Hindawi Publishing Corporation
Boundary Value Problems
Volume 2008, Article ID 236386, 9 pages
doi:10.1155/2008/236386

Research Article
Nonlinear Systems of Second-Order ODEs

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Received 2 February 2007; Accepted 16 November 2007

Recommended by Jean Mawhin

We study existence of positive solutions of the nonlinear system $-(p_1(t,u,v)u')' = h_1(t)f_1(t,u,v)$ in $(0,1)$; $-(p_2(t,u,v)v')' = h_2(t)f_2(t,u,v)$ in $(0,1)$; $u(0) = u(1) = v(0) = v(1) = 0$, where $p_1(t,u,v) = 1/(a_1(t) + c_1g_1(u,v))$ and $p_2(t,u,v) = 1/(a_2(t) + c_2g_2(u,v))$. Here, it is assumed that g_1, g_2 are nonnegative continuous functions, $a_1(t)$, $a_2(t)$ are positive continuous functions, $c_1, c_2 \geq 0$, $h_1, h_2 \in L^1(0,1)$, and that the nonlinearities f_1, f_2 satisfy superlinear hypotheses at *zero* and $+\infty$. The existence of solutions will be obtained using a combination among the method of truncation, a priori bounded and Krasnosel'skii well-known result on fixed point indices in cones. The main contribution here is that we provide a treatment to the above system considering differential operators with nonlinear coefficients. Observe that these coefficients may not necessarily be bounded from below by a positive bound which is independent of u and v.

1. Introduction

We study existence of positive solutions for the following nonlinear system of second-order ordinary differential equations:

$$\begin{gathered}
-\left(\frac{u'}{a_1(t) + c_1g_1(u,v)}\right)' = h_1(t)f_1(t,u,v) \quad \text{in } (0,1), \\
-\left(\frac{v'}{a_2(t) + c_2g_2(u,v)}\right)' = h_2(t)f_2(t,u,v) \quad \text{in } (0,1), \\
u(0) = u(1) = v(0) = v(1) = 0,
\end{gathered} \tag{1.1}$$

where c_1, c_2 are nonnegatives constants, the functions $a_1, a_2 : [0,1] \to (0,+\infty)$ are continuous, the functions $f_1, f_2 : [0,1] \times [0,+\infty)^2 \to [0,+\infty)$ are continuous, and $h_1, h_2 \in L^1(0,1)$. We will suppose the following four hypotheses.

(H_1) We have

$$\lim_{u+v\to 0}\frac{f_1(t,u,v)}{u+v}=0,\qquad \lim_{u+v\to 0}\frac{f_2(t,u,v)}{u+v}=0, \tag{1.2}$$

uniformly for all $t\in[0,1]$.

(H_2) There exist $p,q>1$, $\eta_i>0$, and $0<\alpha_i<\beta_i<1$ for $i=1,2$, such that

$$\begin{aligned} f_1(t,u,v)&\geq \eta_1 u^p \quad \forall u\geq 0,\ t\in(\alpha_1,\beta_1),\\ f_2(t,u,v)&\geq \eta_2 v^q \quad \forall v\geq 0,\ t\in(\alpha_2,\beta_2). \end{aligned} \tag{1.3}$$

(H_3) The functions $g_1,g_2:[0,+\infty)^2\to[0,+\infty)$ are continuous and

$$\lim_{u\to+\infty} g_i(u,u)=+\infty,\quad \text{for } i=1,2. \tag{1.4}$$

In addition, we suppose that there exists an $n_*\in\mathbb{N}$ such that g_1, g_2 are nondecreasing for all $u^2+v^2\geq n_*^2$. Here, g_1, g_2 are nondecreasing, meaning that

$$g_i(u_1,v_1)\leq g_i(u_2,v_2),\quad \text{for } i=1,2, \tag{1.5}$$

whenever $(u_1,v_1)\leq(u_2,v_2)$, where the inequality is understood inside every component.

(H_4) We have

$$\lim_{n\to+\infty}\frac{g_1(n,n)}{n^{r/(p+1)}}=0,\qquad \lim_{n\to+\infty}\frac{g_2(n,n)}{n^{r/(q+1)}}=0, \tag{1.6}$$

where $r=\min\{p-1,q-1\}$.

Here are some comments on the above hypotheses. Hypothesis (H_1) is a superlinear condition at 0 and Hypothesis (H_2) is a local superlinear condition at $+\infty$. About hypothesis (H_3), the fact that g_1, g_2 are unbounded leads us to use the strategy of considering a truncation system. Note that if g_1, g_2 are bounded, we would not need to use that system. Hypothesis (H_4) allows us to have a control on the nonlinear operator in system 1.1.

We remark that, the case when $a_1(s)=a_2(s)=1$ and $g_1(u,v)=g_2(u,v)=0$, systems of type (1.1) have been extensively studied in the literature under different sets of conditions on the nonlinearities. For instance, assuming superlinear hypothesis, many authors have obtained multiplicity of solutions with applications to elliptic systems in annular domains. For homogeneous Dirichlet boundary conditions, see de Figueiredo and Ubilla [1], Conti et al. [2], Dunninger and Wang [3, 4] and Wang [5]. For nonhomogeneous Dirichlet boundary conditions, see Lee [6] and do Ó et al. [7]. Our main goal is to study systems of type (1.1) by considering local superlinear assumptions at $+\infty$ and global superlinear at zero.

The main result is the following.

Theorem 1.1. *Assume hypotheses (H_1) through (H_4). Then system* (1.1) *has at least one positive solution.*

One of the main difficulties here lies in the facts that the coefficients of the differential operators of System (1.1) are nonlinear and that they may not necessarily be bounded from below by a positive bound which is independent of u and v. In order to overcome these difficulties, we introduce a truncation of system (1.1) depending on n so that the new coefficient of the truncation system becomes bounded from below by a uniformly positive constant. (See (2.2).) This allows us to use a fixed point argument for the truncation system. Finally, we show the main result proving that, for n sufficiently large, the solutions of the truncation system are solutions of system (1.1). Observe that, in general, this system has a nonvariational structure.

The paper is organized as follows. In Section 2, we obtain the a priori bounds for the truncation system. In Section 3, we show that the a priori bounds imply a nonexistence result for system (2.4). In Section 4, we introduce a operator of fixed point in cones. In Section 5, we show the existence of positive solutions of the truncation system. In Section 6, we prove the main result, that is, we show the existence of a solution of system (1.1). Finally, in Section 7 we give some remarks.

2. A priori bounds for a truncation system

In this section, we establish a priori bounds for the truncation system. The hypothesis (H_3) allows us to find a $n_{**} \in \mathbb{N}$ so that $n \geq n_{**}$ implies

$$g_1(u,v) \leq g_1(n,n), \qquad g_2(u,v) \leq g_2(n,n), \tag{2.1}$$

for all $u^2+v^2 \leq n^2$. Thus, we can define for every $n \in \mathbb{N}$, such that $n \geq n_{**}$, the functions

$$g_{i,n}(u,v) = \begin{cases} g_i(u,v) & \text{if } u^2+v^2 \leq n^2, \\ g_i\left(\dfrac{nu}{\sqrt{u^2+v^2}}, \dfrac{nv}{\sqrt{u^2+v^2}}\right) & \text{if } u^2+v^2 \geq n^2, \end{cases} \tag{2.2}$$

for $i = 1, 2$.

In the next section, we will prove the existence of a positive solution for the following truncation system:

$$\begin{gathered} -\left(\frac{u'}{a_1(t)+c_1g_{1,n}(u,v)}\right)' = h_1(t)f_1(t,u,v) \quad \text{in } (0,1), \\ -\left(\frac{v'}{a_2(t)+c_2g_{2,n}(u,v)}\right)' = h_2(t)f_2(t,u,v) \quad \text{in } (0,1), \\ u(0)=u(1)=v(0)=v(1)=0. \end{gathered} \tag{2.3}$$

For this purpose we need to establish a priori bounds for solutions of a family of systems parameterized by $\lambda \geq 0$. In fact, for every $n \geq n_{**}$, consider the family

$$\begin{gathered} -\left(\frac{u'}{a_1(t)+c_1g_{1,n}(u,v)}\right)' = h_1(t)f_1(t,u,v)+\lambda \quad \text{in } (0,1), \\ -\left(\frac{v'}{a_2(t)+c_2g_{2,n}(u,v)}\right)' = h_2(t)f_2(t,u,v)+\lambda \quad \text{in } (0,1), \\ u(0)=u(1)=v(0)=v(1)=0. \end{gathered} \tag{2.4}$$

It is not difficult to prove that every solution of system (2.4) satisfies

$$u(t) = \int_0^1 K_{1,n}(t,s)\big(h_1(s) f_1(s,u(s),v(s)) + \lambda\big) ds,$$
$$v(t) = \int_0^1 K_{2,n}(t,s)\big(h_2(s) f_2(s,u(s),v(s)) + \lambda\big) ds. \tag{2.5}$$

Here, $K_{i,n}(t,s)$, $i = 1,2$ are Green's functions given by

$$K_{i,n}(t,s) = \begin{cases} \dfrac{1}{\rho_i} \displaystyle\int_0^t \big(a_i(\tau) + c_i g_{i,n}(u(\tau),v(\tau))\big) \int_s^1 \big(a_i(\tau) + c_i g_{i,n}(u(\tau),v(\tau))\big) & \text{if } 0 \le t \le s \le 1, \\ \dfrac{1}{\rho_i} \displaystyle\int_0^s \big(a_i(\tau) + c_i g_{i,n}(u(\tau),v(\tau))\big) \int_t^1 \big(a_i(\tau) + c_i g_{i,n}(u(\tau),v(\tau))\big) & \text{if } 0 \le s \le t \le 1, \end{cases} \tag{2.6}$$

where ρ_i denotes $\rho_i = \int_0^1 (a_i(\tau) + c_i g_{i,n}(u(\tau)))$.

In order to establish the a priori bound result we need the following two lemmas.

Lemma 2.1. *Assume hypotheses (H_2) and (H_3). Then every solution of system* (2.4) *satisfies*

$$u(t) \ge q_1(t)\|u\|_\infty, \quad v(t) \ge q_2(t)\|v\|_\infty, \quad \forall s \in [0,1], \tag{2.7}$$

where $q_i(t) = (\min\, a_i)t(1-t)/(\|a_i\|_\infty + c_i g_i(n,n))$ *with* $i = 1,2$.

Proof. A simple computation shows that every solution (u,v) satisfies

$$u(s) \ge \hat{q}_1(s,u,v)\|u\|_\infty, \quad v(s) \ge \hat{q}_2(s,u,v)\|v\|_\infty, \quad \forall s \in [0,1], \tag{2.8}$$

where $\hat{q}_i(s,u,v) = (1/\rho_i) \min\, \int_0^s (a_i(\tau) + c_i g_{i,n}(u(\tau),v(\tau))),\ \int_s^1 (a_i(\tau) + c_i g_{i,n}(u(\tau),v(\tau)))\}$.

Since

$$\hat{q}_i(s,u,v) \ge \frac{(\min\, a_i)s(1-s)}{\|a_i\|_\infty + c_i g_i(n,n)}, \quad \text{for } i = 1,2, \tag{2.9}$$

we have that (2.7) is proved. □

Lemma 2.2. *Assume hypotheses (H_2) and (H_3). Then Green's functions satisfy*

$$K_{i,n}(t,s) \ge \frac{(\min a_i)^2}{\|a_i\|_\infty + c_i g_i(n,n)} G(t,s), \quad i = 1,2, \tag{2.10}$$

where

$$G(t,s) = \begin{cases} (1-t)s, & 0 \le s < t \le 1, \\ (1-s)t, & 0 \le t \le s \le 1. \end{cases} \tag{2.11}$$

Theorem 2.3. *Assume hypotheses* (H_2) *and* (H_3). *Then there is a positive constant* B_1 *which does not depend on* λ, *such that for every solution* (u,v) *of system* (2.4), *we have*

$$\|(u,v)\| \leq B_1, \tag{2.12}$$

where $\|(u,v)\| = \|u\|_\infty + \|v\|_\infty$, *with* $\|u\|_\infty = \max_{t\in[0,1]}|u(t)|$.

Proof. By Lemmas 2.1 and 2.2, every solution (u,v) of system (2.4) satisfies

$$\begin{aligned}\|(u,v)\| &\geq \frac{(\min a_1)^2\eta_1}{\|a_1\|_\infty + c_1g_1(n,n)}\int_{\alpha_1}^{\beta_1} h_1(s)u^p(s)ds + \frac{(\min a_2)^2\eta_2}{\|a_2\|_\infty + c_2g_2(n,n)}\int_{\alpha_2}^{\beta_2} h_2(s)v^q(s)ds \\ &\geq \hat{c}\left(\|u\|_\infty^p + \|v\|_\infty^q\right),\end{aligned} \tag{2.13}$$

where $\hat{c} = \min\{((\min a_1)^{p+2}\alpha_1^p(1-\beta_1)^p\eta_1/((\|a_1\|_\infty+c_1g_1(n,n))^{p+1}))\int_{\alpha_1}^{\beta_1} h_1(s)ds, ((\min a_2)^{q+2}\alpha_2^q(1-\beta_2)^q\eta_2/((\|a_2\|_\infty + c_2g_2(n,n))^{q+1}))\int_{\alpha_2}^{\beta_2} h_2(s)ds\}$.

Thus,

$$1 \geq \hat{c}\frac{\|u\|_\infty^p + \|v\|_\infty^q}{\|u\|_\infty + \|v\|_\infty} \tag{2.14}$$

which proves (2.12). □

3. A nonexistence result

In this section, we see that the a priori bounds imply a nonexistence result for system (2.4).

Theorem 3.1. *System* (2.4) *has no solution for all* λ *sufficiently large.*

Proof. Let (u,v) be a solution of system (2.4), in other words,

$$\begin{aligned} u(t) &= \int_0^1 K_{1,n}(t,s)\left(h_1(s)f_1(s,u(s),v(s)) + \lambda\right)ds, \\ v(t) &= \int_0^1 K_{2,n}(t,s)\left(h_2(s)f_2(s,u(s),v(s)) + \lambda\right)ds.\end{aligned} \tag{3.1}$$

Then,

$$\|(u,v)\| \geq \lambda\left(\int_0^1 K_{1,n}\left(s,\frac{1}{2}\right)ds + \int_0^1 K_{2,n}\left(s,\frac{1}{2}\right)ds\right). \tag{3.2}$$

By Theorem 2.3, we know that $\|(u,v)\| \leq B_1$, thus

$$\lambda \leq \frac{B_1}{\int_0^1 K_{1,n}(s,1/2)ds + \int_0^1 K_{2,n}(s,1/2)ds}, \tag{3.3}$$

which proves Theorem 3.1. □

4. Fixed point operators

Consider the following Banach space:

$$X = \mathcal{C}([0,1],\mathbb{R}) \times \mathcal{C}([0,1],\mathbb{R}), \tag{4.1}$$

endowed with the norm $\|(u,v)\| = \|u\|_\infty + \|v\|_\infty$, where $\|u\|_\infty = \max_{t\in[0,1]}|u(t)|$. Define the cone C by

$$C = \{(u,v) \in X : (u,v)(0) = (u,v)(1) = 0,\ \mathrm{y}(u,v) \geq 0\}, \tag{4.2}$$

and the operator $\mathcal{F}_\lambda : X \to X$ by

$$\mathcal{F}_\lambda(u,v)(s) = (\mathcal{A}_\lambda(u,v)(s), \mathcal{B}_\lambda(u,v)(s)), \quad \text{for } s \in [0,1], \tag{4.3}$$

where

$$\begin{aligned} \mathcal{A}_\lambda(u,v)(s) &= \int_0^1 K_{1,n}(s,\tau)(h_1(\tau)f_1(\tau,u(\tau),v(\tau)) + \lambda)d\tau, \\ \mathcal{B}_\lambda(u,v)(s) &= \int_0^1 K_{2,n}(s,\tau)(h_2(\tau)f_2(\tau,u(\tau),v(\tau)) + \lambda)d\tau. \end{aligned} \tag{4.4}$$

Note that a simple calculation shows us that the fixed points of the operator $\mathcal{F}_\lambda$ are the positive solutions of system (2.4).

Lemma 4.1. *The operator $\mathcal{F}_\lambda : X \to X$ is compact, and the cone C is invariant under $\mathcal{F}_\lambda$.*

Proof Outline. The compactness of $\mathcal{F}_\lambda$ follows from the well-known Arzelá-Ascoli theorem. The invariance of the cone C is a consequence of the fact that the nonlinearities are nonnegative.

In Section 5, we will give an existence result of the truncation system (2.3). The proof will be based on the following well-known fixed point result due to Krasnosel'skii, which we state without proof (compare [8, 9]). □

Lemma 4.2. *Let C be a cone in a Banach space, and let $F : C \to C$ be a compact operator such that $F(0) = 0$. Suppose there exists an $r > 0$ verifying*

(a) $u \neq tF(u)$, *for all $\|u\| = r$ and $t \in [0,1]$; suppose further that there exist a compact homotopy $H : [0,1] \times C \to C$ and an $R > r$ such that*

(b) $F(u) = H(0,u)$, *for all $u \in C$;*

(c) $H(t,u) \neq u$, *for all $\|u\| = R$ and $t \in [0,1]$;*

(d) $H(1,u) \neq u$, *for all $\|u\| \leq R$.*

Then F has a fixed point u_0 verifying $r < \|u_0\| < R$.

5. Existence result of truncation system (2.3)

The following is an existence result of the truncation system.

Theorem 5.1. *Assume hipotheses* (H_1) *through* (H_3). *Then there exists a positive solution of system* (2.3).

Proof. We will verify the hypotheses of Lemma 4.2. Let C the cone defined in Section 4 and define the homotopy $\mathcal{H} : [0,1] \times C \to C$ by

$$\mathcal{H}(t,(u,v))(s) = \big(\mathcal{A}_\lambda(t,u,v)(s), \mathcal{B}_\lambda(t,u,v)(s)\big), \quad \text{for } s,t \in [0,1], \tag{5.1}$$

where λ is a sufficiently large parameter, and where

$$\begin{aligned} \mathcal{A}_\lambda(t,u,v)(s) &= \int_0^1 K_{1,n}(s,\tau)\big(h_1(\tau) f_1(\tau,u(\tau),v(\tau)) + t\lambda\big) d\tau, \\ \mathcal{B}_\lambda(t,u,v)(s) &= \int_0^1 K_{2,n}(s,\tau)\big(h_2(\tau) f_2(\tau,u(\tau),v(\tau)) + t\lambda\big) d\tau. \end{aligned} \tag{5.2}$$

Note that $\mathcal{H}(t,u,v)$ is a compact homotopy and that $\mathcal{H}(0,u,v) = \mathcal{F}_0(u,v)$, which verifies (b).

On the other hand, we have

$$\begin{aligned} \|\mathcal{F}_0(u,v)\| \le \big(\|a_1\|_\infty + c_1 g_1(n,n)\big) \int_0^1 h_1(\tau) \frac{f_1(\tau,u(\tau),v(\tau))}{u(\tau)+v(\tau)} d\tau \|(u,v)\| \\ + \big(\|a_2\|_\infty + c_2 g_2(n,n)\big) \int_0^1 h_2(\tau) \frac{f_2(\tau,u(\tau),v(\tau))}{u(\tau)+v(\tau)} d\tau \|(u,v)\|. \end{aligned} \tag{5.3}$$

Taking $\|(u,v)\| = \delta$ with $\delta > 0$ sufficiently small, from hypothesis, we have

$$\|\mathcal{F}_0(u,v)\| < \|(u,v)\|, \tag{5.4}$$

which verifies (a) of Lemma 4.2. By Theorem 2.3, we clearly have (c).

Finally, choosing λ sufficiently large in the homotopy $\mathcal{H}(t,u)$, we see that condition (d) of Lemma 4.2 is satisfied by Theorem 3.1. The proof of Theorem 5.1 is now complete. □

6. Proof of main result Theorem 1.1

The proof of Theorem 1.1 is direct consequence of the following.

Theorem 6.1. *Assume hypotheses* (H_1) *through* (H_4). *Then there exists an* $n_0 \in \mathbb{N}$ *such that every solution* (u,v) *of system* (2.4) *with* $n > n_{**}$ *satisfies*

$$\|(u,v)\| < n_0^2. \tag{6.1}$$

Proof. For otherwise, there would exist a sequence of solutions $\{(u_n, v_n)\}_n$ of system (2.4) such that $\|(u_n, v_n)\| \geq n^2$, for all $n \in \mathbb{N}$ with $n > n_{**}$. Using the same argument as in Theorem 2.3, we would obtain the estimate

$$1 \geq \min\left\{ \frac{(\min a_1)^{p+2} \alpha_1^p (1-\beta_1)^p \eta_1}{(\|a_1\|_\infty + c_1 g_1(n,n))^{p+1}} \int_{\alpha_1}^{\beta_1} h_1(s)ds, \right. \\ \left. \frac{(\min a_2)^{q+2} \alpha_2^q (1-\beta_2)^q \eta_2}{(\|a_2\|_\infty + c_2 g_2(n,n))^{q+1}} \int_{\alpha_2}^{\beta_2} h_2(s)ds \right\} \frac{\|u\|_\infty^p + \|v\|_\infty^q}{\|u\|_\infty + \|v\|_\infty}. \tag{6.2}$$

We have $\|u_n\|_\infty = \sqrt{\|u_n\|_\infty^2 + \|v_n\|_\infty^2} \sin\theta_n$ and $\|v_n\|_\infty = \sqrt{\|u_n\|_\infty^2 + \|v_n\|_\infty^2} \cos\theta_n$ with $\theta_n \in [0, \pi/2]$. Moreover, there exists a constant $c > 0$ such that $\sin^p\theta_n + \cos^q\theta_n > c$. Then

$$\frac{1}{n^{\min\{p-1,q-1\}}} \\ \geq \min\left\{ \frac{(\min a_1)^{p+2} \alpha^p (1-\beta)^p \eta_1 c}{(\|a_1\|_\infty + c_1 g_1(n,n))^{p+1}} \int_{\alpha_1}^{\beta_1} h_1(s)ds, \frac{(\min a_2)^{q+2} \alpha^q (1-\beta)^q \eta_2 c}{(\|a_2\|_\infty + c_2 g_2(n,n))^{q+1}} \int_{\alpha_2}^{\beta_2} h_2(s)ds \right\}, \tag{6.3}$$

which is impossible, since $\lim_{n\to+\infty}(n^{r/(p+1)}/(\|a_1\|_\infty + c_1 g_1(n,n))) = +\infty$ and $\lim_{n\to+\infty}(n^{r/(q+1)}/(\|a_2\|_\infty + c_2 g_2(n,n))) = +\infty$ by hypothesis (H_4). □

7. Remarks

(i) We note that the solutions of nonlinear system (1.1) are of C^1 functions in $[0,1]$ and C^2 almost every where, in $(0,1)$. Note also that when $h_1(t)$, $h_2(t)$ are continuous functions, the solutions of system (1.1) are classic.

(ii) A little modification of our argument may be done to obtain an existence result of the following more general system:

$$-\left(\frac{u'}{a_1(t) + c_1 g_1(u,v)}\right)' = k_1(t,u,v) \quad \text{in } (0,1), \\ -\left(\frac{v'}{a_2(t) + c_2 g_2(u,v)}\right)' = k_2(t,u,v) \quad \text{in } (0,1), \\ u(0) = u(1) = v(0) = v(1) = 0, \tag{7.1}$$

where k_1, k_2 satisfy (H_2). In addition, we must assume that there exist continuous functions $\widehat{f}_1, \widehat{f}_2 : [0,1] \times [0,+\infty)^2 \to [0,+\infty)$ satisfying (H_1) and (H_2), and nonnegative functions $h_1, h_2 \in L^1(0,1)$, so that for all $t \in [0,1]$,

$$k_1(t,u,v) \leq h_1(t)\widehat{f}_1(t,u,v), \qquad k_2(t,u,v) \leq h_2(t)\widehat{f}_2(t,u,v). \tag{7.2}$$

Acknowledgment

The authors are supported by FONDECYT, Grant no. 1040990.

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