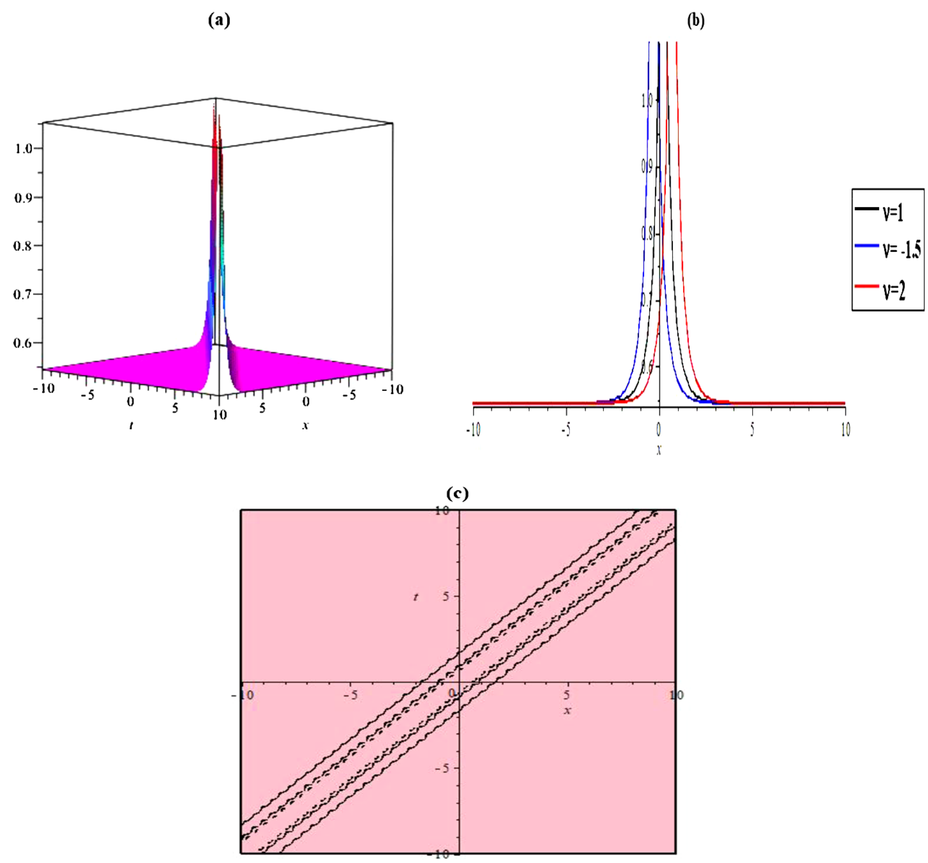

Figure 1 The 3D, 2D, and contour plots of a singular soliton solution $|y_1|$

Graphical illustration is the procedure to show how different parameters relate to each other. In this section, graphs of some obtained solutions are depicted in Figs. 1–4. Each figure contains three subgraphs in which (a), (b), and (c) are representing 3D, 2D, and contour graph, respectively. The parameters are selected in each graph by considering the constraints and definitions of the proposed method and equation. Figures 1–3 represent different shapes of a solution obtained by EHFM while Fig. 4 describes the solutions acquired by generalized Kudryashov's method. The effect of wave velocity ν on the propagation of waves is also checked. The different solutions of perturbed GI equation in the form of singular, dark, bright, and periodic singular solitons are accumulated. The graphs of $|y_1(x,t)|$, $|y_2(x,t)|$, $|y_9(x,t)|$, and $|y_{1,2}(x,t)|$ are shown by Figs. 1–4. The plots of other solutions are ignored to remove the uniformity.

In Fig. 1(a), we observe a singular soliton for $|y_1|$ under the following parameter values: $c = 0.98$, $q = -0.9$, $\lambda = -0.1$, $b = 1$, $a = 0.1$, $h = 1$, $\varpi = 0.98$, $\vartheta = 1$, and $\nu = 1$. This soliton shape shows a localized and solitary wave behavior. Figure 1(b) shows how the velocity affects the propagation of waves, as it is noticed that the wave moves to the right by decreasing the value of ν (black to blue) and on increasing the value of ν the wave shifts to the left (blue to red). The change in the wave velocity also shows the variations in the phase component. To further visualize the singular soliton, Fig. 1(c) depicts a contour graph. This representation helps in understanding the spatial distribution of the wave amplitude and phase.

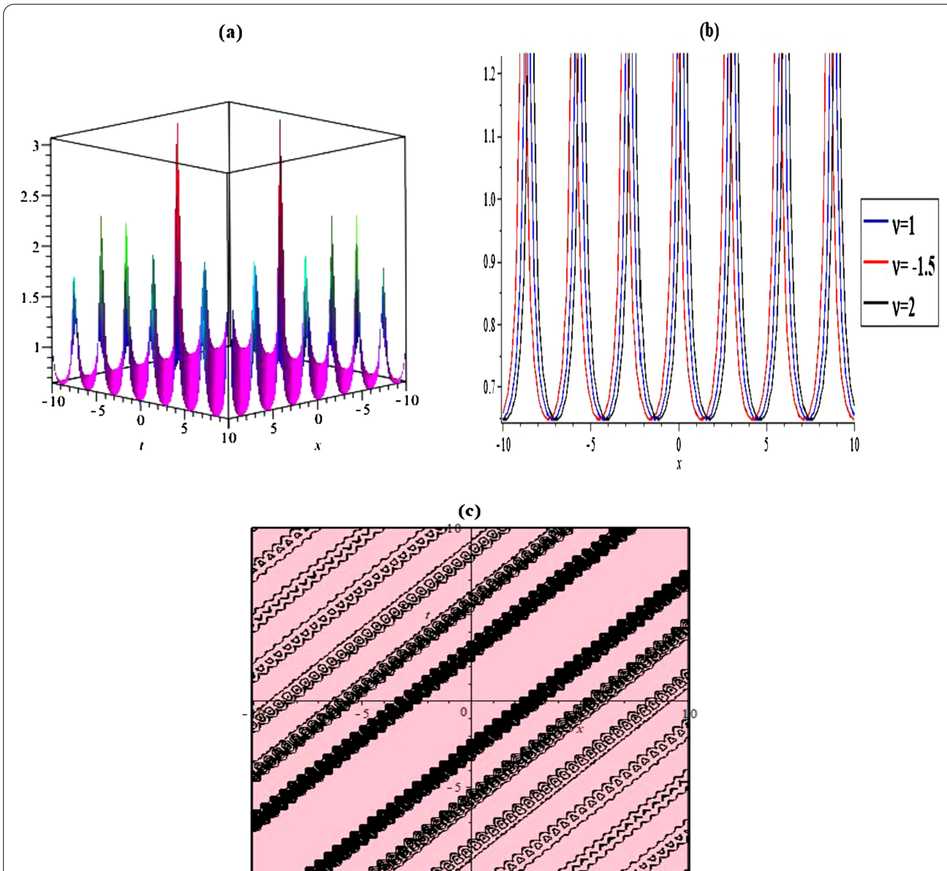

Figure 2 The 3D, 2D, and contour plots of a periodic singular soliton solution $|y_2|$

Figure 2(a) presents the periodic singular soliton of $|y_2|$ for the values of $q = -0.9$, $\lambda = -0.1$, $b = 1$, $a = 0.1$, $h = 1$, $\varpi = 0.98$, $\vartheta = 1$, and $\nu = 1$. This soliton has a periodic, repeating shape and is characterized by specific parameter values. Similarly, Fig. (2b) shows that the position of the wave slightly changes by decreasing and increasing the value of ν. The contour graph of the solution is given in Fig. 2(c), providing a perspective on the soliton's evolution.

Figure 3(a) represents the solution $|y_9|$ demonstrating the dark soliton for the values of $q = -0.9$, $\lambda = -0.1$, $b = 1$, $a = 0.1$, $h = 1$, $\varpi = 0.98$, $\vartheta = 1$, and $\nu = 1$, and exhibits a unique characteristic low-intensity dip in the wave amplitude. On the other hand, when we increase and decrease the value of ν, the wave moves to the right and left, respectively, as shown in Fig. 3(b). By changing the value of ν, the change in the phase component is also noticed. The contour envelope is plotted in Fig. 3(c).

Furthermore, $|y_{1,2}|$ reveals the shape entitled as a bright soliton, having a high-intensity peak. The considered parameters in Fig. 4(a) are $c = 0.98$, $q = -0.9$, $\lambda = -0.1$, $b = 1$, $a = 0.1$, $h = 1$, $\varpi = 0.98$, $\vartheta = 1$, and $\nu = 1$. The 2D profile is outlined in Fig. 4(b), and it is observed that the change in ν shifts the wave to the right or left and also causes variation in the phase component. The contour graph for $|y_{1,2}|$ is delineated in Fig. 4(c).

Collectively, these figures illustrate the diverse behaviors of solitons under the influence of different wave velocities and parameter settings, enriching our knowledge on the dynamics of wave propagation.

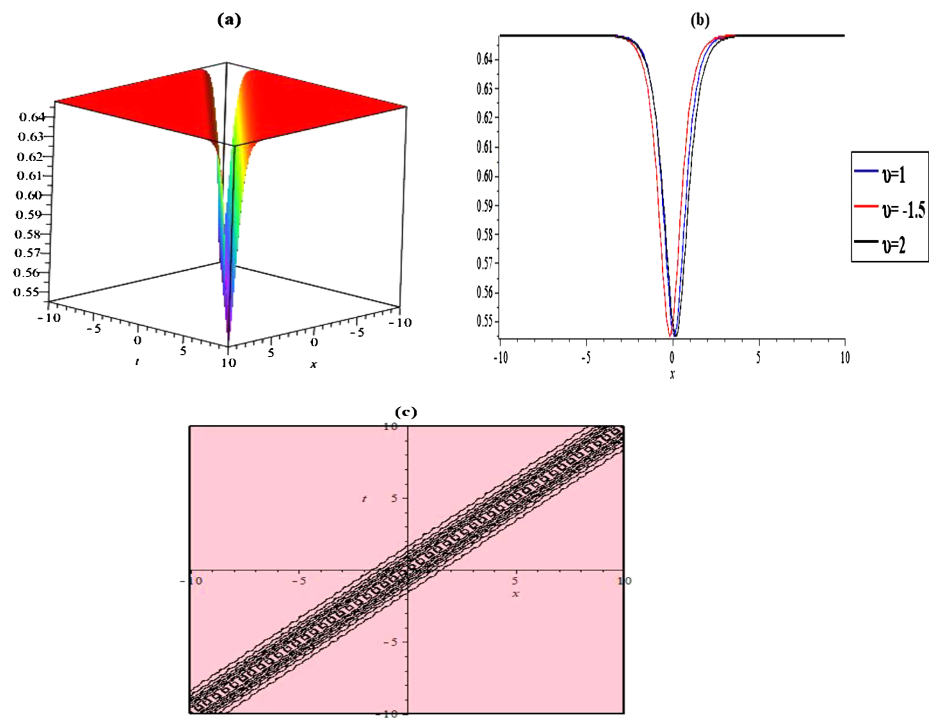

Figure 3 The 3D, 2D, and contour plots of a dark soliton solution $|y_9|$

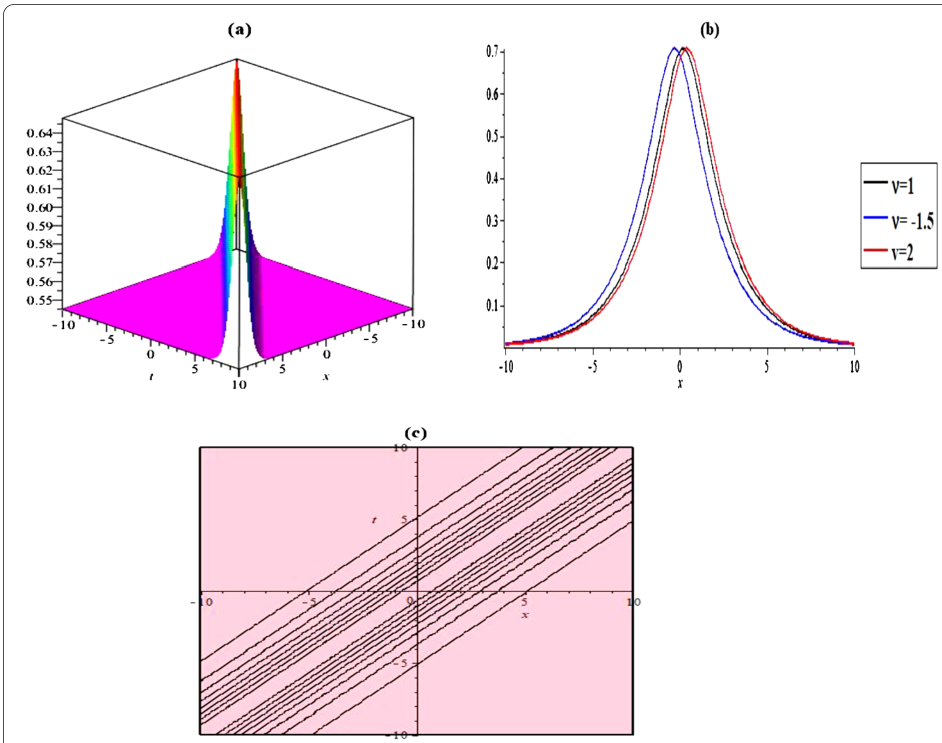

Figure 4 The 3D, 2D, and contour plots of a bright soliton solution $|y_{1,2}|$

8 Conclusions

This research was concentrated on solving the perturbed GI equation with spatio-temporal quintic nonlinearity and velocity dispersion with the help of EHFM and generalized Kudryashov's method particularly in the context of optical fibers. The outcome of this effort is the extraction of various types of solutions, including dark, bright, periodic, and singular periodic solutions. The comparison between achieved and existing solutions confirmed that the attained solutions are distinct from the prior findings in the existing literature. For the verification of solution, Mathematica 11 was used to put these solutions into the governing equation, which confirmed that solutions are accurate. The 3D, 2D, and contour graphs were sketched by using Maple 18. The behavior of waves by changing the value of velocity parameter was also highlighted in detail. Furthermore, the obtained solutions disclosed that the proposed methods are suitable tools for extracting a variety of solutions of nonlinear equations having a high level of potency in nonlinear fields. In the future, an investigation of the Lie symmetry method for this model can help to uncover hidden symmetries and conservation laws within the system and will provide a deeper understanding of its behavior.

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Author contributions
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