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Electrogravitational stability of oscillating streaming fluid cylinder ambient with a transverse varying electric field

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Abstract

The electrogravitational instability of a dielectric oscillating streaming fluid cylinder surrounded by tenuous medium of negligible motion pervaded by transverse varying electric field has been investigated for all the perturbation modes. The model is governed by Mathieu second-order integro-differential equation. Some limiting cases are recovering from the present general one. The self-gravitating force is destabilizing only in the axisymmetric perturbation for long wavelengths, while, the axial electric field interior, the fluid has strong destabilizing effect for all short and long wavelengths. The transverse field is strongly stabilizing. In the case of non-axisymmetric perturbation, the self-gravitating force is stabilizing for short and long waves, while the electric field has stabilizing effect on short waves.

Keywords: electrogravitational stability, oscillating, streaming

1. Introduction

The stability of self-gravitating fluid cylinder has been studied, for the first time, by Chandrasekhar and Fermi [1]. Later on, Chandrasekhar [2] made several extensions as the fluid cylinder is acted by different forces. Radwan [3,4] studied the stability of an ideal hollow jet. Radwan [4] considered that the fluids are penetrated by constant and uniform electric fields. The stability of different cylindrical models under the action of self-gravitating force in addition to other forces has been elaborated by Radwan and Hasan [5,6]. Radwan and Hasan [5] studied the gravitational stability of a fluid cylinder under transverse time-dependent electric field for axisymmetric perturbations. Hasan [7,8] has discussed the stability of oscillating streaming fluid cylinder subject to combined effect of the capillary, self-gravitating, and electrodynamic forces for all axisymmetric and non-axisymmetric perturbation modes. Hasan [7,8] studied the instability of a full fluid cylinder surrounded by self-gravitating tenuous medium pervaded by transverse varying electric field under the combined effect of the capillary, self-gravitating, and electric forces for all the modes of perturbations.

There are many applications of electrohydrodynamic and magnetohydrodynamic stability in several fields of science such as

1. *Geophysics:* the fluid of the core of the Earth and other theorized to be a huge MHD dynamo that generates the Earth's magnetic field because of the motion of the liquid iron.
2. *Astrophysics:* MHD applies quite well to astrophysics since 99% of baryonic matter content of the universe is made of plasma, including stars, the interplanetary medium, nebulae and jets, stability of spiral arm of galaxy, etc. Many astrophysical systems are not in local thermal equilibrium, and therefore require an additional kinematic treatment to describe all the phenomena within the system.
3. *Engineering applications:* there are many forms in engineering sciences including oil and gas extraction process if it surrounded by electric field or magnetic field, gas and steam turbines, MHD power generation systems and magneto-flow meters, etc.

In this article, we aim to investigate the stability of oscillating streaming self-gravitating dielectric incompressible fluid cylinder surrounded by tenuous medium of negligible motion pervaded by transverse varying electric field for all the axisymmetric and non-axisymmetric perturbation modes.

2. Mathematical formulation

Consider a self-gravitating fluid cylinder surrounded by a self-gravitating medium of negligible motion. The cylinder of (radius R_0) dielectric constant $\varepsilon^{(i)}$ while the surrounding medium is being with dielectric constant $\varepsilon^{(e)}$. Fluid is assumed to be incompressible, inviscid, self-gravitating, and pervaded by applied longitudinal electric field.

$$\underline{E}_0^{(i)} = (0,\ 0,\ E_0) \tag{1}$$

The surrounding tenuous medium (being of negligible motion), self-gravitating, and penetrated by transverse varying electric field

$$\underline{E}_0^{(e)} = \left(0,\ \beta E_0 R_0 r^{-1},\ 0\right) \tag{2}$$

where E_0 is the intensity of the electric field in the fluid while β is some parameters satisfy certain conditions. The components of $\underline{E}_0^{(i)}$ and $\underline{E}_0^{(e)}$ are considered along the utilizing cylindrical coordinates $(r,\ \phi,\ z)$ system with z-axis coinciding with the axis of the fluid cylinder. The fluid of the cylinder streams with a periodic velocity

$$\underline{u}_0 = (0,\ 0,\ U\cos\omega t) \tag{3}$$

where ω is constant and U is the speed at time $t = 0$.

The components of electric fields $\underline{E}_0^{(i)}$ and $\underline{E}_0^{(e)}$ are being along (r,ϕ,z) with the z-axis coinciding with the axis of the fluid cylinder (as shown in Figure 1).

The basic equations for investigating the problem under consideration are being the combination of the ordinary hydrodynamic equations, Maxwell equations concerning the electromagnetic theory, and Newtonian self-gravitating equations concerning the self-gravitating matter (see [2,7-10]).

For the problem under consideration, these equations are given as follows.

$$\rho\left(\frac{\partial \underline{u}}{\partial t} + \left(\underline{u}\cdot\nabla\right)\underline{u}\right)^{(i)} = -\nabla P^{(i)} + \rho\nabla V^{(i)} + \frac{1}{2}\nabla\left(\varepsilon^{(i)}\left(\underline{E}^{(i)}\cdot\underline{E}^{(i)}\right)\right) \tag{4}$$

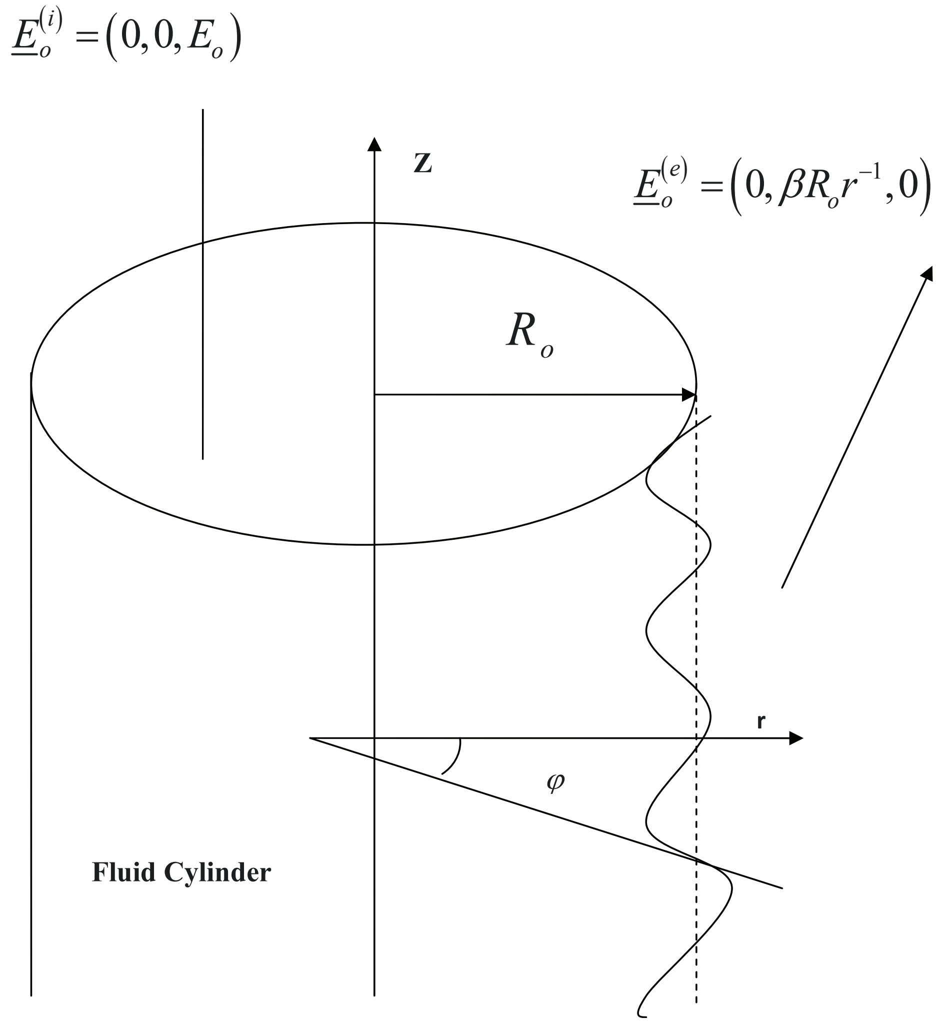

Figure 1 Sketch for gravitational dielectric fluid cylinder.

$$\nabla \cdot \underline{u}^{(i)} = 0 \tag{5}$$

$$\nabla \cdot \left(\varepsilon \underline{E}\right)^{(i,e)} = 0 \tag{6}$$

$$\nabla \wedge \left(\varepsilon^{(i,e)} \underline{E}^{(i,e)}\right) = 0 \tag{7}$$

$$\nabla^2 V^{(i)} = -4\pi \, \rho \, G \tag{8}$$

$$\nabla^2 V^{(e)} = 0 \tag{9}$$

where ρ, $\underline{u}$, and P are the fluid density, velocity vector, and kinetic pressure, respectively, and $\underline{E}^{(i)}$ and $\underline{V}^{(i)}$ are the electric field intensity and self-gravitating potential of the fluid while $\underline{E}^{(e)}$ and $\underline{V}^{(e)}$ are these of tenuous medium surrounding the fluid cylinder, and G is the gravitational constant.

Since the motion of the fluid is irrotational, incompressible motion, the fundamental equations may be written as

$$\nabla^2 \phi^{(i)} = 0 \tag{10}$$

$$\nabla^2 \psi^{(i)} = 0 \tag{11}$$

$$\nabla^2 \psi^{(e)} = 0 \tag{12}$$

where φ and ψ are the potential of the velocity of the fluid and electrical potential.

3. Equilibrium state

In this case, the basic equations are given in the form

$$\nabla^2 V_0^{(i)} = -4\pi\, \rho\, G \tag{13}$$

$$\nabla^2 V_0^{(e)} = 0 \tag{14}$$

$$\nabla^2 \phi_0^{(i)} = 0 \tag{15}$$

where the subscript 0 here and henceforth indicates unperturbed quantities.

Equations 12-14 are solved and moreover the solutions are matched across the fluid cylinder interface at $r = R_0$. The non-singular solution in the unperturbed state is, finally, given as

$$V_0^{(i)} = -\pi\, G\rho\, r^2 \tag{16}$$

$$V_0^{(e)} = -\pi\, G\rho R_0^2 \left[1 + 2\ln\left(\frac{r}{R_0} \right) \right] \tag{17}$$

4. Linearization

For a small wave disturbance across the boundary interface of the fluid, the surface deflection at time t is assumed to be of the form as

$$r = R_0 + \tilde{\eta}, \tag{18}$$

with

$$\tilde{\eta} = \eta\,(t)\, \exp\,(\, i\,(kz + m\varphi)\,) \tag{19}$$

Consequently, any physical quantity $Q(r,\phi,z;t)$ may be expressed as

$$Q\,(r, \varphi, z, t) = Q_0\,(r) + \tilde{\eta}\,(z, \varphi, t) \tag{20}$$

where $\eta(t)$ is the amplitude of the perturbation at an instant time t, k, any real number, is the longitudinal wave number along z-direction while m, an integer, is the azimuthal wave number.

The non-singular solutions of the linearized perturbation equations give φ,V, and ψ as follows:

$$\phi_1^{(i)} = A_1(t) I_m(kr) \exp\left[i(kz + m\varphi)\right], \tag{21}$$

$$V_1^{(i)} = B_1(t) I_m(kr) \exp\left[i(kz + m\varphi)\right], \tag{22}$$

$$V_1^{(e)} = B_2(t) K_m(kr) \exp\left[i(kz + m\varphi)\right] \tag{23}$$

$$\psi_1^{(i)} = C_1(t) I_m(kr) \exp\left[i(kz + m\varphi)\right] \tag{24}$$

$$\psi_1^{(e)} = C_2(t) K_m(kr) \exp\left[i(kz + m\varphi)\right], \tag{25}$$

where $A_1(t)$, $B_1(t)$, $B_2(t)$, $C_1(t)$, and $C_2(t)$ are arbitrary functions of integrations to be determined, while $I_m(kr)$ and $K_m(kr)$ are the modified Bessel functions of the first and second kind of order m.

5. Boundary conditions

The non-singular solutions of the linearized perturbation equation given by the systems (21)-(25) and the solutions (16)-(17) of the unperturbed systems (12)-(14) must satisfy certain boundary conditions. Under the present circumstances, these appropriate boundary conditions could be applied as follows.

(i) Kinematic conditions

The normal component of the velocity vector must be compatible with the velocity of the boundary perturbed surface of the fluid at the level r = R_0. This condition, yield

$$\left(\frac{\partial}{\partial t} + U\cos\omega t\frac{\partial}{\partial z}\right)\tilde{\eta} = \frac{\partial\phi_1^{(i)}}{\partial r} \tag{26}$$

By the use of Equations 18, 19, and 21 for the condition (26), after straight forward calculations, we get

$$A_1(t) = \frac{1}{kI'_m(x)}(\partial_t + ikU)\eta \tag{27}$$

where $x = k R_0$ is, dimensionless, the longitudinal wave number.

(ii) Self-gravitating conditions

The gravitational potential $V = V_0 + \varepsilon V_1 + \ldots$ and its derivative must be continuous across the perturbed boundary fluid surface at $r = R_0$. These conditions are given as

$$\left(V_1 + \tilde{\eta}\frac{\partial V_0}{\partial r}\right)^{(i)} = \left(V_1 + \tilde{\eta}\frac{\partial V_0}{\partial r}\right)^{(e)}, \tag{28}$$

$$\left(\frac{\partial V_1}{\partial r} + \tilde{\eta}\frac{\partial^2 V_0}{\partial r^2}\right)^{(i)} = \left(\frac{\partial V_1}{\partial r} + \tilde{\eta}\frac{\partial^2 V_0}{\partial r^2}\right)^{(e)}. \tag{29}$$

By utilizing Equations 18, 19, 22, and 23 for the conditions (28) and (29), we get

$$B_1(t) = \frac{4\pi G}{k}\,(\rho\, x\, K_m(x)\,\eta) \tag{30}$$

$$B_2(t) = \frac{4\pi G}{k}\,(\rho\, x\, I_m(x)\,\eta) \tag{31}$$

(iii) Electrodynamic condition

The normal component of the electric displacement current and the electric potential ψ perturbed boundary surface at the initial position $r = R_0$. These conditions could be written in the form

$$\left(\psi_1 + \tilde{\eta}\frac{\partial \psi_0}{\partial r}\right)^{(i)} = \left(\psi_1 + \tilde{\eta}\frac{\partial \psi_0}{\partial r}\right)^{(e)} \tag{32}$$

$$\underline{N}.(\varepsilon^{(i)}\underline{E}^{(i)} - \varepsilon^{(e)}\underline{E}^{(e)}) = 0 \tag{33}$$

$$\underline{E} = \underline{E}_0 + \eta\frac{\partial \underline{E}_0}{\partial r} + \underline{E}_1 \tag{34}$$

While $\underline{N}_s$ is, the outward unit vector normal to the interface (18) at $r = R_0$, given by

$$\underline{N}_s = \nabla F(r,\varphi,z;t)\big/|\nabla F(r,\varphi,z;t)| \tag{35}$$

$$F(r,\varphi,z;t) = r - R_o - \tilde{\eta} \tag{36}$$

So that

$$\underline{N}_0 = (1,0,0)\,, \quad \underline{N}_1 = \left(0, \frac{-im}{R_0}, -ik\right)\tilde{\eta} \tag{37}$$

Upon applying these conditions, we get

$$C_1(t) = \frac{-iE_0\varepsilon^{(i)}\eta}{\xi_1}\left(1 + m\beta - \frac{m\beta}{R_0}\right) \tag{38}$$

$$C_2(t) = \frac{-iE_0\varepsilon^{(i)}\eta}{\xi_1}\left(\frac{I_m(x)}{K_m(x)}\right)\left(1 + m\beta - \frac{m\beta}{R_0}\right) \tag{39}$$

where the quantity ξ_1 is given in Appendix 1.

(iv) The dynamical stress condition

The normal component of the total stress across the surface of the coaxial fluid cylinder must be continuous at the initial position at $r = R_0$. This condition is given as follows

$$\rho\left(\frac{\partial \phi_1^{(i)}}{\partial t} + U_0\frac{\partial \phi_1^{(i)}}{\partial z} - V_1^{(i)} - \tilde{\eta}\frac{\partial V_0^{(i)}}{\partial r}\right) + E_0\left(\varepsilon^{(i)}\frac{\partial \psi_1^{(i)}}{\partial z} - \varepsilon^{(e)}\frac{1}{R_0}\frac{\partial}{\partial r}\left(\frac{\beta R_0}{r}\right)\psi_1^{(e)}\right) - \tilde{\eta}\frac{\partial}{\partial r}(E_0\cdot E_0)^{(e)} = 0 \tag{40}$$

By substituting for $\phi_1^{(i)}$, $V_1^{(i)}$, $V_o^{(i)}$, $\psi_1^{(i)}$, $\psi_1^{(e)}$ and $\tilde{\eta}$, after some algebraic calculations, we finally obtain

$$\frac{d^2\eta}{dt^2} + 2\,i\,k\,U_o\,\cos\,\omega t\,\frac{d\eta}{dt} + \left(G\,\beta_{11} - i\,k\omega\,U_o\,\sin\,\omega t - k^2 U_o^2\cos^2\omega t + E_o^2\beta_{12}\right)\eta = 0 \tag{41}$$

where the quantity β_{11} and β_{12} is given in Appendix I.

In order to eliminate the first derivative term, we may use the substitution

$$\eta\,(t) = \eta^*\,(t)\,e^{-\left(\frac{ikU_0}{\omega}\sin\omega t\right)} \tag{42}$$

Equation 41 can be expressed as follows

$$\frac{d^2\eta^*}{dt^2} + \left(G\beta_{11} + E_0^2\beta_{12}\right)\eta^* = 0 \tag{43}$$

Equation 43 is an integro-differential equation governing the surface displacement η^* (t). By means of this relation, we may identify the (in-) stability states and also the self-gravitating and electrodynamic forces influences on the stability of the present model. However in order to do so, it is found more convenient to express this relation in the simple form

$$\left[\frac{d^2}{d\gamma^2} + \left(b - h^2\cos^2\gamma\right)\right]\eta^*\,(t) = 0, \gamma = \omega t \tag{44}$$

where

$$b = \frac{G\beta_{11}}{2} \tag{45}$$

$$h^2 = -\frac{E_0^2\beta_{12}}{\omega^2} \tag{46}$$

Equation 44 has the canonical form

$$\left[\frac{d^2}{d\gamma^2} + \left(a - 2q\cos 2\gamma\right)\right]\eta^*\,(t) = 0 \tag{47}$$

where

$$q = \frac{h^2}{4}, \quad a = b - \left(\frac{h^2}{2}\right) \tag{48}$$

Equation 47 is Mathieu differential equation. The properties of the Mathieu functions are explained and investigated by Melaclan [11]. The solutions of Equation 47, under appropriate restrictions, could be stable and vice versa. The conditions required for periodicity of Mathieu functions are mainly dependent on the correlation between the parameters a and q. However, it is well known, see [11], that (a, q)-plane is divided essentially into two stable and unstable domains separated by the characteristic curves of Mathieu functions. Thence, we can state generally that a solution of Mathieu integro-differential equation is unstable if the point (a, q) say, in the (a, q)-plane lies internal and unstable domain, otherwise it is stable.

6. Discussions and limiting cases

The appropriate solutions of Equation 47 are given in terms of what called ordinary Mathieu functions which, indeed, are periodic in time t with period π and 2π.

Corresponding to extremely small values of q, the first region of instability is bounded by the curves

$$a = \pm q + 1 \tag{49}$$

The conditions for oscillation lead to the problem of the boundary regions of Mathieu functions where Melaclan [11] gives the condition of stability as

$$\left|\Delta(0)\sin^2\left(\frac{\pi a}{2}\right)\right|^{\frac{1}{2}} \leq 1 \tag{50}$$

where Δ(0) is the Hill's determinant.

An approximation criterion for the stability near the neighborhood of the first stable domains of the Mathieu stability domains given by Morse and Feshbach [12] which is valid only for small values of h^2 or q, i.e., the frequency ω of the electric field is very large.

This criterion, under the present circumstances, states that the model is ordinary stable if the restriction

$$h^4 - 16(1-b)h^2 + 32b(1-b) \geq 0 \tag{51}$$

is satisfied where the equality is corresponding to the marginal stability state. The inequality (51) is a quadratic relation in h^2 and could be written as

$$\left(h^2 - \alpha_1\right)\left(h^2 - \alpha_2\right) \geq 0 \tag{52}$$

where α_1 and α_2 are, the two roots of the equality of the relation (51), being

$$\alpha_1 = 8(1-b) - \Delta \tag{53}$$

$$\alpha_2 = 8(1-b) + \Delta \tag{54}$$

with

$$\Delta^2 = 32(1-b)(2-3b) \tag{55}$$

The electrogravitational stability and instability investigations analysis should be carried out in the following two cases

(i). $0 < b < 2/3$

In this case Δ^2 is positive and therefore the two roots α_1 and α_2 of the equality (51) are real. Now, we will show that both α_1 and α_2 are positive. If α_1 α + *ve* then α_1 must be negative and this means that

$$8(1-b) \leq b \tag{56}$$

or alternatively

$$64(1-b)^2 \leq 32(1-b)(2-3b)$$

From which we get

$$2b \geq 3b \tag{57}$$

and this is contradiction, so α_1 must be positive and consequently $\alpha_2 \geq 0$ as well (noting that $\alpha_2 > \alpha_1$). This means that both the quantities (h^2 -α_1) and (h^2 -α_2) are negative and that in turn show that the inequality (51) is identically satisfied.

(ii). $2/3 < b < 1$

In this case, in which $b < 1$ and simultaneously $3b > 2$, it is found that Δ^2 is negative, i.e., Δ is imaginary; therefore, the two roots α_1 and α_2 are complex. We may prove that the inequality (51) is satisfied as follows.

Let h^2 - c and $\alpha_{1,2} = c_1$ - ic_2 where c, c_1, and c_2 are real, so

$$\begin{aligned}\left(h^2 - \alpha_1\right)\left(h^2 - \alpha_2\right) &= \left[-c - (c_1 + ic_2)\right]\left[-c - (c_1 - ic_2)\right] \\ &= c^2 + 2cc_2 + c_1^2 + c_2^2 \\ &= (c + c_1)^2 + c_2^2 = +\text{ve}\end{aligned} \tag{58}$$

which is positive definite.

By an appeal to the cases (i) and (ii), we deduce that the model is stable under the restrictions

$$0 < b < 1 \tag{59}$$

This means that the model is stable if there exists a critical value ω_0 of the electric field frequency ω such that $\omega > \omega_0$ where ω_0 is given by

$$\pi G\rho^{(i)}\left(\frac{xI'_0(x)}{I_0(x)}\right)\left(I_0(x)K_0(x) - \frac{1}{2}\right) > 0 \tag{60}$$

One has to mention here that if $\omega = 0$, $\beta = 0$, and $E_0 = 0$ and we suppose that

$$\gamma(t) = (\text{const})\exp(\sigma t) \tag{61}$$

The second-order integro-differential equation of Mathieu equation (41) yields

$$\sigma^2 = 4\pi G\rho^{(i)}\left(\frac{xI'_0(x)}{I_0(x)}\right)\left(I_0(x)K_0(x) - \frac{1}{2}\right) \tag{62}$$

where σ is the temporal amplification and note by the way that $\left(4\pi G\rho^i\right)^{-\frac{1}{2}}$ has a unit of time. The relation (62) is identical to the gravitational dispersion relation derived for the first time by Chandrasekhar and Fermi [1]. In fact, they [1] have used a totally different technique rather than that used here. They have used the method of representing the solenoidal vectors in terms of poloidal and toroidal vector fields for axisymmetric perturbation.

To determine the effect of ω, it is found more convenient to investigate the eigenvalue relation (62) since the right side of it is the same the middle side of (60).

Taking into account the recurrence relation of the modified Bessel's functions and their derivatives, we see, for x α 0, that

$$\left(\frac{xI'_0(x)}{I_0(x)}\right) > 0 \tag{63}$$

and

$$(I_0(x)K_0(x)) > \frac{1}{2}, \text{ or } (I_0(x)K_0(x)) < \frac{1}{2} \tag{64}$$

based on the values of x.

Now, returning to the relation (62), we deduce that the determining of the sign $\sigma^2/(4\pi G\rho^i)$ is identified if the sign of the quantity

$$Q_o(x) = \left(I_0(x)K_0(x) - \frac{1}{2}\right) \tag{65}$$

is identified.

Here, it is found that the quantity $Q_0(x)$ may be positive or negative depending on $x \,\alpha\, 0$ values. Numerical investigations and analysis of the relation (62) reveal that σ^2 is positive for small values of x while it is negative in all other values of x. In more details, it is unstable in the domain $0 < x < 1.0667$ while it is stable in the domains $1.0667 \le x < \infty$ where the equality is corresponding to the marginal stability state.

From the foregoing discussions, investigations, and analysis, we conclude (on using (65) for (62)) that the quantity

$$L^2 = \left(\frac{xI'_0(x)}{I_0(x)}\right)\left(I_0(x)K_0(x) - \frac{1}{2}\right), \quad L = \frac{\sigma}{(4\pi G\rho)^{\frac{1}{2}}} \tag{66}$$

has the following properties

$$\left.\begin{array}{l} L^2 \le 0 \text{ in the ranges } \quad 1.0667 \le x < \infty \\ L^2 > 0 \text{ in the range } \quad 0 < x < 1.0667 \end{array}\right\} \tag{67}$$

Now, returning to the relation (60) concerning the frequency ω_0 of the periodic electric field

$$\frac{\omega^2}{(4\pi G\rho)} > \left[\left(\frac{xI'_0(x)}{I_0(x)}\right)\left(\frac{1}{2} - I_0(x)K_0(x)\right)\right] > 0. \tag{68}$$

Therefore, we deduce that the electrodynamic force (with a periodic time electric field) has stabilizing influence and could predominate and overcoming the self-gravitating destabilizing influence of the dielectric fluid cylinder dispersed in a dielectric medium of negligible motion.

However, the self-gravitating destabilizing influence could not be suppressed whatever is the greatest value of the magnitude and frequency of the periodic electric field because the gravitational destabilizing influence will persist.

7. Numerical discussions

If we assume that $\omega = 0$ and consider the condition (61), then the second-order integro-differential equation of Mathieu equation (47) yields

$$\frac{\sigma^2}{4\pi G\rho} = \left(\frac{xI'_0(x)}{I_0(x)}\right)\left(I_0(x)K_0(x) - \frac{1}{2}\right) - M\left(\frac{xI'_0(x)}{I_0(x)}\right)\left[\frac{xI'_0(x)K_0(x)}{[I'_0(x)K_0(x) - \varepsilon I'_0(x)K'_0(x)]} - \varepsilon^e\beta^2\right] = 0 \tag{69}$$

where

$$M = \left(\frac{E_0}{E_s}\right)^2, \quad E_s^2 = \frac{4\pi G(\rho)^2 R_0^2}{\varepsilon^{(i)}} \tag{70}$$

and

$$\varepsilon = \left(\varepsilon^{(e)} / \varepsilon^{(i)}\right) \tag{71}$$

To verify and confirm the foregoing analytical results, the relation (69) has been inserted in the computer and computed. This has been done for several values of β as $\beta < 1$, $\beta = 1$, and $\beta > 1$ in the wide domain $0 \le x \le 0.5$. The numerical data of instability corresponding $\sigma / \left(4\pi G\rho^i\right)\frac{1}{2}$ and those of stability corresponding to $\zeta / \left(4\pi G\rho^i\right)\frac{1}{2}$ are collected and tabulated and presented graphically (see Figures 2, 3, 4, 5, and 6). There are many features and properties in this numerical presentation as we see in the following:

(i) For $\beta = 0.5$ corresponding to $M = 0.1, 0.3, 0.5, 0.7, 1.0$, and 1.5 it is found that the electrogravitational unstable domains are $0 < x < 1.1175$, $0 < x < 1.19759$, $0 < x < 1.27235$, $0 < x$ 1.29599, $0 < x < 1.362741$, and $0 < x < 1.3978$, the neighboring stable domains are $1.1175 \le x < \infty$, $1.19759 \le x < \infty$, $1.27235 \le x < \infty$, $1.29599 \le x < \infty$, $1.362741 \le x < \infty$, and $1.3978 \le x < \infty$, where the equalities correspond to the marginal stability states (see Figure 2).

(ii) For $\beta = 1.0$ corresponding to M = 0.1, 0.3, 0.5, 0.7, 1.0, and 1.5 it is found that the electrogravitational unstable domains are $0 < x < 1.22669$, $0 < x < 1.5266$, $0 < x < 1.750969$, $0 < x < 1.90513$, $0 < x < 2.05422$, and $0 < x < 2.19341$, the neighboring stable domains are $1.22669 \le x < \infty$, $1.5266 \le x < \infty$, $1.750969 \le x < \infty$, $1.90513 \le x < \infty$, $2.05422 \le x < \infty$, and $2.19341 \le x < \infty$, where the equalities correspond to the marginal stability states (see Figure 3).

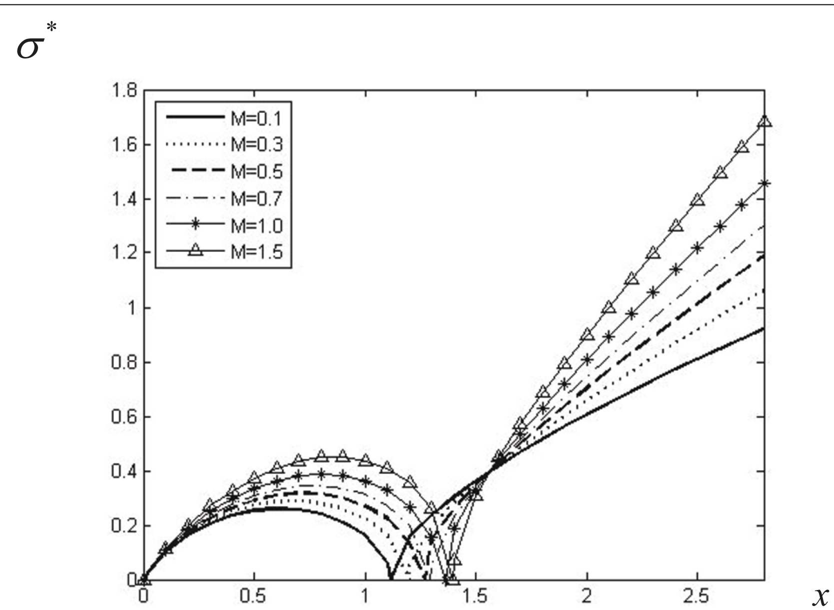

Figure 2 Electrogravitational stable and unstable domains for β = 0.5.

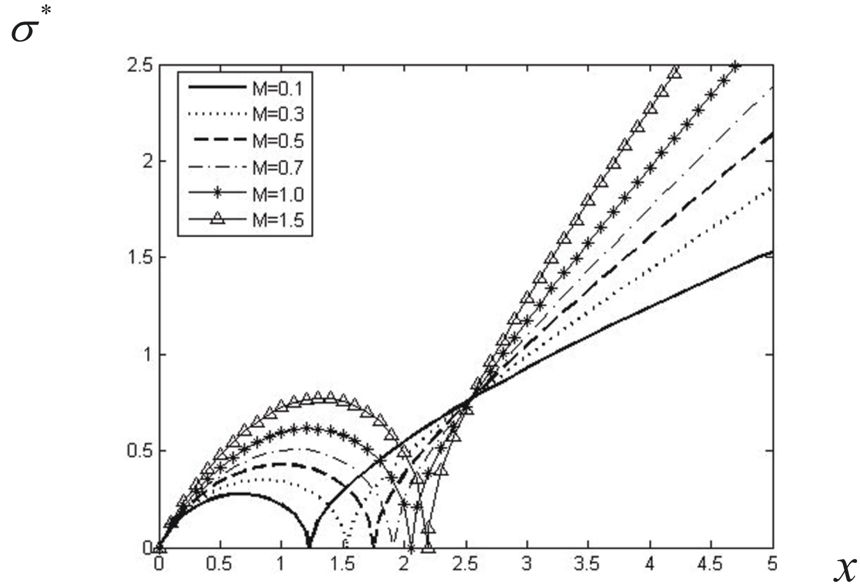

Figure 3 Electrogravitational stable and unstable domains for β = 1.0.

(iii) For β = 1.5 corresponding to M = 0.1, 0.3, 0.5, 0.7, 1.0, and 1.5 it is found that the electrogravitational unstable domains are $0 < x < 1.35924$, $0 < x < 1.9735$, $0 < x < 2.3982$, $0 < x < 2.6563$, $0 < x < 2.8835$, and $0 < x < 3.0798$, the neighboring stable domains are $1.35924 \le x < \infty$, $1.9735 \le x < \infty$, $2.3982 \le x < \infty$, $2.6563 \le x < \infty$, $2.8835 \le x < \infty$, and $3.0798 \le x < \infty$, where the equalities correspond to the marginal stability states (see Figure 4).

(iv) For β = 2.5, corresponding to M = 0.1, 0.3, 0.5, 0.7, 1.0, and 1.5 it is found that the electrogravitational fluid cylinder is completely stable not only for short wavelengths, but also for very long wavelengths and the gravitational unstable domains are completely suppressed (see Figure 5).

(v) For β = 3.0, corresponding to M = 0.1, 0.3, 0.5, 0.7, 1.0 and 1.5 it is found that the electrogravitational fluid cylinder is completely stable not only for short

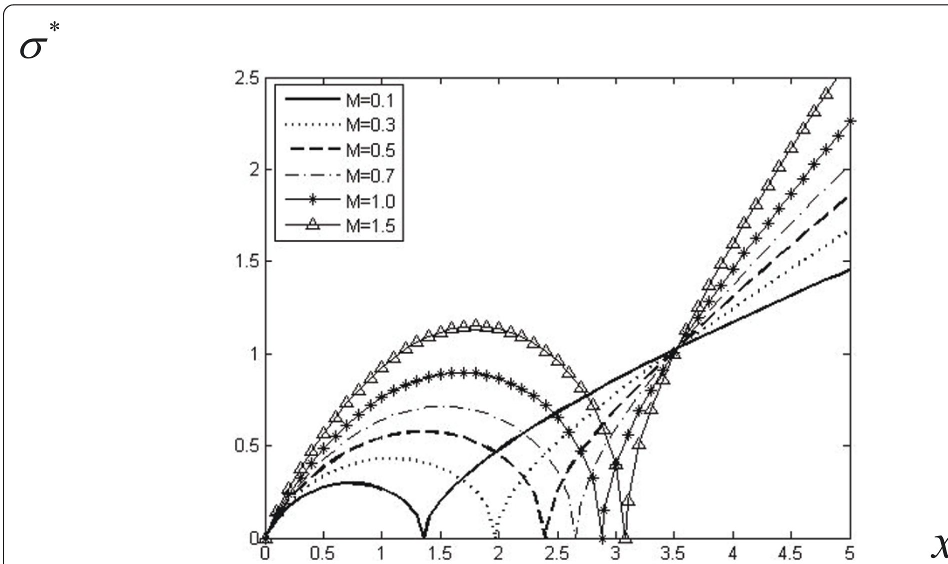

Figure 4 Electrogravitational stable and unstable domains for β = 1.5.

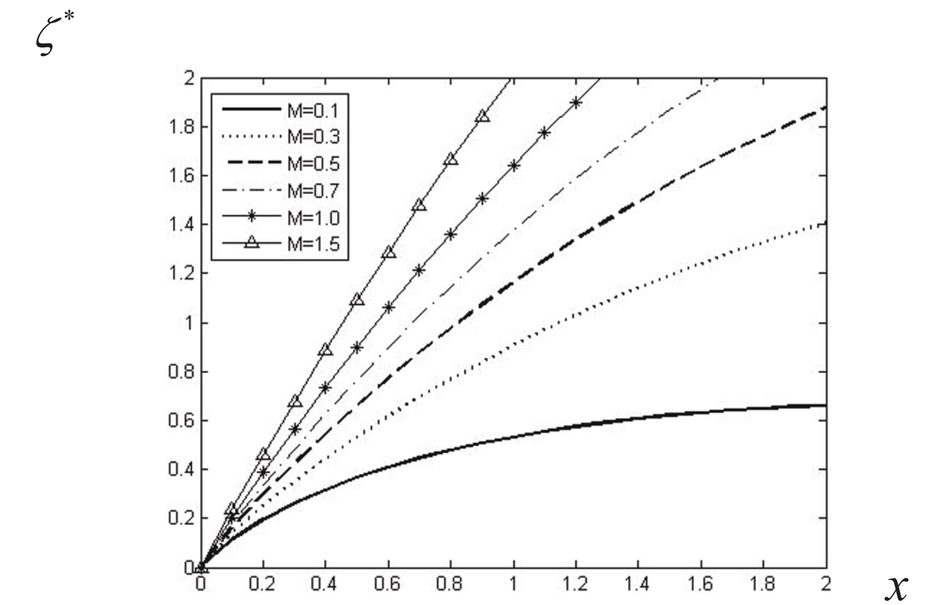

Figure 5 Electrogravitational stable domains for β = 2.5.

wavelengths, but also for very long wavelengths and the gravitational unstable domains are completely suppressed (see Figure 6).

8. Conclusion

From the presented numerical results, we may deduce the following. For the same value of M, it is found that the unstable domains are increasing with increasing of β

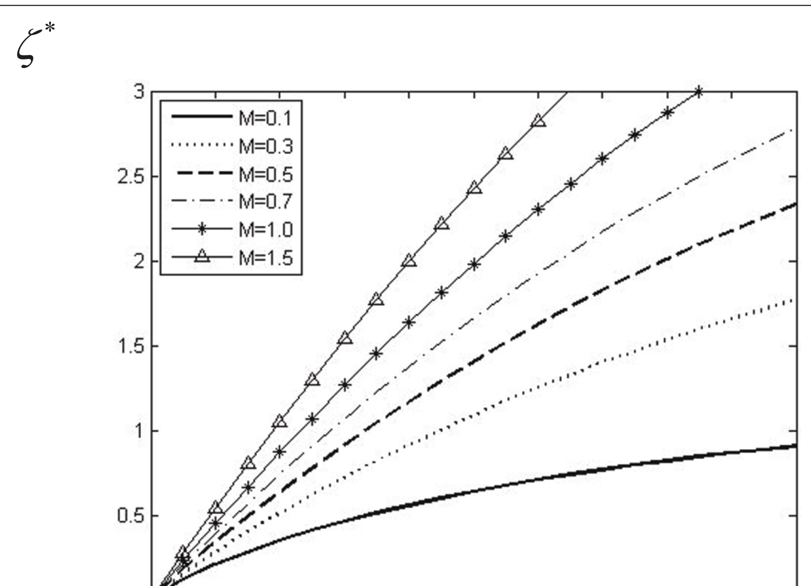

Figure 6 Electrogravitational stable domains for β = 3.0.

values. This means that the influence of electric field has a destabilizing effect for all short and long wavelengths.

If $\beta > 2.0$, then the model is completely stable not only for short wave lengths, but also for long wave lengths.

Appendix I

$$\xi_1 = \varepsilon^{(i)} I'_m(x) K_m(x) - \varepsilon^{(e)} I_m(x) K'_m(x)$$

$$\beta_{11} = \frac{2\pi\rho R_o k I'_m(x) - 4\pi\rho x I'_m(x) K_m(x)}{I_m(x)}$$

$$\beta_{12} = \frac{k^2 \left(\varepsilon^{(i)}\right)^2 I'_m(x)}{\xi_1 \rho} \left(1 + m\beta - \frac{m\beta}{R_o}\right) + \frac{2\beta^2 k I'_m(x)}{R_o \rho I_m(x)} + \frac{i\varepsilon^{(i)}\varepsilon^{(e)} I'_m(x) k}{\xi_1 \rho} \left(\frac{m\beta}{R_o}\right) \left(1 + m\beta - \frac{m\beta}{R_o}\right)$$

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Competing interests
The authors declare that they have no competing interests.

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