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Research Article

Several Existence Theorems of Monotone Positive Solutions for Third-Order Multipoint Boundary Value Problems

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Using fixed point index theory, we obtain several sufficient conditions of existence of at least one positive solution for third-order m-point boundary value problems.

1. Introduction

We are concerned with the existence of positive solutions for the following third-order multipoint boundary value problems:

$$\begin{gathered} u'''(t)+h(t)f\left(t,u(t),u'(t)\right)=0, \quad \text{a.e. } t\in[0,1], \\ u'(0)=u''(0)=0, \qquad u(1)=\sum_{i=1}^{m-2}\alpha_i u(\xi_i), \end{gathered} \tag{1.1}$$

where $0<\xi_1<\xi_2<\cdots<\xi_{m-2}<1$, $\alpha_i>0(i=1,2,\dots,m-2)$, $0<\sum_{i=1}^{m-2}\alpha_i<1$,$h(t)$ may be singular at any point of $[0,1]$ and $f(t,u,v)$ satisfies Carathéodory condition.

Third-order boundary value problem arises in boundary layer theory, the study of draining and coating flows. By using the Leray-Schauder continuation theorem, the coincidence degree theory, Guo-Krasnoselskii fixed point theorem, the Leray-Schauder nonlinear alternative theorem, and upper and lower solutions method, many authors have studied certain boundary value problems for nonlinear third-order ordinary differential equations. We refer the reader to [1–7] and references cited therein. By using the Leray-Schauder nonlinear alternative theorem, Zhang et al. [1] studied the existence of at least one nontrivial solution for the following third-order eigenvalue problems:

$$\begin{gathered} u'''(t) = \lambda f(t,u,u'), \quad 0 < t < 1, \\ u(0) = u'(\eta) = u''(0) = 0, \end{gathered} \tag{1.2}$$

where $\lambda > 0$ is a parameter, $1/2 \le \eta < 1$ is a constant, and $f : [0,1] \times R \times R \to R$ is continuous.

By using Guo-Krasnoselskii fixed point theorem, Guo et al. [2] investigated the existence of at least one positive solution for the boundary value problems

$$\begin{gathered} u'''(t) + a(t)f(u(t)) = 0, \quad 0 < t < 1, \\ u(0) = u'(0) = 0, \qquad u'(1) = \alpha u'(\eta), \end{gathered} \tag{1.3}$$

where $0 < \eta < 1$, $1 < \alpha < 1/\eta$, and $a(t)$ and $f(u)$ are continuous.

The aim of this paper is to establish some results on existence of monotone positive solutions for problems (1.1). To do this, we give at first the associated Green function and its properties. Then we obtain several theorems of existence of monotone positive solutions by using the fixed point index theory. Our results differ from those of [1–3] and extend the results of [1–3]. Particularly, we do not need any continuous assumption on the nonlinear term, which is essential for the technique used in [1–3].

We always suppose the following conditions are satisfied:

(C_1) $\alpha_i > 0$ $(i = 1,2,\dots,m-2)$, $\sum_{i=1}^{m-2}\alpha_i < 1$, $1 = \xi_0 < \xi_1 < \xi_2 < \cdots < \xi_{m-1} = 1$;

(C_2) $h(t) \in L^1[0,1]$, $h(t) \ge 0$, a.e. $t \in [0,1]$, $\int_0^{\xi_{m-2}} h(t)dt > 0$;

(C_3) $f : [0,1] \times [0,\infty) \times (-\infty,0] \to [0,\infty)$ satisfies Carathéodory conditions, that is, $f(\cdot,u,v)$ is measurable for each fixed $u \in [0,\infty), v \in (-\infty,0]$ and $f(t,\cdot,\cdot)$ is continuous for a.e. $t \in [0,1]$;

(C_4) for any $r, r' > 0$, there exists $\Phi(t) \in L^\infty[0,1]$ such that $f(t,u,v) \le \Phi(t)$, where $(u,v) \in [0,r] \times [-r',0]$, a.e. $t \in [0,1]$.

2. Preliminary lemmas

Lemma 2.1 (Krein-Rutman [8]). *Let K be a reproducing cone in a real Banach space X and let $L : X \to X$ be a compact linear operator with $L(K) \subseteq K$. $r(L)$ is the spectral radius of L. If $r(L) > 0$, then there exists $\varphi_1 \in K \setminus \{0\}$ such that $L\varphi_1 = r(L)\varphi_1$.*

Lemma 2.2 [9]. *Let X be a Banach space, P a cone in X, and $\Omega(P)$ a bounded open subset in P. Suppose that $A : \overline{\Omega(P)} \to P$ is a completely continuous operator. Then the following results hold.*

(1) *If there exists $u_0 \in P \setminus \{0\}$ such that $u \ne Au + \lambda u_0$, for all $u(t) \in \partial\Omega(P)$, $\lambda \ge 0$, then the fixed point index $i(A,\Omega(P),P) = 0$.*

(2) *If $0 \in \Omega(P)$ and $Au \ne \lambda u, \forall u(t) \in \partial\Omega(P), \lambda \ge 1$, then the fixed point index $i(A, \Omega(P),P) = 1$.*

We can easily get the following lemmas.

Lemma 2.3. *Suppose* $\sum_{i=1}^{m-2}\alpha_i \neq 1$. *If* $y(t) \in L^1[0,1]$, *then the problem*

$$\begin{gathered} u'''(t)+y(t)=0, \quad a.e.\ t\in[0,1], \\ u'(0)=u''(0)=0, \qquad u(1)=\sum_{i=1}^{m-2}\alpha_i u(\xi_i) \end{gathered} \tag{2.1}$$

has a unique solution:

$$\begin{aligned} u(t) = &-\frac{1}{2}\int_0^t (t-s)^2 y(s)ds + \frac{1}{2(1-\sum_{i=1}^{m-2}\alpha_i)}\int_0^1 (1-s)^2 y(s)ds \\ &-\frac{1}{2(1-\sum_{i=1}^{m-2}\alpha_i)}\sum_{i=1}^{m-2}\alpha_i\int_0^{\xi_i}(\xi_i-s)^2 y(s)ds. \end{aligned} \tag{2.2}$$

Lemma 2.4. *Suppose* $0<\sum_{i=1}^{m-2}\alpha_i<1$, $y(t)\in L^1[0,1]$, $y(t)\geq 0$. *Then the unique solution of (2.1) satisfies* $u(t)\geq 0$, $u'(t)\leq 0$.

Lemma 2.5. *Suppose* $0<\sum_{i=1}^{m-2}\alpha_i<1$. *The Green function for the boundary value problem*

$$\begin{gathered} -u'''(t)=0, \quad 0<t<1, \\ u'(0)=u''(0)=0, \qquad u(1)=\sum_{i=1}^{m-2}\alpha_i u(\xi_i) \end{gathered} \tag{2.3}$$

is given by

$$G(t,s)=\begin{cases} \dfrac{(1-s)^2-\sum_{j=\omega}^{m-2}\alpha_j(\xi_j-s)^2-(1-\sum_{i=1}^{m-2}\alpha_i)(t-s)^2}{2(1-\sum_{i=1}^{m-2}\alpha_i)}, \\ \qquad 0\leq t\leq 1,\ \xi_{\omega-1}\leq s\leq \min\{\xi_\omega,t\},\ \omega=1,2,\ldots,m-1, \\ \dfrac{(1-s)^2-\sum_{j=\omega}^{m-2}\alpha_j(\xi_j-s)^2}{2(1-\sum_{i=1}^{m-2}\alpha_i)}, \\ \qquad 0\leq t\leq 1,\ \max\{\xi_{\omega-1},t\}\leq s\leq \xi_\omega,\ \omega=1,2,\ldots,m-1. \end{cases} \tag{2.4}$$

Obviously, $G(t,s)$ *is nonnegative and continuous in* $[0,1]\times[0,1]$, *and*

$$G(t,s)\geq \frac{(1-\xi_{m-2})^2}{2(1-\sum_{i=1}^{m-2}\alpha_i)}, \quad t,s\in[0,\xi_{m-2}]. \tag{2.5}$$

3. Main results

Let $X=C^1[0,1]$ with norm $\|x\|=\max_{t\in[0,1]}|x(t)|+\max_{t\in[0,1]}|x'(t)|$. Clearly, $(X,\|\cdot\|)$ is a Banach space. Take $P=\{u\in X \mid u\geq 0,\ u'\leq 0\}$, $P_r=\{u\in P\mid \|u\|<r\}$, $r>0$. Obviously, P is a cone in X and P_r is an open bounded subset in P.

Lemma 3.1. *P is a reproducing cone in X.*

Proof. Let $x \in X$, then $x' \in C[0,1]$ and $x' = x^+ - x^-$, where $x^+ = \max\{x'(t),0.\}, x^- = \max\{-x'(t),0.\}$. Obviously, $x^+, x^- \in C[0,1]$ and $x^+ \geq 0$, $x^- \geq 0$. Integrating $x' = x^+ - x^-$ from t to 1, we get

$$x(t) = -\int_t^1 x^+(s)ds + \int_t^1 x^-(s)ds + x(1). \tag{3.1}$$

If $x(1) \geq 0$, let $x_1(t) = \int_t^1 x^-(s)ds + x(1)$, $x_2(t) = \int_t^1 x^+(s)ds$, then $x_1, x_2 \in P$, and $x = x_1 - x_2$. If $x(1) < 0$, let $x_1(t) = \int_t^1 x^-(s)ds$, $x_2(t) = \int_t^1 x^+(s)ds - x(1)$, then $x_1, x_2 \in P$, and $x = x_1 - x_2$. The proof is completed. □

Define operators $A : P \to X$, $L : X \to X$ as follows:

$$\begin{aligned} Au &= \int_0^1 G(t,s)h(s)f(s,u(s),u'(s))ds, \\ Lu &= \int_0^1 G(t,s)h(s)(u(s) - u'(s))ds. \end{aligned} \tag{3.2}$$

By Lemma 2.3, we get that if $u(t) \in P \setminus \{0\}$ is a fixed point of A, then $u(t)$ is a monotone positive solution of (1.1). Assume (C_1)–(C_4) hold, then we can easily get that $A : P \to P$ and $L : P \to P$ are completely continuous by the absolute continuity of integral, Ascoli-Arzela theorem, Lemmas 2.3, 2.4, and 2.5.

Lemma 3.2. *Suppose* (C_1)–(C_2) *hold; then* $r(L) > 0$.

Proof. Take $u(t) \equiv 1$. For $t \in [0, \xi_{m-2}]$ we get

$$\begin{aligned} Lu(t) &= \int_0^1 G(t,s)h(s)ds \geq \int_0^{\xi_{m-2}} G(t,s)h(s)ds \geq \frac{(1-\xi_{m-2})^2}{2(1-\sum_{i=1}^{m-2}\alpha_i)} \int_0^{\xi_{m-2}} h(s)ds := l > 0. \\ L^2u(t) &\geq \int_0^1 G(t,s)h(s)Lu(s)ds \geq \int_0^{\xi_{m-2}} G(t,s)h(s)Lu(s)ds \geq l^2. \end{aligned} \tag{3.3}$$

By mathematical induction, it can be proved that

$$L^n u(t) \geq l^n, \quad \forall t \in [0, \xi_{m-2}]. \tag{3.4}$$

Hence

$$\|L^n\|^{1/n} \geq l, \quad r(L) = \lim_{n\to\infty} \|L^n\|^{1/n} \geq l > 0. \tag{3.5}$$

The proof is completed. □

By Lemma 2.1, we get that L has an eigenfunction $\varphi \in P \setminus \{0\}$ corresponding to $r(L)$. Let $\mu = 1/r(L)$.

For convenience, we make the following definitions:

$$\overline{f}(u,v) = \sup_{t\in[0,1]\setminus E} f(t,u,v), \quad \underline{f}(u,v) = \inf_{t\in[0,1]\setminus E} f(t,u,v),$$
$$f_{c,0} = \max\left\{\liminf_{u\to 0^+}\left\{\inf_{v\in[-c,0]}\frac{\underline{f}(u,v)}{u-v}\right\},\ \liminf_{v\to 0^-}\left\{\inf_{u\in[0,c]}\frac{\underline{f}(u,v)}{u-v}\right\}\right\}, \tag{3.6}$$
$$f^\infty = \max\left\{\limsup_{u\to\infty}\left\{\sup_{v\in R^-}\frac{\overline{f}(u,v)}{u-v}\right\},\ \limsup_{v\to-\infty}\left\{\sup_{u\in R^+}\frac{\overline{f}(u,v)}{u-v}\right\}\right\},$$

where $c>0$, $R^+=[0,\infty)$, $R^-=(-\infty,0]$, $E\subset[0,1]$ with null Lebesgue measure.

Lemma 3.3. *Suppose (C_1)–(C_4) hold. In addition, suppose $0\le f^\infty<\mu$, then there exists $r_0>0$ such that*

$$i(A,P_r,P)=1 \quad \textit{for each } r>r_0. \tag{3.7}$$

Proof. Let $\varepsilon>0$ be small enough such that $f^\infty<\mu-\varepsilon$. Then there exists $r_1>0$ such that

$$f(t,u,v)\le(\mu-\varepsilon)(u-v) \quad \text{for } u>r_1,\ \text{ or } v<-r_1,\ \text{ a.e. } t\in[0,1]. \tag{3.8}$$

By (C_4), there exists $\Phi\in L^\infty[0,1]$ such that

$$f(t,u,v)\le\Phi(t) \quad \text{for } u,v\in[0,r_1]\times[-r_1,0],\ \text{a.e. } t\in[0,1]. \tag{3.9}$$

So we get that for all $u\in R^+, v\in R^-$, a.e. $t\in[0,1]$,

$$f(t,u,v)\le(\mu-\varepsilon)(u-v)+\Phi(t). \tag{3.10}$$

Since $1/\mu$ is the spectrum radius of L, $(I/(\mu-\varepsilon)-L)^{-1}$ exists. Let

$$C=\left\|\int_0^1 G(t,s)h(s)\Phi(s)ds\right\|, \qquad r_0=\left\|\left(\frac{1}{\mu-\varepsilon}I-L\right)^{-1}\frac{C}{\mu-\varepsilon}e^{1-t}\right\|. \tag{3.11}$$

We will show that for $r>r_0$,

$$Au\neq\lambda u \quad \text{for each } u\in\partial P_r,\ \lambda\ge 1. \tag{3.12}$$

In fact, if not, there exist $u_0\in\partial P_r$, $\lambda_0\ge 1$ such that $Au_0=\lambda_0u_0$. This, together with (3.10) and Lemma 2.4, implies

$$\begin{aligned} u_0 &\le \lambda_0u_0 = Au_0 \le (\mu-\varepsilon)Lu_0+C,\\ u_0' &\ge \lambda_0u_0' = (Au_0)' \ge (\mu-\varepsilon)(Lu_0)'-C. \end{aligned} \tag{3.13}$$

Thus,

$$\left(\frac{1}{\mu-\varepsilon}I-L\right)u_0(t)\le\frac{C}{\mu-\varepsilon}e^{1-t}, \qquad \left(\left(\frac{1}{\mu-\varepsilon}I-L\right)u_0(t)\right)'\ge\left(\frac{C}{\mu-\varepsilon}e^{1-t}\right)'. \tag{3.14}$$

So, we get

$$\frac{C}{\mu-\varepsilon}e^{1-t}-\left(\frac{1}{\mu-\varepsilon}I-L\right)u_0(t)\in P. \tag{3.15}$$

It follows from $((1/(\mu-\varepsilon))I-L)^{-1}=\sum_{n=0}^{\infty}(\mu-\varepsilon)^{n+1}L^n$ and $L(P)\subset P$ that

$$u_0(t)\le\left(\frac{1}{\mu-\varepsilon}I-L\right)^{-1}\frac{C}{\mu-\varepsilon}e^{1-t},\qquad u_0'(t)\ge\left[\left(\frac{1}{\mu-\varepsilon}I-L\right)^{-1}\frac{C}{\mu-\varepsilon}e^{1-t}\right]'. \tag{3.16}$$

Therefore, we have $\|u_0\|\le r_0<r$; this is a contradiction.

By (2) of Lemma 2.2, we get that $i(A,P_r,P)=1$, for each $r>r_0$. The proof is completed. □

Lemma 3.4. *Suppose* (C_1)–(C_4) *hold and there exists* $c>0$ *satisfying* $\mu<f_{c,0}\le\infty$, *then there exists* $0<\rho_0\le c$ *such that for* $\rho\in(0,\rho_0]$, *if* $u\neq Au$ *for* $u\in\partial P_\rho$, *then* $i(A,P_\rho,P)=0$.

Proof. Let $\varepsilon>0$ be small enough such that $f_{c,0}>\mu+\varepsilon$. Then there exists $0<\rho_0\le c$ such that

$$f(t,u,v)\ge(\mu+\varepsilon)(u-v)\quad\text{for } 0\le u\le\rho_0,\ -\rho_0\le v\le 0,\ \text{a.e. } t\in[0,1]. \tag{3.17}$$

Let $\rho\in(0,\rho_0]$. Considering of (1) of Lemma 2.2, we need only to prove that

$$u\neq Au+\lambda\varphi\quad\text{for each } u\in\partial P_\rho,\ \lambda>0, \tag{3.18}$$

where $\varphi\in P\setminus\{0\}$ is the eigenfunction of L corresponding to $r(L)$.

In fact, if not, there exist $u_0\in\partial P_\rho$, $\lambda_0>0$ such that $u_0=Au_0+\lambda_0\varphi$. This implies $u_0\ge\lambda_0\varphi$ and $u_0'\le\lambda_0\varphi'$. Let

$$\lambda^*=\sup\{\lambda\mid u_0\ge\lambda\varphi,\ u_0'\le\lambda\varphi'\}. \tag{3.19}$$

Clearly, $\infty>\lambda^*\ge\lambda_0>0$, $u_0\ge\lambda^*\varphi$, $u_0'\le\lambda^*\varphi'$. Therefore, we get $u_0-\lambda^*\varphi\in P$. It follows from $L(P)\subset P$ that

$$\mu Lu_0\ge\lambda^*\mu L\varphi=\lambda^*\varphi,\qquad \mu(Lu_0)'\le\lambda^*\mu(L\varphi)'=\lambda^*(\varphi)'. \tag{3.20}$$

By (3.17) and Lemma 2.4, we get

$$Au_0\ge(\mu+\varepsilon)Lu_0,\qquad (Au_0)'\le(\mu+\varepsilon)(Lu_0)'. \tag{3.21}$$

So, we have

$$\begin{aligned} u_0&=Au_0+\lambda_0\varphi\ge(\mu+\varepsilon)Lu_0+\lambda_0\varphi\ge(\lambda^*+\lambda_0)\varphi,\\ (u_0)'&=(Au_0)'+\lambda_0(\varphi)'\le(\mu+\varepsilon)(Lu_0)'+\lambda_0(\varphi)'\le(\lambda^*+\lambda_0)(\varphi)', \end{aligned} \tag{3.22}$$

which contradict the definition of λ^*. So, Lemma 3.4 holds. □

In the following theorems, we always suppose (C_1)–(C_4) hold.

Theorem 3.5. *Assume that there exists $c > 0$ such that $\mu < f_{c,0} \leq \infty$, and $0 \leq f^{\infty} < \mu$, then (1.1) have at least one positive solution.*

Proof. It follows from $0 \leq f^{\infty} < \mu$ and Lemma 3.3 that there exists $r > 0$ such that $i(A, P_r, P) = 1$. By $\mu < f_{c,0} \leq \infty$ and Lemma 3.4, we get that there exists $0 < \rho < \min\{r, c\}$ such that either there exists $u \in \partial P_\rho$ satisfying $u = Au$ or $i(A, P_\rho, P) = 0$. In the second case, A has a fixed point $u \in P$ with $\rho < \|u\| < r$ by the properties of index. The proof is completed. □

Theorem 3.6. *Assume that the following assumptions are satisfied.*

(H_1) *There exists $c > 0$ such that $\mu < f_{c,0} \leq \infty$.*

(H_2) *There exists $\rho_1 > 0$ such that*

$$f(t,u,v) \leq m_0 \rho_1 \quad \text{for } u \in [0, \rho_1],\ v \in [-\rho_1, 0],\ \text{a.e. } t \in [0,1], \tag{3.23}$$

where $m_0 = 1/\| \int_0^1 G(t,s)h(s)ds\|$.

Then (1.1) have at least one positive solution.

Proof. For $u \in \partial P_{\rho_1}$, by (3.23) and Lemma 2.4, we obtain

$$\begin{aligned}
\|Au\| &= \max_{t\in[0,1]} Au + \max_{t\in[0,1]} (-Au)' \\
&= \max_{t\in[0,1]} \int_0^1 G(t,s)h(s)f(s,u(s),u'(s))\,ds + \max_{t\in[0,1]} \left(- \int_0^1 G(t,s)h(s)f(s,u(s),u'(s))\,ds \right)' \\
&\leq m_0\rho_1 \left[\max_{t\in[0,1]} \int_0^1 G(t,s)h(s)ds + \max_{t\in[0,1]} \left(- \int_0^1 G(t,s)h(s)ds \right)' \right] \leq \rho_1.
\end{aligned} \tag{3.24}$$

This implies $Au \neq \lambda u$ for each $u \in \partial P_{\rho_1}, \lambda > 1$. If $Au \neq u$ for $u \in \partial P_{\rho_1}$, by (2) of Lemma 2.2 we get $i(A, P_{\rho_1}, P) = 1$.

It follows from $\mu < f_{c,0} \leq \infty$ and Lemma 3.4 that there exists $0 < \rho < \min\{c, \rho_1\}$ such that either there exists $u \in \partial P_\rho$ satisfying $u = Au$ or $i(A, P_\rho, P) = 0$.

Suppose $Au \neq u$ for $u \in \partial P_{\rho_1} \cup \partial P_\rho$ (otherwise the proof is completed), by the properties of index we get that A has a fixed point $u \in P$ satisfying $\rho < \|u\| < \rho_1$. So Theorem 3.6 holds. □

Theorem 3.7. *Assume that the following assumptions are satisfied.*

(H_3) $0 \leq f^{\infty} < \mu$.

(H_4) *There exists $\rho_2 > 0$ such that*

$$f(t,u,v) \geq M_0 \rho_2 \quad \text{for } u \in [0, \rho_2],\ v \in [-\rho_2, 0],\ \text{a.e. } t \in [0,1], \tag{3.25}$$

where $M_0 = 1/\min_{t\in[0,\xi_{m-2}]}[\int_0^1 G(t,s)h(s)ds - (\int_0^1 G(t,s)h(s)ds)']$.

Then (1.1) have at least one positive solution.

Proof. For $u \in \partial P_{\rho_2}$, $t \in [0, \xi_{m-2}]$, by (3.25) and Lemma 2.4 we get

$$\begin{aligned} Au - (Au)' &= \int_0^1 G(t,s)h(s)f(s,u(s),u'(s))\,ds - \left(\int_0^1 G(t,s)h(s)f(s,u(s),u'(s))\,ds\right)' \\ &\geq M_0\rho_2\left[\int_0^1 G(t,s)h(s)ds - \left(\int_0^1 G(t,s)h(s)ds\right)'\right] \geq \rho_2. \end{aligned} \tag{3.26}$$

This implies $u \neq Au + \lambda\varphi$, for $u \in \partial P_{\rho_2}, \lambda > 0$, where $\varphi \in P \setminus \{0\}$ is the eigenfunction of L corresponding to $r(L)$. Suppose $u \neq Au$, for $u \in \partial P_{\rho_2}$ (otherwise, the proof is completed), by (1) of Lemma 2.2 we get $i(A, P_{\rho_2}, P) = 0$.

By $0 \leq f^\infty < \mu$ and Lemma 3.3, we get that there exists $r > \rho_2$ such that $i(A, P_r, P) = 1$. By the properties of index, we get that A has a fixed point u satisfying $\rho_2 < \|u\| < r$. The proof is completed. □

Theorem 3.8. *Assume that there exist ρ_1, ρ_2 satisfying $0 < \rho_2 < \rho_1 m_0/M_0$ such that (3.23) and (3.25) hold, where m_0, M_0 are the same as in Theorems 3.6 and 3.7. Then (1.1) have at least one positive solution.*

Proof. By the proving process of Theorems 3.6 and 3.7, we can easily get this result. □

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