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Unique solvability for the non-Newtonian magneto-micropolar fluid

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Abstract

In this paper, a motion of an incompressible non-Newtonian magneto-micropolar fluid is considered. We assume that the stress tensor has a p-structure, and we establish the global in time existence and uniqueness of the weak solutions with $p \geq \frac{5}{2}$ in three dimensions.

1 Introduction and main results

This paper is concerned about the existence and uniqueness of the weak solutions to the non-Newtonian magneto-micropolar fluid equations in $\Omega \times (0,T)$, which are described by

$$\begin{cases} u_t + (u\cdot\nabla)u - \operatorname{div}(|\boldsymbol{e}(u)|^{p-2}\boldsymbol{e}(u)) + \nabla(\pi + \frac{1}{2}|b|^2) = \chi \operatorname{rot}\omega + (b\cdot\nabla)b, \\ \omega_t + (u\cdot\nabla)\omega - \mu\Delta\omega + 2\chi\omega = \chi \operatorname{rot} u, \\ b_t - \lambda\Delta b + (u\cdot\nabla)b - (b\cdot\nabla)u = 0, \\ \operatorname{div} u = \operatorname{div} b = 0, \end{cases} \tag{1.1}$$

here $\Omega \in R^3$ is an open-bounded domain with Lipschitz boundaries, and the unknowns u, ω, b, π denote the velocity of the fluid, the micro-rotational velocity, magnetic field and hydrostatic pressure, respectively. χ, μ, λ are positive numbers associated with properties of the material: χ is the vortex viscosity, μ is spin viscosity and $\frac{1}{\lambda}$ is the magnetic Reynold. In (1.1), $\boldsymbol{e}(u)$ is the symmetric part of the velocity gradient, *i.e.*,

$$2\boldsymbol{e}(u) = \nabla u + (\nabla u)^T.$$

To (1.1) we append the following initial and boundary conditions

$$u(x,0) = u_0(x), \qquad \omega(x,0) = \omega_0(x), \qquad b(x,0) = b_0(x), \quad \forall x \in \Omega, \tag{1.2}$$

$$u(x,t) = \omega(x,t) = b(x,t) = 0, \quad \forall (x,t) \in \Sigma_T, \tag{1.3}$$

where $\Sigma_T = \partial\Omega \times (0,T)$.

The theory of micropolar fluid was first proposed by Eringen [1] in 1966, which enabled us to consider some physical phenomena that cannot be treated by the classical Navier-Stokes equations for the viscous incompressible fluid, for example, the motion of animal blood, liquid crystals and dilute aqueous polymer solutions, *etc.*

If $p = 2$, system (1.1) reduces to the classical magneto-micropolar fluid equations, and there are many earlier results concerning the weak and strong solvability in a bounded domain $\Omega \in R^3$. For strong solutions, Galdi and Rionero [2] stated, without proof, the results of existence and uniqueness of strong solutions. Rojas-Medar [3] studied it and established the local in time existence and uniqueness of strong solutions by the spectral Galerkin method. In 1999, Ortega-Torres and Rojas-Medar [4] proved global existence of strong solutions with the small initial values. For weak solutions, Rojas-Medar and Boldrini [5] proved the existence of weak solutions, and in the 2D case, also proved the uniqueness of the weak solutions.

On the other hand, there are few existence results about the non-Newtonian magneto-micropolar fluid, *i.e.*, the $p \neq 2$ case. In a recent work, Gunzburger *et al.* [6] studied the reduced problem (with both $\omega = 0$ and $\chi = 0$), and gave the global unique solvability of the first initial-boundary value problem in a bounded two or three-dimensional domain. Improved results are proved for the periodic boundary condition case.

In this paper, we will prove the global existence and uniqueness of the weak solutions for the full system (1.1)-(1.3) under the condition that $p \geq \frac{5}{2}$. These results are based on the Galerkin method and a series of uniform estimates, which do not depend on the parameters.

Throughout this work, we use a standard notation $L^p(\Omega)$ (normed $\|\cdot\|_p$) for Lebesgue L^p-spaces, as well as $W^{k,p}(\Omega)$ (normed $\|\cdot\|_{k,p}$) for the usual Sobolev spaces. As usual, $C_0^\infty(\Omega)$ denotes the set of all C^∞-functions with the compact support in Ω. Given $T > 0$ and a Banach space X, we denote by $L^q(0,T;X)$ Bochner spaces, which are equipped with the norm

$$\|\cdot\|_{L^q(0,T;X)} := \left(\int_0^T \|\cdot\|_X^q \, ds\right)^{\frac{1}{q}}.$$

We also introduce the following functional vector spaces:

$$\begin{aligned}
&\mathcal{V} \equiv \{u \in C_0^\infty(\Omega), \operatorname{div} u = 0\},\\
&H \equiv \text{the closure of } \mathcal{V} \text{ in } L^2(\Omega),\\
&V_p \equiv \text{the closure of } \mathcal{V} \text{ in } W^{1,p}(\Omega).
\end{aligned}$$

We next introduce the definition of a weak solution for problems (1.1)-(1.3).

Definition 1.1 We say that (u,ω,b) is a weak solution to problems (1.1)-(1.3) if

$$\begin{aligned}
&u \in L^\infty(0,T;V_p), \qquad u_t \in L^2(0,T;H),\\
&\omega \in L^\infty\bigl(0,T;W^{1,2}(\Omega)\bigr) \cap L^2\bigl(0,T;W^{2,2}(\Omega)\bigr),\\
&b \in L^\infty(0,T;V_2) \cap L^2\bigl(0,T;W^{2,2}(\Omega)\bigr),\\
&\omega_t \in L^2\bigl(0,T;W^{1,2}(\Omega)\bigr), \qquad b_t \in L^2(0,T;H)
\end{aligned} \tag{1.4}$$

satisfy

$$\begin{aligned}
&\langle u_t,\phi\rangle + \bigl\langle \bigl|\boldsymbol{e}(u)\bigr|^{p-2}\boldsymbol{e}(u), \boldsymbol{e}(\phi)\bigr\rangle = \chi\langle \operatorname{rot}\omega,\phi\rangle + \bigl\langle (b\cdot\nabla)b,\phi\bigr\rangle - \bigl\langle (u\cdot\nabla)u,\phi\bigr\rangle\\
&\quad \text{for all } \phi \in V_p,
\end{aligned} \tag{1.5}$$

$$\langle \omega_t, \psi\rangle + \mu\langle \nabla\omega, \nabla\psi\rangle = \chi\langle \operatorname{rot} u, \psi\rangle - \big\langle (u\cdot\nabla)\omega, \psi\big\rangle - 2\chi\langle\omega,\psi\rangle$$
$$\text{for all } \psi \in W_0^{1,2}(\Omega), \tag{1.6}$$
$$\langle b_t, \eta\rangle + \lambda\langle \nabla b, \nabla\eta\rangle = \big\langle (b\cdot\nabla)u, \eta\big\rangle - \big\langle (u\cdot\nabla)b, \eta\big\rangle$$
$$\text{for all } \eta \in V_2, \tag{1.7}$$

where the symbol $\langle\cdot,\cdot\rangle$ denotes a generic duality pairing.

The following theorem gives the main results of this paper.

Theorem 1.1 *Let $\Omega \in R^3$ be an open-bounded domain with a Lipschitz boundary $\partial\Omega$. Assume that $p \geq \frac{5}{2}$, $u_0 \in W^{1,p}(\Omega)$, $\omega_0, b_0 \in W^{1,2}(\Omega)$. Then, for $\forall T \in (0, +\infty)$, there exists a unique weak solution to problem* (1.1)-(1.3) *in the sense of Definition* 1.1.

Remark 1.1 If (1.4)-(1.5) hold, it could be easy to introduce the pressure $\pi \in L^\infty(0,T; L^{p'}(\Omega))$, $p' = p/(p-1)$. This will be done at the end of Section 3.

For latter use, let us state some useful inequalities.

Lemma 1.1 (See [7]) (Korn's inequality) *Let $1 < p < \infty$. Then there exists a constant $C_p = C_p(\Omega)$ such that*

$$C_p\|v\|_{1,p} \leq \big\|\boldsymbol{e}(v)\big\|_p \quad \text{for all } v \in W_0^{1,p}(\Omega), \tag{1.8}$$

where $\Omega \in R^n$ is open and bounded with a Lipschitz boundary.

Lemma 1.2 (See [8]) (On negative norm) *Let $1 < p < \infty$, and let $v \in W_0^{1,p}(\Omega)$. Then there exists a constant C such that*

$$C\|v\|_p \leq \|v\|_{-1,p} + \|\nabla v\|_{-1,p}.$$

Lemma 1.3 (See [9]) *Let $v, w \in W^{1,p}(\Omega)$. For each $2 < p < \infty$, there exists a constant $C' = C'(p) > 0$ such that*

$$\big\|\boldsymbol{e}(v) - \boldsymbol{e}(w)\big\|_p^p \leq C'\int_\Omega \big(\big|\boldsymbol{e}(v)\big|^{p-2}\boldsymbol{e}(v) - \big|\boldsymbol{e}(w)\big|^{p-2}\boldsymbol{e}(w)\big)\big(\boldsymbol{e}(v) - \boldsymbol{e}(w)\big).$$

By using Hölder's inequality and the imbedding inequality, we could arrive at

$$\|u\|_m \leq C(m,r)\|\nabla u\|_r + \tilde{C}(m,r)\|u\|_2 \tag{1.9}$$

with

$$\begin{cases} m \leq 3r/(3-r) & \text{for } r \in [1,3), \\ \text{for any } m < \infty & \text{for } r = 3, \\ m = \infty & \text{for } r > 3. \end{cases}$$

Here, $\tilde{C}(m,r) = 0$ if $u|_{\partial\Omega} = 0$ or $\int_\Omega u\,dx = 0$. We will also apply the so-called multiplicative inequalities

$$\|u\|_q \le C(q)\|\nabla u\|_2^\alpha \cdot \|u\|_2^{1-\alpha} + \hat{C}(q)\|u\|_2 \tag{1.10}$$

with

$$\alpha = 3\left(\frac{1}{2} - \frac{1}{q}\right) \in [0,1], \quad q \in [2,6].$$

If $u|_{\partial\Omega} = 0$ or $\int_\Omega u\,dx = 0$, then $\hat{C}(q) = 0$.

Finally, the paper is organized as follows. In Section 2, we focus on the derivation of the *priori* estimates for the smooth solutions. On the bases of these estimates, in Section 3, we get the existence result with the help of the Galerkin method. The aim of Section 4 is to give the uniqueness criterion.

2 The *priori* estimates

Let (u,ω,b) be a smooth solution to system (1.1)-(1.3). The goal of this section is to derive some *priori* estimates about it. In all the following sections, we always assume that $p \ge \frac{5}{2}$ holds.

Setting $\phi = u$ in (1.5), $\psi = \omega$ in (1.6), $\eta = b$ in (1.7), and observing that $\langle (u\cdot\nabla)u, u\rangle = \langle (u\cdot\nabla)\omega,\omega\rangle = \langle (u\cdot\nabla)b, b\rangle = 0$, we obtain

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\|u\|_2^2 + \|\boldsymbol{e}(u)\|_p^p = \chi\langle \operatorname{rot}\omega, u\rangle + \langle (b\cdot\nabla)b, u\rangle, \\
&\frac{1}{2}\frac{d}{dt}\|\omega\|_2^2 + \mu\|\nabla\omega\|_2^2 + 2\chi\|\omega\|_2^2 = \chi\langle \operatorname{rot} u, \omega\rangle, \\
&\frac{1}{2}\frac{d}{dt}\|b\|_2^2 + \lambda\|\nabla b\|_2^2 = \langle (b\cdot\nabla)u, b\rangle.
\end{aligned}$$

Adding the identities above, noting that $\langle (b\cdot\nabla)b, u\rangle + \langle (b\cdot\nabla)u, b\rangle = 0$, and Korn's inequality (1.8), we get

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\left(\|u\|_2^2 + \|\omega\|_2^2 + \|b\|_2^2\right) + \left(C_p\|\nabla u\|_p^p + \mu\|\nabla\omega\|_2^2 + 2\chi\|\omega\|_2^2 + \lambda\|\nabla b\|_2^2\right) \\
&\quad = \chi\langle \operatorname{rot}\omega, u\rangle + \chi\langle \operatorname{rot} u, \omega\rangle \\
&\quad \le C\chi\left(\|\nabla\omega\|_2\cdot\|u\|_2 + \|\nabla u\|_2\cdot\|\omega\|_2\right) \\
&\quad \le \varepsilon\left(\|\nabla\omega\|_2^2 + \|\nabla u\|_p^p\right) + C_\varepsilon\left(\|\omega\|_2^2 + \|u\|_2^2\right) \quad (\text{for } p \ge 2).
\end{aligned}$$

After choosing ε properly small, integrating over $(0,t)$, $t \in (0,T]$, the Gronwall's inequality yields that

$$\sup_{t\in(0,T)}\left(\|u\|_2^2 + \|\omega\|_2^2 + \|b\|_2^2\right) + \int_0^T \left(\|\nabla u\|_p^p + \|\nabla\omega\|_2^2 + \|\nabla b\|_2^2\right) \le C_1, \tag{2.1}$$

where C_1 is a constant depending on the time T and $\|u_0\|_2$, $\|w_0\|_2$, $\|b_0\|_2$.

Next, we derive the higher order estimates for ω and b. Setting $\psi = -\Delta\omega$ in (1.6), we find

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|\nabla\omega\|_2^2 + \mu\|\Delta\omega\|_2^2 &= \langle (u\cdot\nabla)\omega, \Delta\omega\rangle + 2\chi\langle\omega,\Delta\omega\rangle - \chi\langle \operatorname{rot} u, \Delta\omega\rangle \\ &\le \left|\langle (u\cdot\nabla)\omega, \Delta\omega\rangle\right| + \varepsilon\|\Delta\omega\|_2^2 + C_\varepsilon\left(\|\omega\|_2^2 + \|\nabla u\|_2^2\right) \end{aligned} \tag{2.2}$$

for the first term on the right hand side, we compute by the divergence free conditions

$$\begin{aligned}\left|\langle (u\cdot\nabla)\omega, \Delta\omega\rangle\right| &= \left|\sum_{ijk}\langle \partial_k u_i\,\partial_i\omega_j, \partial_k\omega_j\rangle\right| \\ &\le C\int_\Omega |\nabla u||\nabla\omega|^2 \le C\|\nabla u\|_p\cdot\|\nabla\omega\|_{\frac{2p}{p-1}}^2 \\ &\le C\|\nabla u\|_p\cdot\left(\|\nabla\omega\|_2^{\frac{2p-3}{p}}\cdot\|\Delta\omega\|_2^{\frac{3}{p}} + \|\nabla\omega\|_2^2\right) \\ &\le \varepsilon\|\Delta\omega\|_2^2 + C_\varepsilon\|\nabla u\|_p^{\frac{2p}{2p-3}}\cdot\|\nabla\omega\|_2^2 + C\|\nabla u\|_p\cdot\|\nabla\omega\|_2^2, \end{aligned} \tag{2.3}$$

where Hölder's, Young's inequality and (1.9), (1.10) have been used.

Inserting (2.3) into (2.2), choosing $\varepsilon = \frac{\mu}{4}$ and integrating over $(0,t)$, $t\in(0,T]$, we have

$$\begin{aligned}&\frac{1}{2}\|\nabla\omega\|_2^2 + \frac{\mu}{2}\int_0^t\|\Delta\omega\|_2^2 \\ &\quad\le C\int_0^t\left(\|\nabla u\|_p^{\frac{2p}{2p-3}} + \|\nabla u\|_p\right)\cdot\|\nabla\omega\|_2^2 + C\int_0^t\left(\|\omega\|_2^2 + \|\nabla u\|_2^2\right) + \frac{1}{2}\|\nabla\omega_0\|_2^2 \\ &\quad\overset{(2.1)}{\le} C\int_0^t\left(\|\nabla u\|_p^{\frac{2p}{2p-3}} + \|\nabla u\|_p\right)\cdot\|\nabla\omega\|_2^2 + \frac{1}{2}\|\nabla\omega_0\|_2^2 + C_1, \end{aligned} \tag{2.4}$$

since $p \ge \frac{5}{2}$, we have

$$\frac{2p}{2p-3} \le p. \tag{2.5}$$

Gronwall's inequality and estimate (2.1) now provide the bound

$$\sup_{t\in(0,T)}\|\nabla\omega\|_2^2 + \int_0^T\|\Delta\omega\|_2^2 \le C_2, \tag{2.6}$$

where C_2 is a constant depending on the time T, C_1 and $\|\nabla\omega_0\|_2$.

Next, set $\eta = -\Delta b$ in (1.7) to discover

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|\nabla b\|_2^2 + \lambda\|\Delta b\|_2^2 &= \langle (u\cdot\nabla)b, \Delta b\rangle - \langle (b\cdot\nabla)u, \Delta b\rangle \\ &\le \left|\langle (u\cdot\nabla)b, \Delta b\rangle\right| + \left|\langle (b\cdot\nabla)u, \Delta b\rangle\right|. \end{aligned} \tag{2.7}$$

Reasoning similar to (2.3), we could find

$$\left|\langle (u\cdot\nabla)b, \Delta b\rangle\right| \le \varepsilon\|\Delta b\|_2^2 + C_\varepsilon\|\nabla u\|_p^{\frac{2p}{2p-3}}\cdot\|\nabla b\|_2^2 + C\|\nabla u\|_p\cdot\|\nabla b\|_2^2. \tag{2.8}$$

For the second term on the right hand side of (2.7), we compute

$$\begin{aligned}
&\left|\left\langle (b\cdot\nabla)u,\Delta b\right\rangle\right| \\
&\quad\le \varepsilon\|\Delta b\|_2^2 + C_\varepsilon\left\|(b\cdot\nabla)u\right\|_2^2 \\
&\quad\le \varepsilon\|\Delta b\|_2^2 + C_\varepsilon\|\nabla u\|_p^2\cdot\|b\|_{\frac{2p}{p-2}}^2 \\
&\quad\le \varepsilon\|\Delta b\|_2^2 + C_\varepsilon\|\nabla u\|_p^2\cdot\|\nabla b\|_{\frac{6p}{5p-6}}^2 \\
&\quad\le \varepsilon\|\Delta b\|_2^2 + C_\varepsilon\|\nabla u\|_p^2\cdot\left(\|\nabla b\|_2^{\frac{2(2p-3)}{p}}\cdot\|\Delta b\|_2^{\frac{2(3-p)}{p}} + \|\nabla b\|_2^2\right) \\
&\quad\le \varepsilon\|\Delta b\|_2^2 + \delta\|\Delta b\|_2^2 + C_{\varepsilon\delta}\|\nabla u\|_p^{\frac{2p}{2p-3}}\cdot\|\nabla b\|_2^2 + C_\varepsilon\|\nabla u\|_p^2\cdot\|\nabla b\|_2^2,
\end{aligned} \tag{2.9}$$

where we have used Hölder's, Young's inequality and (1.9), (1.10). Choosing ε and δ properly small, inserting (2.8)-(2.9) into (2.7) and integrating it over $(0,t)$, $t\in(0,T]$, we find

$$\frac{1}{2}\|\nabla b\|_2^2 + \frac{\lambda}{2}\int_0^t\|\Delta b\|_2^2 \le C\int_0^t\left(\|\nabla u\|_p^{\frac{2p}{2p-3}} + \|\nabla u\|_p^2\right)\cdot\|\nabla b\|_2^2 + \frac{1}{2}\|\nabla b_0\|_2^2. \tag{2.10}$$

Observing (2.5) and estimate (2.1), then Gronwall's inequality yields

$$\sup_{t\in(0,T)}\|\nabla b\|_2^2 + \int_0^T\|\Delta b\|_2^2 \le C_3, \tag{2.11}$$

where C_3 is a constant depending on the time T, C_1 and $\|\nabla b_0\|_2$.

Reasoning analogously to (2.6) and (2.11), it is easy to see that identity (1.6) with $\psi=\omega_t$, (1.7) with $\eta=b_t$, with the help of (2.6) and (2.11), guarantee the estimate

$$\int_0^T\|\omega_t\|_2^2 \le C_4, \qquad \int_0^T\|b_t\|_2^2 \le C_5, \tag{2.12}$$

where C_4, C_5 are both constants depending only on the time T and some norm of the initial values.

In fact (we here only take b_t as an example), set $\eta=b_t$ in (1.7), we deduce that

$$\begin{aligned}
\|b_t\|_2^2 + \frac{\lambda}{2}\frac{d}{dt}\|\nabla b\|_2^2 &\le \left|\left\langle (b\cdot\nabla)u,b_t\right\rangle\right| + \left|\left\langle (u\cdot\nabla)b,b_t\right\rangle\right| \\
&\le \varepsilon\|b_t\|_2^2 + C_\varepsilon\left(\left\|(b\cdot\nabla)u\right\|_2^2 + \left\|(u\cdot\nabla)b\right\|_2^2\right).
\end{aligned} \tag{2.13}$$

Now, we compute, by using Hölder's, Young's inequality and (1.9), (1.10)

$$\begin{aligned}
\left\|(b\cdot\nabla)u\right\|_2^2 &\le \|\nabla u\|_p^2\cdot\|b\|_{\frac{2p}{p-2}}^2 \\
&\le C\left(\|\nabla u\|_p^p + \|b\|_{\frac{2p}{p-2}}^{\frac{2p}{p-2}}\right) \\
&\le C\left(\|\nabla u\|_p^p + \|\nabla b\|_{\frac{6p}{5p-6}}^{\frac{2p}{p-2}}\right) \\
&\le C\|\nabla u\|_p^p + C\left(\|\nabla b\|_2^{\frac{2(2p-3)}{p-2}}\cdot\|\Delta b\|_2^{\frac{2(3-p)}{p-2}} + \|\nabla b\|_2^{\frac{2p}{p-2}}\right)
\end{aligned} \tag{2.14}$$

and

$$\begin{aligned}\big\|(u\cdot\nabla)b\big\|_2^2 &\le \|u\|^2_{\frac{3p}{3-p}}\cdot\|\nabla b\|^2_{\frac{6p}{5p-6}}\\ &\le C\|\nabla u\|_p^2\cdot\|\nabla b\|^2_{\frac{6p}{5p-6}}\\ &\le C\big(\|\nabla u\|_p^p+\|\nabla b\|^{\frac{2p}{p-2}}_{\frac{6p}{5p-6}}\big)\\ &\le C\|\nabla u\|_p^p + C\big(\|\nabla b\|_2^{\frac{2(2p-3)}{p-2}}\cdot\|\Delta b\|_2^{\frac{2(3-p)}{p-2}}+\|\nabla b\|_2^{\frac{2p}{p-2}}\big). \end{aligned}\tag{2.15}$$

Combining (2.13)-(2.15), by choosing $\varepsilon=\frac{1}{2}$, we arrive at

$$\frac{1}{2}\|b_t\|_2^2+\frac{\lambda}{2}\frac{d}{dt}\|\nabla b\|_2^2\le C\big(\|\nabla u\|_p^p+\|\nabla b\|_2^{\frac{2(2p-3)}{p-2}}\cdot\|\Delta b\|_2^{\frac{2(3-p)}{p-2}}+\|\nabla b\|_2^{\frac{2p}{p-2}}\big),$$

noting that $p\ge\frac{5}{2}$, so $2(3-p)/(p-2)\le 2$, and now estimate (2.1), (2.11) and Gronwall's inequality imply the estimate of b_t in (2.12).

In the following, we will derive the bound for u_t. Setting $\eta=u_t$ in (1.5), we deduce that

$$\begin{aligned}&\|u_t\|_2^2+\frac{1}{p}\frac{d}{dt}\big\|\boldsymbol{e}(u)\big\|_p^p\\ &\quad\le\big|\big\langle(b\cdot\nabla)b,u_t\big\rangle\big|+\big|\big\langle(u\cdot\nabla)u,u_t\big\rangle\big|+\chi\big|\langle\operatorname{rot}\omega,u_t\rangle\big|\\ &\quad\le\varepsilon\|u_t\|_2^2+C_\varepsilon\big(\big\|(b\cdot\nabla)b\big\|_2^2+\big\|(u\cdot\nabla)u\big\|_2^2+\|\nabla\omega\|_2^2\big).\end{aligned}$$

Integrating it over $(0,t)$, $t\in(0,T]$, by choosing $\varepsilon=\frac{1}{2}$ and Korn's inequality, we have

$$\frac{1}{2}\int_0^t\|u_t\|_2^2+C_p\|\nabla u\|_p^p\le C\int_0^t\big(\big\|(b\cdot\nabla)b\big\|_2^2+\big\|(u\cdot\nabla)u\big\|_2^2\big)+C_1.\tag{2.16}$$

Now, we compute, by (2.11)

$$\begin{aligned}\int_0^t\big\|(b\cdot\nabla)b\big\|_2^2&\le\int_0^t\|b\|_6^2\cdot\|\nabla b\|_3^2\\ &\le C\int_0^t\|\nabla b\|_2^2\cdot\big(\|\nabla b\|_2\cdot\|\Delta b\|_2+\|\nabla b\|_2^2\big)\\ &\le C\sup_{\tau\in(0,t)}\|\nabla b\|_2^3\int_0^t\|\Delta b\|_2+C\sup_{\tau\in(0,t)}\|\nabla b\|_2^4\cdot t\\ &\le C_3^4(1+T),\end{aligned}\tag{2.17}$$

$$\int_0^t\big\|(u\cdot\nabla)u\big\|_2^2\le\int_0^t\|u\|^2_{\frac{2p}{p-2}}\cdot\|\nabla u\|_p^2\le C\int_0^t\|\nabla u\|_p^4.\tag{2.18}$$

Inserting (2.17)-(2.18) into (2.16), by appealing to Korn's inequality, it follows that

$$\|\nabla u\|_p^p+\int_0^t\|u_t\|_2^2\le C\int_0^t\|\nabla u\|_p^p\cdot\|\nabla u\|_p^{4-p}+C_6+C\|\nabla u_0\|_p^p,\tag{2.19}$$

where C_6 depends on T and C_3. Now, Gronwall's inequality and (2.1) yield that

$$\sup_{t\in(0,T)} \|\nabla u\|_p + \int_0^T \|u_t\|_2^2 \le C_7, \tag{2.20}$$

where C_7 depends on the time T, C_1, C_3 and $\|\nabla u_0\|_p$.

3 Approximate solutions and existence result

In this section, we show the existence of a weak solution to the system (1.1)-(1.3) via the Galerkin approximations. For this purpose, we take the set $\{\phi^i\}_{i=1}^\infty$ formed by the eigenvectors ϕ^i, $i = 1, 2, \dots$, of the Stokes operator and the set $\{\psi^i\}_{i=1}^\infty$ formed by the eigenvectors ψ^i, $i = 1, 2, \dots$, of the Laplace operator. According to the Appendix of [7], the functions $\{\phi^i\}_{i=1}^\infty$ form a basis in the space V_p, and $V_2 \cap W^{2,2}(\Omega)$. Setting $R_k = \operatorname{span}\{\phi^1, \phi^2, \dots, \phi^k\}$ and $S_k = \operatorname{span}\{\psi^1, \psi^2, \dots, \psi^k\}$, we construct the Galerkin approximations $\{u^k, \omega^k, b^k\}$ being of the form

$$u^k(x,t) = \sum_{i=1}^k a_i^k(t)\phi^i(x); \qquad \omega^k(x,t) = \sum_{i=1}^k c_i^k(t)\psi^i(x); \qquad b^k(x,t) = \sum_{i=1}^k d_i^k(t)\phi^i(x),$$

where $\boldsymbol{a}^k := (a_1^k, \dots, a_k^k)$, $\boldsymbol{c}^k := (c_1^k, \dots, c_k^k)$, $\boldsymbol{d}^k := (d_1^k, \dots, d_k^k)$ solve the system of ordinary equations

$$\langle u_t^k, \phi\rangle + \langle |\boldsymbol{e}(u^k)|^{p-2}\boldsymbol{e}(u^k), \boldsymbol{e}(\phi)\rangle = \chi\langle \operatorname{rot}\omega^k, \phi\rangle + \langle (b^k\cdot\nabla)b^k, \phi\rangle - \langle (u^k\cdot\nabla)u^k, \phi\rangle \quad \text{for all } \phi \in R_k, \tag{3.1}$$

$$\langle \omega_t^k, \psi\rangle + \mu\langle \nabla\omega^k, \nabla\psi\rangle = \chi\langle \operatorname{rot} u^k, \psi\rangle - \langle (u^k\cdot\nabla)\omega^k, \psi\rangle - 2\chi\langle \omega^k, \psi\rangle \quad \text{for all } \psi \in S_k, \tag{3.2}$$

$$\langle b_t^k, \phi\rangle + \lambda\langle \nabla b^k, \nabla\phi\rangle = \langle (b^k\cdot\nabla)u^k, \phi\rangle - \langle (u^k\cdot\nabla)b^k, \phi\rangle \quad \text{for all } \phi \in R_k. \tag{3.3}$$

Moreover, we require that u^k, ω^k, b^k satisfy the following initial conditions

$$u^k|_{t=0} = \sum_{i=1}^k (u_0, \phi^i)\phi^i, \qquad \omega^k|_{t=0} = \sum_{i=1}^k (\omega_0, \psi^i)\psi^i, \qquad b^k|_{t=0} = \sum_{i=1}^k (b_0, \phi^i)\phi^i. \tag{3.4}$$

The local solvability is guaranteed by the Carathéodory theorem, and the global unique solvability follows from the fact that

$$\sup_{t\in(0,T)} \|u^k(t)\|_2^2 = \sup_{t\in(0,T)} \sum_{i=1}^k (a_i^k(t))^2 \le C,$$

$$\sup_{t\in(0,T)} \|\omega^k(t)\|_2^2 = \sup_{t\in(0,T)} \sum_{i=1}^k (c_i^k(t))^2 \le C,$$

$$\sup_{t\in(0,T)} \|b^k(t)\|_2^2 = \sup_{t\in(0,T)} \sum_{i=1}^k (d_i^k(t))^2 \le C$$

with upper bounds C that do not depend on k. Moreover, we have for u^k, ω^k and b^k the same estimates for all norms we have obtained in Section 2. More precisely, we have

$$\begin{aligned}
&\sup_{t\in(0,T)}\|u^k\|_2, \qquad \sup_{t\in(0,T)}\|\nabla u^k\|_p, \qquad \int_0^T \|u_t\|_2^2 \le C,\\
&\sup_{t\in(0,T)}\|\omega^k\|_{1,2}, \qquad \int_0^t \|\omega^k\|_{2,2}^2, \qquad \int_0^T \|\omega_t\|_2^2 \le C,\\
&\sup_{t\in(0,T)}\|b^k\|_{1,2}, \qquad \int_0^t \|b^k\|_{2,2}^2, \qquad \int_0^T \|b_t\|_2^2 \le C
\end{aligned} \tag{3.5}$$

with a constant C that does not depend on k.

Uniform estimates (3.5) imply that there exists a subsequence of $\{u^k\}$, $\{\omega^k\}$ and $\{b^k\}$ (not relabeled) such that

$$\begin{aligned}
&u^k \rightharpoonup u, \quad \text{weak-* in } L^\infty(0,T;H)\cap L^\infty(0,T;V_p),\\
&\omega^k \rightharpoonup \omega, \quad \text{weakly in } L^2\bigl(0,T;W^{2,2}(\Omega)\bigr) \text{ and weak-* in } L^\infty\bigl(0,T;W^{1,2}(\Omega)\bigr),\\
&b^k \rightharpoonup b, \quad \text{weakly in } L^2\bigl(0,T;V_2\cap W^{2,2}(\Omega)\bigr) \text{ and weak-* in } L^\infty(0,T;V_2),\\
&u_t^k \rightharpoonup u_t, \qquad b_t^k \rightharpoonup b_t, \quad \text{weakly in } L^2(0,T;H),\\
&\omega_t^k \rightharpoonup \omega_t, \quad \text{weakly in } L^2\bigl(0,T;L^2(\Omega)\bigr),\\
&\bigl|\boldsymbol{e}(u^k)\bigr|^{p-2}\boldsymbol{e}(u^k) \rightharpoonup \Lambda, \quad \text{weakly in } L^{p'}\bigl(0,T;L^{p'}(\Omega)\bigr),
\end{aligned}$$

where $p' = p/(p-1)$. Therefore, by making use of the Aubin-Lions lemma (see Lions [10], Theorem 1.5.1), we have

$$\begin{aligned}
&u^k \to u, \quad \text{strongly in } L^2(0,T;H),\\
&\omega^k \to \omega, \quad \text{strongly in } L^2\bigl(0,T;W^{1,2}(\Omega)\bigr),\\
&b^k \to b, \quad \text{strongly in } L^2(0,T;V_2).
\end{aligned}$$

With the convergence above, it is easy to pass to the limit as $k\to\infty$ in (3.1)-(3.3) to find

$$\begin{aligned}
&\langle u_t,\phi\rangle + \bigl\langle \Lambda, \boldsymbol{e}(\phi)\bigr\rangle = \chi\langle \operatorname{rot}\omega,\phi\rangle + \bigl\langle (b\cdot\nabla)b,\phi\bigr\rangle - \bigl\langle (u\cdot\nabla)u,\phi\bigr\rangle\\
&\quad \text{for all } \phi\in V_p, \qquad (3.6)\\
&\langle \omega_t,\psi\rangle + \mu\langle\nabla\omega,\nabla\psi\rangle = \chi\langle \operatorname{rot} u,\psi\rangle - \bigl\langle (u\cdot\nabla)\omega,\psi\bigr\rangle - 2\chi\langle\omega,\psi\rangle\\
&\quad \text{for all } \psi\in W^{2,2}(\Omega), \qquad (3.7)\\
&\langle b_t,\phi\rangle + \lambda\langle\nabla b,\nabla\phi\rangle = \bigl\langle (b\cdot\nabla)u,\phi\bigr\rangle - \bigl\langle (u\cdot\nabla)b,\phi\bigr\rangle\\
&\quad \text{for all } \phi\in V_2. \qquad (3.8)
\end{aligned}$$

Next, to complete the existence proof, we need to verify that

$$\Lambda = \bigl|\boldsymbol{e}(u)\bigr|^{p-2}\boldsymbol{e}(u). \tag{3.9}$$

By Lemma 1.3, we have

$$\begin{aligned}\|\boldsymbol{e}(u^k)-\boldsymbol{e}(u)\|_p^p &\le C'\int_\Omega \big(|\boldsymbol{e}(u^k)|^{p-2}\boldsymbol{e}(u^k)-|\boldsymbol{e}(u)|^{p-2}\boldsymbol{e}(u)\big)\big(\boldsymbol{e}(u^k)-\boldsymbol{e}(u)\big)\\ &= C'\int_\Omega |\boldsymbol{e}(u^k)|^{p-2}\boldsymbol{e}(u^k)\boldsymbol{e}(u^k)-\int_\Omega |\boldsymbol{e}(u)|^{p-2}\boldsymbol{e}(u)\big(\boldsymbol{e}(u^k)-\boldsymbol{e}(u)\big)\\ &\quad -\int_\Omega |\boldsymbol{e}(u^k)|^{p-2}\boldsymbol{e}(u^k)\boldsymbol{e}(u)\\ &\overset{(3.1)}{=} \chi\langle \operatorname{rot}\omega^k,u^k\rangle+\big\langle (b^k\cdot\nabla)b^k,u^k\big\rangle-\langle u_t^k,u^k\rangle\\ &\quad -\int_\Omega |\boldsymbol{e}(u)|^{p-2}\boldsymbol{e}(u)\big(\boldsymbol{e}(u^k)-\boldsymbol{e}(u)\big)-\int_\Omega |\boldsymbol{e}(u^k)|^{p-2}\boldsymbol{e}(u^k)\boldsymbol{e}(u).\end{aligned}$$

Considering $\lim_{k\to\infty}$ of this identity together with (3.6) implies that

$$\lim_{k\to\infty}\|\boldsymbol{e}(u^k)-\boldsymbol{e}(u)\|_p^p\le 0,$$

and thus (3.9) follows.

Having the estimates

$$\|u\|_{L^\infty(0,T;W_0^{1,p}(\Omega))},\qquad \|\omega\|_{L^\infty(0,T;W_0^{1,2}(\Omega))},\qquad \|b\|_{L^\infty(0,T;W_0^{1,2}(\Omega))}\le C, \tag{3.10}$$

we can now introduce the pressure from (1.5). For $t\in(0,T]$, define the functional $F\in W^{-1,p'}(\Omega)$ as

$$\begin{aligned}\langle F,\xi\rangle &:= \big\langle \operatorname{div}\big(|\boldsymbol{e}(u)|^{p-2}\boldsymbol{e}(u)\big),\xi\big\rangle+\chi\langle\operatorname{rot}\omega,\xi\rangle-\langle b\times\operatorname{rot}b,\xi\rangle\\ &\quad -\big\langle (u\cdot\nabla)u,\xi\big\rangle-\langle u_t,\xi\rangle.\end{aligned}$$

We have

$$\langle F,\xi\rangle=0,\quad \forall\xi\in V_p,\ \text{a.e. } t\in(0,T].$$

By using De Rahm's theorem (see [11], Lemma 2.7), we obtain a function $\pi\in L^{p'}(\Omega)$ such that

$$F=-\nabla\pi,\quad \text{a.e. } t\in(0,T].$$

Moreover, due to estimates (3.10),

$$\|\nabla\pi\|_{W^{-1,p'}(\Omega)}\le C,\quad \text{a.e. } t\in(0,T].$$

Then, by Lemma 1.2, there is a generic constant C, depending only on the data such that

$$\|\pi\|_{p'}\le C,\quad \text{a.e. } t\in(0,T].$$

Now, we complete the proof of the existence part of Theorem 1.1.

4 Uniqueness criterion

Let (u_1, ω_1, b_1) and (u_2, ω_2, b_2) be both solutions of the problem. Then, for their difference $\bar{u} = u_1 - u_2$, $\bar{\omega} = \omega_1 - \omega_2$, $\bar{b} = b_1 - b_2$, we have

$$\begin{aligned}
&\langle \bar{u}_t, \phi\rangle + \bigl\langle \bigl|\boldsymbol{e}(u_1)\bigr|^{p-2}\boldsymbol{e}(u_1) - \bigl|\boldsymbol{e}(u_2)\bigr|^{p-2}\boldsymbol{e}(u_2), \boldsymbol{e}(\phi)\bigr\rangle \\
&\quad = \chi\langle \operatorname{rot}\bar{\omega}, \phi\rangle + \bigl\langle (b_1\cdot\nabla)\bar{b}, \phi\bigr\rangle \\
&\qquad + \bigl\langle (\bar{b}\cdot\nabla)b_2, \phi\bigr\rangle - \bigl\langle (u_1\cdot\nabla)\bar{u}, \phi\bigr\rangle - \bigl\langle (\bar{u}\cdot\nabla)u_2, \phi\bigr\rangle \quad \text{for all } \phi \in V_p, \qquad (4.1)\\
&\langle \bar{\omega}_t, \psi\rangle + \mu\langle \nabla\bar{\omega}, \nabla\psi\rangle + 2\chi\langle \bar{\omega}, \psi\rangle = \chi\langle \operatorname{rot}\bar{u}, \psi\rangle - \bigl\langle (u_1\cdot\nabla)\bar{\omega}, \psi\bigr\rangle - \bigl\langle (\bar{u}\cdot\nabla)\omega_2, \psi\bigr\rangle \\
&\quad \text{for all } \psi \in W_0^{1,2}(\Omega), \qquad (4.2)\\
&\langle \bar{b}_t, \eta\rangle + \lambda\langle \nabla\bar{b}, \nabla\eta\rangle = \bigl\langle (b_1\cdot\nabla)\bar{u}, \eta\bigr\rangle + \bigl\langle (\bar{b}\cdot\nabla)u_2, \eta\bigr\rangle - \bigl\langle (u_1\cdot\nabla)\bar{b}, \eta\bigr\rangle - \bigl\langle (\bar{u}\cdot\nabla)b_2, \eta\bigr\rangle \\
&\quad \text{for all } \eta \in V_2. \qquad (4.3)
\end{aligned}$$

Taking $\phi = \bar{u}$ in (4.1), by Lemma 1.3 and the fact that $\langle (u_1\cdot\nabla)\bar{u}, \bar{u}\rangle = 0$ and $p > 2$, we obtain

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\|\bar{u}\|_2^2 + C_p\|\nabla\bar{u}\|_p^p &\le \chi\langle \operatorname{rot}\bar{\omega}, \bar{u}\rangle + \bigl\langle (\bar{b}\cdot\nabla)b_2, \bar{u}\bigr\rangle - \bigl\langle (\bar{u}\cdot\nabla)u_2, \bar{u}\bigr\rangle + \bigl\langle (b_1\cdot\nabla)\bar{b}, \bar{u}\bigr\rangle \\
&\equiv \sum_{i=1}^{3} J_i + \bigl\langle (b_1\cdot\nabla)\bar{b}, \bar{u}\bigr\rangle
\end{aligned}$$

for each J_i, $i = 1, 2, 3$, it follows from Hölder's, Young's inequality and (1.9), (1.10) that

$$\begin{aligned}
J_1 &\le \chi\bigl|\langle \operatorname{rot}\bar{\omega}, \bar{u}\rangle\bigr| \le \chi\|\nabla\bar{\omega}\|_2\cdot\|\bar{u}\|_2 \le \varepsilon\|\nabla\bar{\omega}\|_2^2 + C_\varepsilon\|\bar{u}\|_2^2, \\
J_2 &\le \bigl|\bigl\langle (\bar{b}\cdot\nabla)b_2, \bar{u}\bigr\rangle\bigr| \le \|\nabla b_2\|_6\cdot\|\bar{b}\|_3\cdot\|\bar{u}\|_2 \le C\bigl(\|\Delta b_2\|_2 + \|\nabla b_2\|_2\bigr)\cdot\|\nabla\bar{b}\|_2\cdot\|\bar{u}\|_2 \\
&\le \varepsilon\|\nabla\bar{b}\|_2^2 + C_\varepsilon\bigl(\|\Delta b_2\|_2^2 + \|\nabla b_2\|_2^2\bigr)\cdot\|\bar{u}\|_2^2, \\
J_3 &\le \bigl|\bigl\langle (\bar{u}\cdot\nabla)u_2, \bar{u}\bigr\rangle\bigr| \le \|\nabla u_2\|_p\cdot\|\bar{u}\|_{\frac{2p}{p-1}}^2 \le \|\nabla u_2\|_p\cdot\|\bar{u}\|_2^{\frac{2p-3}{p}}\cdot\|\nabla\bar{u}\|_2^{\frac{3}{p}} \\
&\le \varepsilon\|\nabla\bar{u}\|_2^2 + C_\varepsilon\|\nabla u_2\|_p^{\frac{2p}{2p-3}}\cdot\|\bar{u}\|_2^2,
\end{aligned}$$

so, we have

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\|\bar{u}\|_2^2 + C_p\|\nabla\bar{u}\|_p^p \\
&\quad \le \varepsilon\bigl(\|\nabla\bar{u}\|_2^2 + \|\nabla\bar{\omega}\|_2^2 + \|\nabla\bar{b}\|_2^2\bigr) \\
&\qquad + C_\varepsilon\bigl(1 + \|\nabla b_2\|_2^2 + \|\Delta b_2\|_2^2 + \|\nabla u_2\|_p^{\frac{2p}{2p-3}}\bigr)\cdot\|\bar{u}\|_2^2 + \bigl\langle (b_1\cdot\nabla)\bar{b}, \bar{u}\bigr\rangle. \qquad (4.4)
\end{aligned}$$

Taking $\psi = \bar{\omega}$ in (4.2) and noting that $\langle (u_1\cdot\nabla)\bar{\omega}, \bar{\omega}\rangle = 0$, it follows

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\|\bar{\omega}\|_2^2 + \mu\|\nabla\bar{\omega}\|_2^2 + 2\chi\|\bar{\omega}\|_2^2 &= \chi\langle \operatorname{rot}\bar{u}, \bar{\omega}\rangle - \bigl\langle (\bar{u}\cdot\nabla)\omega_2, \bar{\omega}\bigr\rangle \\
&\equiv \sum_{i=1}^{2} I_i,
\end{aligned}$$

and for I_i, $i = 1, 2$,

$$I_1 \leq \chi \|\nabla \bar{u}\|_2 \cdot \|\bar{\omega}\|_2 \leq \varepsilon \|\nabla \bar{u}\|_2^2 + C_\varepsilon \|\bar{\omega}\|_2^2,$$

$$\begin{aligned} I_2 &\leq \|\nabla \omega_2\|_6 \cdot \|\bar{\omega}\|_3 \cdot \|\bar{u}\|_2 \leq C\big(\|\Delta \omega_2\|_2 + \|\nabla \omega_2\|_2\big) \cdot \|\nabla \bar{\omega}\|_2 \cdot \|\bar{u}\|_2 \\ &\leq \varepsilon \|\nabla \bar{\omega}\|_2^2 + C_\varepsilon \big(\|\Delta \omega_2\|_2^2 + \|\nabla \omega_2\|_2^2\big) \cdot \|\bar{u}\|_2^2, \end{aligned}$$

thus, we obtain

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\|\bar{\omega}\|_2^2 + \mu \|\nabla \bar{\omega}\|_2^2 &\leq \varepsilon \big(\|\nabla \bar{u}\|_2^2 + \|\nabla \bar{\omega}\|_2^2\big) + C_\varepsilon \|\bar{\omega}\|_2^2 \\ &\quad + C_\varepsilon \big(\|\Delta \omega_2\|_2^2 + \|\nabla \omega_2\|_2^2\big) \cdot \|\bar{u}\|_2^2. \end{aligned} \tag{4.5}$$

Similarly, by taking $\eta = \bar{b}$ in (4.3), reasoning analogous as above, we could get

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\|\bar{b}\|_2^2 + \lambda \|\nabla \bar{b}\|_2^2 &\leq \varepsilon \|\nabla \bar{b}\|_2^2 + C_\varepsilon \big(\|\Delta b_2\|_2^2 + \|\nabla b_2\|_2^2\big) \cdot \|\bar{u}\|_2^2 \\ &\quad + C_\varepsilon \|\nabla u_2\|_p^{\frac{2p}{2p-3}} \cdot \|\bar{b}\|_2^2 + \big\langle (b_1 \cdot \nabla)\bar{u}, \bar{b}\big\rangle. \end{aligned} \tag{4.6}$$

Adding (4.4)-(4.6) and observing that $\langle (b_1 \cdot \nabla)\bar{b}, \bar{u}\rangle + \langle (b_1 \cdot \nabla)\bar{u}, \bar{b}\rangle = 0$, after choosing ε properly small, we finally get

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\big(\|\bar{u}\|_2^2 + \|\bar{\omega}\|_2^2 + \|\bar{b}\|_2^2\big) + C\big(\|\nabla \bar{u}\|_2^2 + \|\nabla \bar{\omega}\|_2^2 + \|\nabla \bar{b}\|_2^2\big) \\ &\quad \leq CF(t) \cdot \big(\|\bar{u}\|_2^2 + \|\bar{\omega}\|_2^2 + \|\bar{b}\|_2^2\big) \end{aligned}$$

with

$$F(t) = 1 + \|\nabla b_2\|_2^2 + \|\Delta b_2\|_2^2 + \|\nabla u_2\|_p^{\frac{2p}{2p-3}} + \|\nabla \omega_2\|_2^2 + \|\Delta \omega_2\|_2^2.$$

Since $2p/(2p-3) \leq p$ for $p \geq \frac{5}{2}$, then Gronwall's inequality and the estimates obtained in Section 2 yield that

$$\bar{u} = \bar{\omega} = \bar{b} = 0 \quad \text{for } t \in [0, T].$$

This completes the proof of the theorem.

Competing interests
The author declares that they have no competing interests.

Authors' contributions
The author completed the paper. The author read and approved the final manuscript.

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