Pan et al. Boundary Value Problems 2012, 2012:42
http://www.boundaryvalueproblems.com/content/2012/1/42

Boundary Value Problems
a SpringerOpen Journal

RESEARCH Open Access

Global solutions to a class of nonlinear damped wave operator equations

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Abstract

This study investigates the existence of global solutions to a class of nonlinear damped wave operator equations. Dividing the differential operator into two parts, variational and non-variational structure, we obtain the existence, uniformly bounded and regularity of solutions.

Mathematics Subject Classification 2000: 35L05; 35A01; 35L35.

Keywords: nonlinear damped wave operator equations, global solutions, uniformly bounded, regularity

1 Introduction

In recent years, there have been extensive studies on well-posedness of the following nonlinear variational wave equation with general data:

$$\begin{cases} \partial_t{}^2 u - c(u)\partial_x\left(c(u)\partial_x u\right) = 0 \text{ in } (0,\infty)\times \mathbf{R}, \\ u|_{t=0} = u_0 \quad \text{on } \mathbf{R}, \\ \partial_t u|_{t=0} = u_1 \quad \text{on } \mathbf{R}, \end{cases} \tag{1.1}$$

where $c(\cdot)$ is given smooth, bounded, and positive function with $c'(\cdot) \geq 0$ and $c'(u_0) > 0, u_0 \in H^1(\mathbf{R}), u_1(x) \in L^2(\mathbf{R})$. Equation (1.1) appears naturally in the study for liquid crystals [1-4]. In addition, Chang et al. [5], Su [6] and Kian [7] discussed globally Lipschitz continuous solutions to a class one dimension quasilinear wave equations

$$\begin{cases} u_{tt} - \left(p\left(\rho(x), u_x\right)\right)_x = \rho(x)h\left(\rho(x), u, u_x\right), \\ u(x,0) = u_0(x), \\ u_t(x,0) = \omega_0(x), \end{cases} \tag{1.2}$$

where $(x,t) \in \mathbf{R} \times \mathbf{R}^+$, $u_0(x), \omega_0(x) \in \mathbf{R}$. Furthermore, Nishihara [8] and Hayashi [9] obtained the global solution to one dimension semilinear damped wave equation

$$\begin{cases} u_{tt} + u_t - u_{xx} = f(u), (t,x) \in \mathbf{R}^+ \times \mathbf{R}^+ \\ (u, u_t)(0,x) = (u_0, u_1)(x). \end{cases} \tag{1.3}$$

Ikehata [10] and Vitillaro [11] proved global existence of solutions for semilinear damped wave equations in $\mathbf{R}^N$ with noncompactly supported initial data or in the energy space, in where the nonlinear term $f(u) = |u|^p$ or $f(u) = 0$ is too special; some authors [12-14] discussed the regularity of invariant sets in semilinear wave equation, but they didn't refer to any the initial value condition of it. Unfortunately, it is difficulty to classify a class wave operator equations, since the differential operator structure is too complex to identify whether have variational property. Our aim is to classify a class of nonlinear damped wave operator equations in order to research them more extensively and go beyond the results of [12].

In this article, we are interested in the existence of global solutions of the following nonlinear damped wave operator equations:

$$\begin{cases} \frac{d^2u}{dt^2} + k\frac{du}{dt} = G(u), \, k > 0 \\ u(x,0) = \varphi(x), \\ u_t(x,0) = \psi(x), \end{cases} \tag{1.4}$$

where $G : X_2 \times \mathbf{R}^+ \to X_1^*$ is a mapping, $X_2 \subset X_1$, X_1, X_2 are Banach spaces and X_1^* is the dual spaces of X_1, $\mathbf{R}^+ = [0, \infty)$, $u = u(x,t)$. If $k > 0$, (1.4) is called damped wave equation. We obtain the existence, uniformly bounded and regularity of solutions by dividing the differential operator $G(u)$ into two parts, variational and non-variational structure.

2 Preliminaries

First we introduce a sequence of function spaces:

$$\begin{cases} X \subset H_2 \subset X_2 \subset X_1 \subset H, \\ X_2 \subset H_1 \subset H, \end{cases} \tag{2.1}$$

where H, H_1, H_2 are Hilbert spaces, X is a linear space, X_1, X_2 are Banach spaces and all inclusions are dense embeddings. Suppose that

$$\begin{cases} L : X \to X_1 \text{ is one to one dense linear operator,} \\ \langle Lu, v\rangle_H = \langle u, v\rangle_H, \quad \forall u, v \in X. \end{cases} \tag{2.2}$$

In addition, the operator L has an eigenvalue sequence

$$Le_k = \lambda_k e_k, \quad (k = 1, 2, \ldots) \tag{2.3}$$

such that $\{e_k\} \subset X$ is the common orthogonal basis of H and H_2. We investigate the existence of global solutions of the Equation (1.4), so we need define its solution. Firstly, in Banach space X, introduce

$$L^p((0,T),X) = \left\{ u : (0,T) \to X | \int_0^T \|u\|^p dt < \infty \right\},$$

where $p = (p_1, p_2, \ldots, p_m)$,$p_i \geq 1 (1 \leq i \leq m)$,

$$\|u\|^p = \sum_{k=1}^m |u|_k^{p_k},$$

where $|\cdot|_k$ is semi-norm in X, and $\|\cdot\|_X = \sum_{i=1}^m |\cdot|_i$. Similarly, we can define

$$W^{1,p}((0,T),X) = \left\{ u : (0,T) \to X | u, \quad u' \in L^p((0,T),X) \right\}.$$

Let $L^p_{\text{loc}}((0,\infty),X) = \left\{ u(t) \in X | u \in L^p((0,T),X), \forall T > 0 \right\}$..

Definition 2.1. Set $(\phi, \psi) \in X_2 \times H_1$, $u \in W^{1,\infty}_{\text{loc}}((0,\infty), H_1) \bigcap L^\infty_{\text{loc}}((0,\infty),X_2)$ is called a globally weak solution of (1.4), if for $\forall v \in X_1$, it has

$$\langle u_t, v\rangle_H + k\langle u, v\rangle_H = \int_0^t \langle Gu, v\rangle dt + k\langle \varphi, v\rangle_H + \langle \psi, v\rangle_H. \tag{2.4}$$

Definition 2.2. Let Y_1, Y_2 be Banach spaces, the solution $u(t, \phi, \psi)$ of (1.4) is called uniformly bounded in $Y_1 \times Y_2$, if for any bounded domain $\Omega_1 \times \Omega_2 \subset Y_1 \times Y_2$, there exists a constant C which only depends the domain $\Omega_1 \times \Omega_2$, such that

$$\|u\|_{Y_1} + \|u_t\|_{Y_2} \leq C, \quad \forall(\varphi, \psi) \in \Omega_1 \times \Omega_2 \text{ and } t \geq 0.$$

Definition 2.3. A mapping $G : X_2 \to X_1^*$ is called weakly continuous, if for any sequence $\{u_n\} \subset X_2$, $u_n \rightharpoonup u_0$ in X_2,

$$\lim_{n\to\infty} \langle G(u_n), v\rangle = \langle G(u_0), v\rangle, \quad \forall v \in X_1.$$

Lemma 2.1. [15]*Let H_2, H be Hilbert spaces, and $H_2 \subset H$ be a continuous embedding. Then there exists a orthonormal basis $\{e_k\}$ of H, and also is one orthogonal basis of H_2.*

Proof. Let $I : H_2 \to H$ be imbedded. According to assume I is a linear compact operator, we define the mapping $A : H_2 \to H$ as follows

$$\langle Au, v\rangle_{H_2} = \langle Iu, v\rangle_H = \langle u, v\rangle_H, \quad \forall v \in H_2.$$

obviously, $A : H_2 \to H_2$ is linear symmetrical compact operator and positive definite. Therefore, A has a complete eigenvalue sequence $\{\lambda_k\}$ and eigenvector sequence$\{\tilde{e}_k\} \subset H_2$ such that

$$A\tilde{e}_k = \lambda_k \tilde{e}_k, \quad k = 1, 2, \ldots,$$

and $\{\tilde{e}_k\}$ is orthogonal basis of H_2. Hence

$$\langle \tilde{e}_i, \tilde{e}_j\rangle_H = \langle A\tilde{e}_i, \tilde{e}_j\rangle_{H_2} = \lambda_i \langle \tilde{e}_i, \tilde{e}_j\rangle_{H_2} = 0, \quad \text{if } i \neq j.$$

it implies $\{\tilde{e}_i\}$ is also orthogonal sequence of H. Since $H_2 \subset H$ is dense, $\{\tilde{e}_i\}$ is also orthogonal sequence of H, so $\{e_i\} = \{\tilde{e}_i / \|\tilde{e}_i\|_H\}$ is norm orthogonal basis of H. The proof is completed.

Now, we introduce an important inequality

Lemma 2.2. [16] (Gronwall inequality) *Let $x(t)$, $y(t)$, $z(t)$ be real function on $[a, b]$, where $x(t) \geq 0, \forall a \leq t \leq b$, $z(t) \in C[a, b]$, $y(t)$ is differentiable on $[a, b]$. If the inequality as follows is hold*

$$z(t) \leq y(t) + \int_a^t x(\tau) z(\tau) d\tau, \quad a \leq t \leq b, \tag{2.5}$$

then

$$z(t) \leq y(a) e^{\int_a^t x(s) ds} + \int_a^t e^{\int_a^t x(\tau)} \frac{dy}{ds} ds. \tag{2.6}$$

3 Main results

Suppose that $G = A + B : X_2 \times \mathbf{R}^+ \to X_1^*$. Throughout of this article, we assume that

(i) There exists a function $F \in C^1 : X_2 \to \mathbf{R}^1$ such that

$$\langle Au, Lv\rangle = \langle -DF(u), v\rangle, \quad \forall u, v \in X \tag{3.1}$$

(ii) Function F is coercive, if

$$F(u) \to \infty \Leftrightarrow \|u\|_{X_2} \to \infty \tag{3.2}$$

(iii) B as follows

$$|\langle Bu, Lv\rangle| \le C_1 F(u) + C_2 \|v\|_{H_1}^2, \quad \forall u, v \in X \tag{3.3}$$

for some $g \in L^1_{\text{loc}}(0, \infty)$.

Theorem 3.1. *Set $G : X_2 \times \mathbf{R}^+ \to X_1^*$is weakly continuous, $(\phi, \psi) \in X_2 \times H_1$, then we obtain the results as follows:*

(1) If $G = A$ satisfies the assumption (i) and (ii), then there exists a globally weak solution of (1.4)

$$u \in W^{1,\infty}_{\text{loc}}\left((0,\infty), H_1 \bigcap L^\infty_{\text{loc}}((0,\infty), X_2)\right)$$

and u is uniformly bounded in $X_2 \times H_1$;

(2) If $G = A + B$ satisfies the assumption (i), (ii) and (iii), then there exists a globally weak solution of (1.4)

$$u \in W^{1,\infty}_{\text{loc}}((0,\infty), H_1) \bigcap L^\infty_{\text{loc}}((0,\infty), X_2);$$

(3) Furthermore, if $G = A + B$ satisfies

$$|\langle Gu, v\rangle| \le \frac{1}{2}\|v\|_H^2 + CF(u) + g(t) \tag{3.4}$$

for some $g \in L^1_{\text{loc}}(0,\infty)$, then $u \in W^{2,2}_{\text{loc}}((0,\infty), H)$.

Proof. Let $\{e_k\} \subset X$ be the public orthogonal basis of H and H_2, satisfies (2.3).
Note

$$\begin{cases} X_n = \left\{ \sum_{i=1}^n \alpha_i e_i | \alpha_i \in \mathbf{R}^1 \right\}, \\ \widetilde{X}_n = \left\{ \sum_{j=1}^n \beta_j(t) e_j | \beta_j \in C^2[0,\infty) \right\}. \end{cases} \tag{3.5}$$

From the assumption, we know $LX_n = X_n, L\widetilde{X_n} = \widetilde{X_n}$, apply the Galerkin method to make truncate in $\widetilde{X_n}$:

$$\begin{cases} \frac{d^2u_i}{dt^2} + k\frac{du_i}{dt} = \left\langle G(u_n), e_i \right\rangle,\ 1 \le i \le n \\ u_i(x,0) = \langle \varphi,\ \ e_i \rangle_H, \\ u'_i(x,0) = \langle \psi,\ e_i \rangle_H \end{cases} \tag{3.6}$$

there exists $u_n = \sum_{i=1}^{n} u_i(t)e_i \in C^2\left((0,\infty), X_n\right)$ for any $v \in \widetilde{X_n}$ satisfies

$$\int_0^t \left\langle \frac{d^2u_n}{dt^2} + k\frac{du_n}{dt}, v \right\rangle_H dt = \int_0^t \langle Gu_n,\ \ v\rangle\, dt \tag{3.7}$$

for any $v \in\ X_n$, it yields that

$$\left\langle \frac{du_n}{dt}, v \right\rangle_H + k\langle u_n, v\rangle_H = \int_0^t \langle Gu_n, v\rangle dt + k\langle \varphi, v\rangle_H + \langle \psi, v\rangle_H \tag{3.8}$$

(1) If $G = A, u_n \in \widetilde{X_n}$ substitute $v = \frac{d}{dt}Lu_n$ into (3.7), we get

$$\int_0^t \left\langle \frac{d^2u_n}{dt^2} + k\frac{du_n}{dt}, \frac{d}{dt}Lu_n \right\rangle_{H_1} dt = \int_0^t \left\langle Gu_n,\ \frac{d}{dt}\ Lu_n \right\rangle dt$$

combine condition (2.2) with (3.1), we get

$$\int_0^t \int_\Omega \frac{d^2u_n}{dt^2}\frac{du_n}{dt}dxdt + \int_0^t \int_\Omega k\frac{du_n}{dt}\frac{du_n}{dt}dxdt + \int_0^t DF(u_n)\frac{du_n}{dt}dxdt = 0$$

$$\int_0^t \frac{1}{2}\frac{d}{dt}\left\|\frac{du_n}{dt}\right\|_{H_1}^2 dt + k\int_0^t \left\|\frac{du_n}{dt}\right\|_{H_1}^2 dt + \int_0^t \frac{d}{dt}F(u_n)dt = 0$$

$$\frac{1}{2}\left\|\frac{du_n}{dt}\right\|_{H_1}^2 - \frac{1}{2}\|\psi_n\|_{H_1}^2 + k\int_0^t \left\|\frac{du_n}{dt}\right\|_{H_1}^2 dt + F(u_n) - F(\varphi_n) = 0$$

consequently, we get

$$F(u_n) + \frac{1}{2}\left\|u'_n\right\|_{H_1}^2 + k\int_0^t \left\|u'_n\right\|_{H_1}^2 dt = F(u_n) + \frac{1}{2}\|\psi_n\|_{H_1}^2. \tag{3.9}$$

Assume $\phi \in\ H_2$, $combine(2.2)with(2.3)$, we know $\{e_n\}$ is also the orthogonal basis of H_1, then $\phi_n \to \phi$ in H_2, $\psi_n \to \psi$ in H_1, owing to $H_2 \subset X_2$ is embedded, so

$$\begin{cases} \varphi_n \to \varphi \ in\ X_2 \\ \psi_n \to \psi\ in\ X_1 \end{cases} \tag{3.10}$$

due to the condition (3.6), from (3.9)and (3.10) we easily know

$$\{u_n\} \subset W_{\text{loc}}^{1,\infty}((0,\infty), H_1) \bigcap L_{\text{loc}}^{\infty}((0,\infty), X_2)\ is\ bounded.$$

consequently, assume that

$$u_n \rightharpoonup u_0 \ in \ W^{1,\infty}_{\text{loc}}((0,\infty),H_1)\bigcap L^{\infty}_{\text{loc}}((0,\infty),X_2) \ a.e. \ t>0$$

i.e. $u_n \rightharpoonup u_0$ *in* X_2 *a.e.* $t > 0$, and G is weakly continuous, so

$$\lim_{n\to\infty} \langle Gu_n, v\rangle = \langle Gu_0, v\rangle \, .$$

By (3.8), we have

$$\lim_{n\to\infty}\left[\left\langle \frac{du_n}{dt}\right\rangle_H + k\langle u_n, v\rangle_H\right] = \lim_{n\to\infty}\int_0^t \langle Gu_n, v\rangle\, dt + k\langle \varphi, v\rangle_H + \langle \psi, v\rangle_H$$

$$\left\langle \frac{du_0}{dt}, v\right\rangle_H + k\langle u_0, v\rangle_H = \int_0^t \langle Gu_0, v\rangle\, dt + k\langle \varphi, v\rangle_H + \langle \psi, v\rangle_H$$

it indicates for any $v \in \bigcup_{n=1}^{\infty} X_n \subset X_2$, it holds. Hence, for any $v \in X_2$, *we* have

$$\left\langle \frac{du_0}{dt}, v\right\rangle_H + k\langle u_0, v\rangle_H = \int_0^t \langle Gu_0, v\rangle\, dt + k\langle \varphi, v\rangle_H + \langle \psi, v\rangle_H. \tag{3.11}$$

Consequently, u_0 is a globally weak solution of (1.4).

Furthermore, by (3.9) and (3.10), for any $R > 0$, there exists a constant C such that if

$$\|\varphi\|_{X_2} + \|\psi\|_{H_1} \le R \tag{3.12}$$

then the weak solution $u(t, \phi, \psi)$ of (1.4) satisfies

$$\left\|u(t,\varphi,\psi)\right\|_{X_2} + \left\|u_t(t,\varphi,\psi)\right\|_{H_1} \le C. \ \ \forall t \ge 0 \tag{3.13}$$

Assume $(\phi,\psi) \in X_2 \times H_1$ satisfies (3.12), by $H_2 \subset X_2$ is dense. May fix $\phi_n \in H_2$ such that

$$\|\varphi_n\|_{X_2} + \|\psi\|_{H_1} \le R, \quad \lim_{n\to\infty} \varphi_n = \varphi \text{ in } X_2$$

by (3.13), the solution $\{u(t, \phi_n, \psi)\}$ of (1.4) is bounded in $W^{1,\infty}_{\text{loc}}((0,\infty),H_1)\bigcap L^{\infty}_{\text{loc}}((0,\infty),X_2)$ a.e. $t > 0$.

Therefore, assume $u(t, \phi_n, \psi) \rightharpoonup u$ in $W^{1,\infty}_{\text{loc}}((0,\infty),H_1)\bigcap L^{\infty}_{\text{loc}}((0,\infty),X_2)$ then $u(t)$ is a weak solution of (1.4), it satisfies uniformly bounded of (3.13). So the conclusion (1) is proved.

(2) If $G = A + B, u_n \in \widetilde{X_n}$, substitute $v = \frac{d}{dt}Lu_n$ into (3.7), we get

$$\int_0^t \left[\left\langle \frac{d^2u_n}{dt^2}, \frac{d}{dt}Lu_n\right\rangle_{H_1}\right] + k\left\langle \frac{du_n}{dt}, \frac{d}{dt}Lu_{n1}\right\rangle_{H_1} dt$$

$$= \int_0^t \left[\left\langle Au_n, \frac{du_n}{dt}\right\rangle + \left\langle Bu_n, \frac{du_n}{dt}\right\rangle\right] dt$$

combine the condition (2.2) and (3.1), we have

$$\int_0^t\int_\Omega \frac{d^2u_n}{dt^2}\frac{du_n}{dt}dxdt + k\int_0^t\int_\Omega \frac{du_n}{dt^2}\frac{du_n}{dt}dxdt + \int_0^t \left\langle DF(u_n)\frac{du_n}{dt}\right\rangle dt$$
$$= \int_0^t \left\langle Bu_n, \frac{du_n}{dt}\right\rangle dt$$
$$\int_0^t \frac{1}{2}\frac{d}{dt}\left\|\frac{du_n}{dt}\right\|_{H_1}^2 dt + k\int_0^t \left\|\frac{du_n}{dt}\right\|_{H_1}^2 dt + \int_0^t \frac{d}{dt}F(u_n)dt$$
$$= \int_0^t \left\langle Bu_n, \frac{du_n}{dt}\right\rangle dt$$
$$\frac{1}{2}\|u'_n\|_{H_1^2} - \frac{1}{2}\|\psi_n\|_{H_1}^2 + k\int_0^t \|u'_n\|_{H_1}^2 dt + F(u_n) + F(\varphi_n)$$
$$= \int_0^t \left\langle Bu_n, \frac{du_n}{dt}\right\rangle dt$$

consequently, we have

$$F(u_n) + \frac{1}{2}\|u'_n\|_{H_1}^2 + k\int_0^t \|u'_n\|_{H_1}^2 dt = \int_0^t \left\langle Bu_n, \frac{du_n}{dt}\right\rangle dt + F(\varphi_n) + \frac{1}{2}\|\psi_n\|_{H_1}^2 \quad (3.14)$$

by the condition (3.3),(3.14)implies

$$F(u_n) + \frac{1}{2}\|u'_n\|_{H_1}^2 \le C\int_0^t \left[F(u_n) + \frac{1}{2}\|u'_n\|_{H_1}^2\right]dt + f(t) \quad (3.15)$$

where $f(t) = \int_0^t g(\tau)dt + \frac{1}{2}\|\psi\|_{H_1}^2 + \sup_n F(\varphi_n)$..
by Gronwall inequality [Lemma(2.2)], from (3.15) we easily know:

$$F(u_n) + \frac{1}{2}\|u'_n\|_{H_1}^2 \le f(0)e^{C^t} + \int_0^t f'(\tau)e^{C(t-\tau)}d\tau \quad (3.16)$$

it implies that, for any $0 < T < \infty$

$$\{u_n\} \subset W^{1,\infty}\left((0,T),X_2\right)\bigcap L^\infty\left((0,T),X_2\right) \text{ is bounded.}$$

now, use the same way as (1), we can obtain the result (2).

(3) If the condition (3.4) is hold, $u_n \in \widetilde{X}_n$, substitute $v = \frac{d^2u}{dt^2}$ into (3.7), we can get

$$\int_0^t \left[\left\langle \frac{d^2u_n}{dt^2}, \frac{d^2u_n}{dt^2}\right\rangle_H + k\left\langle \frac{du_n}{dt}, \frac{d^2u_n}{dt^2}\right\rangle_H\right]dt = \int_0^t \left\langle Gu_n, \frac{d^2u_n}{dt^2}\right\rangle dt$$

then

$$\int_0^t \left\langle \frac{d^2u_n}{dt^2}, \frac{d^2u_n}{dt^2} \right\rangle_H dt + \frac{k}{2}\int_0^t \frac{d}{dt}\left\|u'_n(t)\right\|_H^2 dt$$

$$\leq \int_0^t \left[\frac{1}{2}\left\|u''_n(t)\right\|_H^2 + CF(u_n) + g(t)\right] dt$$

$$\int_0^t \left\langle \frac{d^2u_n}{dt^2}, \frac{d^2u_n}{dt^2} \right\rangle_H dt + \frac{k}{2}\left\|u'_n\right\|_H^2$$

$$\leq \frac{k}{2}\left\|\psi_n\right\|_H^2 + \int_0^t \left[\frac{1}{2}\left\|\frac{d^2u_n}{dt^2}\right\|_H^2 + CF(u_n) + g(\tau)\right] d\tau$$

by (3.16), it implies that

$$\int_0 t\left\|\frac{d^2u_n}{dt^2}\right\|_H^2 d\tau \leq C, \quad (C > 0)$$

consequently, for any $0 < T < \infty$

$$\{u_n\} \subset W^{2,2}((0,T),H) \text{ is bounded.}$$

it implies that $u \in W^{2,2}((0,T), H)$, the main theorem (3.1) has been proved.

Acknowledgements
The author is very grateful to the anonymous referees whose careful reading of the manuscript and valuable comments enhanced presentation of the manuscript. Foundation item: the National Natural Science Foundation of China (No. 10971148).

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Authors' contributions
All authors typed, read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 6 October 2011 Accepted: 13 April 2012 Published: 13 April 2012

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doi:10.1186/1687-2770-2012-42
Cite this article as: Pan *et al.*: Global solutions to a class of nonlinear damped wave operator equations. *Boundary Value Problems* 2012 **2012**:42.