Sun et al. *Boundary Value Problems* 2013, 2013:259
http://www.boundaryvalueproblems.com/content/2013/1/259

Boundary Value Problems
a SpringerOpen Journal

RESEARCH Open Access

Extinction in finite time of solutions to the nonlinear diffusion equations involving $p(x,t)$-Laplacian operator

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Abstract
The aim of this paper is to study the extinction of solutions of the initial boundary value problem for $u_t = \operatorname{div}(|\nabla u|^{p(x,t)-2}\nabla u) + b(x,t)|u|^q - a_0 u$. The authors discuss how the relations of $p(x,t)$ and dimension N affect the properties of extinction in finite time.
MSC: 35K35; 35K65; 35B40

Keywords: nonlinear diffusion equations; $p(x,t)$-Laplacian operator; extinction

1 Introduction

In this paper, we consider the following nonlinear degenerate parabolic equation:

$$\begin{cases} u_t = \operatorname{div}(|\nabla u|^{p(x,t)-2}\nabla u) + b(x,t)|u|^q - a_0 u, & (x,t) \in \Omega \times (0,T) = Q_T, \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T) = \Gamma_T, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $Q_T = \Omega \times (0,T]$, Γ_T denotes the lateral boundary of the cylinder Q_T, $a_0 > 0$. It will be assumed throughout the paper that the exponent $p(x,t)$ is continuous in $Q = \overline{Q_T}$ with logarithmic module of continuity

$$1 < p^- = \inf_{(x,t)\in Q} p(x,t) \le p(x,t) \le p^+ = \sup_{(x,t)\in Q} p(x,t) < \infty, \tag{1.2}$$

$$\forall z = (x,t) \in Q_T, \xi = (y,s) \in Q_T, |z-\xi| < 1, \quad \bigl|p(z) - p(\xi)\bigr| \le \omega\bigl(|z-\xi|\bigr), \tag{1.3}$$

where

$$\limsup_{\tau\to 0^+} \omega(\tau)\ln\frac{1}{\tau} = C < +\infty.$$

Model (1.1) may describe some properties of electro-rheological fluids which change their mechanical properties dramatically when an external electric field is applied. The variable exponent p in model (1.1) is a function of the external electric field $|\overrightarrow{E}|^2$, which satisfies the quasi-static Maxwell equations $\operatorname{div}(\varepsilon_0 \overrightarrow{E} + \overrightarrow{p}) = 0$, $\operatorname{Curl}(\overrightarrow{E}) = 0$, where ε_0 is the dielectric constant in vacuum and the electric polarization $\overrightarrow{p}$ is linear in $\overrightarrow{E}$, *i.e.*, $\overrightarrow{p} = \lambda\overrightarrow{E}$. For more complete physical background, the readers may refer to [1–3].

These models include parabolic or elliptic equations which are nonlinear with respect to gradient of the thought solution and with variable exponents of nonlinearity; see [4–8] and references therein. Besides, another important application is the image processing where the anisotropy and nonlinearity of the diffusion operator and convection terms are used to underline the borders of the distorted image and to eliminate the noise [9–11]. When p is a fixed constant, authors in [12–14] studied extinctions in finite time, blowing-up in finite time of solutions. Due to the lack of homogeneity and the gap between norm and modular, some methods in [12–14] fail in solving our problems. In order to overcome some difficulties, we have to search for some new methods and techniques. To the best of our knowledge, there are only a few works about parabolic equations with variable exponents of nonlinearity. In [4], applying Galerkin's method, Antontsev and Shmarev obtained the existence and uniqueness of weak solutions with the assumption that the function $a(u)$ in $\operatorname{div}(a(u)|\nabla u|^{p(x)-2}\nabla u)$ was bounded. In the case when the function $a(u)$ in $\operatorname{div}(a(u)|\nabla u|^{p(x)-2}\nabla u)$ might be not upper bounded, the authors in [15, 16] applied the method of parabolic regularization and Galerkin's method to prove the existence of weak solutions. In this paper, we apply energy methods and Gronwall inequalities to prove that the solution vanishes in finite time. Moreover, we obtain the critical exponent of extinction in finite time.

The outline of this paper is the following. In Section 2, we shall introduce the function spaces of Orlicz-Sobolev type, give the definition of a weak solution to the problem; Section 3 will be devoted to the proof of the extinction of the solution obtained in Section 2.

2 Preliminaries

We will state some properties of variable exponent spaces and give the definition of a weak solution to the problem. Let us introduce the Banach spaces.

$$
\begin{aligned}
&\mathbf{L}^{p(x,t)}(Q_T) = \left\{ u(x,t) \middle| u \text{ is measurable in } Q_T, A_{p(\cdot)}(u) = \iint_{Q_T} |u|^{p(x,t)}\,dx\,dt < \infty \right\}, \\
&\qquad \|u\|_{p(\cdot)} = \inf\{\lambda > 0, A_{p(\cdot)}(u/\lambda) \le 1\}; \\
&\mathbf{V}_t(\Omega) = \{u | u \in L^2(\Omega) \cap W_0^{1,1}(\Omega), |\nabla u| \in L^{p(x,t)}(\Omega)\}, \\
&\qquad \|u\|_{\mathbf{V}_t(\Omega)} = \|u\|_{2,\Omega} + \|\nabla u\|_{p(\cdot)\Omega}; \\
&\mathbf{W}(Q_T) = \{u : [0,T] \mapsto \mathbf{V}_t(\Omega) | u \in L^2(Q_T), |\nabla u| \in L^{p(x,t)}(Q_T), u = 0 \text{ on } \Gamma_T\}, \\
&\qquad \|u\|_{\mathbf{W}(Q_T)} = \|u\|_{2,Q_T} + \|\nabla u\|_{p(x),Q_T}
\end{aligned}
$$

and denote by $\mathbf{W}'(Q_T)$ the dual of $\mathbf{W}(Q_T)$ with respect to the inner product in $L^2(Q_T)$.

For the sake of simplicity, we first state some results about the properties of the Luxemburg norm.

Lemma 2.1 [17, 18] *For any* $u \in L^{p(x)}(\Omega)$,

$$
\begin{aligned}
&(1) \quad \|u\|_{p(x)} < 1\ (= 1; > 1) \quad \Leftrightarrow \quad A_{p(\cdot)}(u) < 1\ (= 1; > 1); \\
&(2) \quad \|u\|_{p(x)} < 1 \quad \Rightarrow \quad \|u\|_{p(x)}^{p^+} \le A_{p(\cdot)}(u) \le \|u\|_{p(x)}^{p^-}; \\
&\qquad \|u\|_{p(x)} \ge 1 \quad \Rightarrow \quad \|u\|_{p(x)}^{p^-} \le A_{p(\cdot)}(u) \le \|u\|_{p(x)}^{p^+}; \\
&(3) \quad \|u\|_{p(x)} \to 0 \quad \Leftrightarrow \quad A_{p(\cdot)}(u) \to 0; \qquad \|u\|_{p(x)} \to \infty \quad \Leftrightarrow \quad A_{p(\cdot)}(u) \to \infty.
\end{aligned}
$$

Lemma 2.2 [17, 18] *If $p \in C(\overline{\Omega})$, $1 < p^- < p^+ < \infty$, then $L^{p(x)}(\Omega)$ and $W^{1,p(x)}(\Omega)$ are uniformly convex. Hence they are reflexive.*

Lemma 2.3 ([17, 18], Poincaré's inequality) *There exists a constant $C = C(p^{\pm}, |\Omega|) > 0$ such that for any $u \in W_0^{1,p(x)}(\Omega)$,*

$$\|u\|_{p(x)} \leq C\|\nabla u\|_{p(x)}.$$

Definition 2.1 A function $u \in \mathbf{W}(Q_T) \cap L^\infty(0, T; L^\infty(\Omega))$ is called a weak solution of problem (1.1) if every test-function

$$\xi \in Z = \{\eta(x,t) | \eta \in \mathbf{W}(Q_T) \cap L^\infty(0, T; L^2(\Omega)), \eta_t \in \mathbf{W}'(Q_T)\}$$

and for every $t_1 < t_2 \in (0, T]$, the following identity holds:

$$\int_{t_1}^{t_2} \int_\Omega \left[u\xi_t - |\nabla u|^{p(x,t)-2}\nabla u \nabla \xi + b(x,t)|u|^q \xi - a_0 u\xi\right] \mathrm{d}x\,\mathrm{d}t = \int_\Omega u\xi\,\mathrm{d}x\Big|_{t_1}^{t_2}. \tag{2.1}$$

3 Main results and their proofs

Our main results read as follows.

Theorem 3.1 *Assume that $p(x,t)$ satisfies conditions* (1.2)-(1.3) *and $0 < q < 1$, $b \in L^1(0, T; L^\infty(\Omega))$, $a_0 > 0$, then problem* (1.1) *has at lease one weak solution $u(x,t)$ in the sense of Definition* 2.1.

Proof Define the operator

$$\langle Lu, \varphi\rangle = \int_\Omega \langle |\nabla u|^{p(x,t)-2}\nabla u \nabla \varphi - b(x,t)|u|^q \varphi + a_0 u\varphi \rangle \,\mathrm{d}x + \int_\Omega u_t \varphi\,\mathrm{d}x, \quad \varphi \in V_t(\Omega).$$

According to [4, 15], we know that $\{\varphi_k(x)\} \subset V_+(\Omega) = \{u(x) | u \in L^2(\Omega) \cap W_0^{1,1}(\Omega), |\nabla u| \in L^{p^+}\}$ such that $\bigcup_{n=1}^\infty V_n$ is dense in $V_+(\Omega)$ with

$$V_n = \operatorname{Span}\{\varphi_1, \varphi_2, \ldots, \varphi_n\}.$$

We construct the approximate solution $u^{(m)}(x,t)$ as follows:

$$u^{(m)}(x,t) = \sum_{i=1}^m C_k^{(m)}(t)\varphi_k(x),$$

where the coefficient $C_k^{(m)}(t)$ may be obtained by solving the following identities:

$$\langle Lu^{(m)}, \varphi_k \rangle = 0, \quad k = 1, 2, \ldots, m. \tag{3.1}$$

Problem (1.1) generates the systems of m ODE

$$\begin{cases} (C_k^m(t))' = G_k(t, C_1^m(t) \cdots C_m^m(t)), \\ C_k^m(0) = \int_\Omega u_0(x)\varphi_k\,\mathrm{d}x, \quad k = 1, 2, \ldots, m. \end{cases} \tag{3.2}$$

If all the conditions in Theorem 3.1 hold, the functions G_k are continuous in all arguments. □

In order to prove this theorem, we need the following lemmas.

Lemma 3.1 *The approximate solutions $u^{(m)}$ of problem* (1.1) *satisfy the following inequality on the interval*:

$$\|u^{(m)}(\cdot,\tau)\|_{2,\Omega}^2 + \iint_{Q_\tau} |\nabla u^{(m)}|^{p(x,t)}\,\mathrm{d}x\,\mathrm{d}t \le C.$$

Proof By Peano's theorem, for every finite m, system (3.1) has solutions C_i^m, $i = 1, 2, \dots, m$, on the interval $(0, T_0)$.

Multiplying each of equations (3.1) by $C_k^m(t)$ and summing over $k = 1, 2, \dots, m$, we arrive at the relations

$$\begin{aligned}
&\frac{1}{2}\|u^{(m)}\|_{L^2(\Omega)}^2\Big|_{t=0}^{t=\tau} + \iint_{Q_\tau} |\nabla u^{(m)}|^{p(x,t)}\,\mathrm{d}x\,\mathrm{d}t \\
&\quad= \iint_{Q_\tau} b|u^{(m)}|^q u^{(m)}\,\mathrm{d}x\,\mathrm{d}t - a_0\iint_{Q_\tau} |u^{(m)}|^2\,\mathrm{d}x\,\mathrm{d}t \\
&\quad\le C_0\int_0^\tau \left(\int_\Omega |b|^{\frac{2}{1-q}}\,\mathrm{d}x\right)\mathrm{d}t + a_0\int_0^\tau\int_\Omega |u^{(m)}|^2\,\mathrm{d}x\,\mathrm{d}t - a_0\iint_{Q_\tau}|u^{(m)}|^2\,\mathrm{d}x\,\mathrm{d}t \\
&\quad\le C_0\int_0^\tau\int_\Omega |b|^{\frac{2}{1-q}}\,\mathrm{d}x\,\mathrm{d}t.
\end{aligned}$$

This completes the proof of Lemma 3.1. □

The rest of the argument is similar to that in [4], we omit it here. In order to prove the locally extinction of weak solutions, we need to prove that the solution remains bounded.

Theorem 3.2 *Assume that the conditions in Theorem* 3.1 *hold, then the solution of problem* (1.1) *remains bounded, i.e.*,

$$\|u\|_{L^\infty(Q_T)} \le K(T) = \left\{(1-q)\int_0^T \|b\|_{L^\infty(Q_T)}\,\mathrm{d}t + \|u_0\|_{L^\infty(\Omega)}^{1-q}\right\}^{\frac{1}{1-q}}.$$

Proof For any fixed $M > 0$, define

$$u_M = \max\{-M, \min\{u, M\}\}.$$

For any integer $k \in N^*$, we choose u_M^{2k-1} as a test-function in (2.1). Letting $t_2 = t + h$, $t_1 = t$, with $t, t + h \in (0, T)$ in (2.1), we have

$$\begin{aligned}
&\frac{1}{2k}\int_t^{t+h}\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_\Omega u_M^{2k}\,\mathrm{d}x\right)\mathrm{d}t + \int_t^{t+h}\int_\Omega (2k-1)u_M^{2(k-1)}|\nabla u_M|^{p(x,t)}\,\mathrm{d}x\,\mathrm{d}t \\
&\quad= \int_t^{t+h}\int_\Omega b(x,t)u_M^{2k-1}|u_M|^q\,\mathrm{d}x\,\mathrm{d}t - a_0\int_t^{t+h}\int_\Omega u_M^{2k}\,\mathrm{d}x\,\mathrm{d}t.
\end{aligned}$$

Multiplying the above identity by $\frac{1}{h}$, letting $h \to 0^+$ and applying Lebesgue's dominated convergence theorem, we have that for every $t \in (0,T)$,

$$\begin{aligned}
&\frac{1}{2k}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u_M^{2k}\,\mathrm{d}x + \int_\Omega (2k-1)u_M^{2(k-1)}|\nabla u_M|^{p(x,t)}\,\mathrm{d}x \\
&\quad = \int_\Omega b(x,t)u_M^{2k+q-1}\,\mathrm{d}x - a_0\int_\Omega u_M^{2k}\,\mathrm{d}x \\
&\quad \le \|u_M\|_{2k}^{\frac{2k+q-1}{2k}} \cdot \|b\|_{\frac{2k}{1-q}}^{\frac{2k}{1-q}} - a_0\|u_M\|_{2k}^{2k}.
\end{aligned}$$

We have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|u_M\|_{2k} \le \|u_M\|_{2k}^{\frac{q}{2k}} \cdot \|b\|_{\frac{2k}{1-q}}^{\frac{2k}{1-q}} - a_0\|u_M\|_{2k}.$$

By Gronwall's inequality, we get

$$\|u_M\|_{2k} \le \left\{(1-q)\int_0^T \|b\|_{\frac{2k}{1-q}}^{\frac{2k}{1-q}}\,\mathrm{d}t + \|u_0\|^{1-q}\right\}^{\frac{1}{1-q}}. \tag{3.3}$$

Letting $k \to +\infty$ in (3.3), we have

$$\|u_M\|_{\infty,\Omega} \le \left\{(1-q)\int_0^T \left\|b(\cdot,t)\right\|_{\infty,\Omega}\,\mathrm{d}t + \|u_0\|_{\infty,\Omega}^{1-q}\right\}^{\frac{1}{1-q}} \triangleq K(T),$$

We choose $M > K(T)$, then $u_M = u$ a.e. in Q_T. □

Theorem 3.3 (Extinction) ($N > 2$) *Suppose* $\frac{2N}{N+2} < p^- < \frac{2N-1}{N+1}$.

(1) $1 < p^- < p^+ < \frac{(N-1)p^-}{Np^- - N + p^-}$, *then there exists* $T_1^* \triangleq T_1^*(p^\pm, N, q, |\Omega|, |b|_{L^{\frac{2}{1-q}}}, a_0)$ *such that every nonnegative solution of problem* (1.1) *vanishes in finite time, i.e.,*

$$\lim_{t\to T_1^*}\|u\|_{L^2(\Omega)} = 0;$$

(2) $1 < p^- < \frac{(N-1)p^-}{Np^- - N + p^-} < p^+ < \frac{Np^-}{Np^- + p^- - N}$, *then there exists* $T_2^* \triangleq T_2^*(p^\pm, N, q, |\Omega|, |b|_{L^{\frac{2}{1-q}}}, a_0)$ *such that every nonnegative solution of problem* (1.1) *vanishes in finite time, i.e.,*

$$\lim_{t\to T_2^*}\|u\|_{L^r(\Omega)} = 0, \quad \text{with } r = \frac{p^+(N - p^- - Np^-) + (N+1)p^-}{p^-} > 2.$$

Theorem 3.4 (Extinction) ($N > 2$) *Assume that* $\frac{2N-1}{N+1} < p^- < p^+ < \frac{Np^-}{Np^- + p^- - N} < 2$, *then there exists* $T_3^* \triangleq T_3^*(p^\pm, N, q, |\Omega|, |b|_{L^{\frac{2}{1-q}}}, a_0)$ *such that every nonnegative solution of problem* (1.1) *vanishes in finite time, i.e.,*

$$\lim_{t\to T_3^*}\|u\|_{L^r(\Omega)} = 0, \quad \text{with } 1 < r = 1 + \frac{p^+(N - p^-) - Np^-(p^+ - 1)}{p^-} < 2.$$

Proof Case 1. $\frac{2N}{N+2} < p^- < \frac{2N-1}{N+1}$, $1 < p^- < p^+ < \frac{(N-1)p^-}{Np^- - N + p^-}$.

Choosing u as a test-function in (2.1), we have

$$\frac{1}{2}\int_0^t\int_\Omega \frac{\mathrm{d}}{\mathrm{d}t}|u|^2\,\mathrm{d}x\,\mathrm{d}t + \int_0^t\int_\Omega |\nabla u|^{p(x,t)}\,\mathrm{d}x\,\mathrm{d}t = \int_0^t\int_\Omega b|u|^{1+q}\,\mathrm{d}x - a_0\int_0^t\int_\Omega u^2\,\mathrm{d}x,$$

$$\begin{aligned}\int_0^t\int_\Omega |\nabla u|^{p(x,\tau)}\,\mathrm{d}x\,\mathrm{d}\tau &\overset{\text{Lemma 2.1}}{\geq} C_0\min\bigl\{|\nabla u|^{p^-}_{p(\cdot,t)}\cdot|\nabla u|^{p^+}_{p(\cdot,t)}\bigr\}\\ &\geq C_0\min\bigl\{M^{p^+-p^-},1\bigr\}|\nabla u|^{p^+}_{p^-}\\ &\geq C_1|\nabla u|^{p^+}_{p^-}.\end{aligned} \tag{3.4}$$

Applying the embedding theorem $W^{1,p^-}\hookrightarrow L^2$, we have

$$\begin{aligned}&\left(\int_\Omega u^2\,\mathrm{d}x\right)^2 \leq C_0\left(\int_\Omega |\nabla u|^{p^-}\,\mathrm{d}x\right)^{\frac{1}{p^-}},\\ &\int_0^t\int_\Omega b\bigl|u^{q+1}\bigr|\,\mathrm{d}x \leq \left(\int_0^t\int_\Omega b^{\frac{2}{1-q}}\,\mathrm{d}x\,\mathrm{d}t\right)^{\frac{1-q}{2}}\cdot\left(\int_0^t\int_\Omega |u|^2\,\mathrm{d}x\,\mathrm{d}t\right)^{\frac{q+1}{2}},\\ &F(t) = \int_0^t\int_\Omega |u|^2\,\mathrm{d}x\,\mathrm{d}t.\end{aligned} \tag{3.5}$$

Moreover, we have

$$F'(t) + C_0F(t)^{\frac{p^+}{2}} + a_0F(t) \leq C_1F(t)^{\frac{q+1}{2}}.$$

This conclusion follows from Gronwall's inequality.

Case 2. $\frac{2N}{N+2} < p^- < \frac{2N-1}{N+1}$, $1 < p^- < \frac{(N-1)p^-}{Np^- - N + p^+} < p^+ < \frac{Np^-}{Np^- - N + p^-}$.

Choosing u^m as a test-function with $m = 1 + \frac{p^+(N-p^-)-Np^-(p^+-1)}{p^-}$, we have

$$\begin{aligned}&\frac{1}{2}\int_0^t\int_\Omega \frac{\mathrm{d}}{\mathrm{d}t}|u|^{m+1}\,\mathrm{d}x\,\mathrm{d}t + (m-1)\int_0^t\int_\Omega u^m|\nabla u|^{p(x,t)}\,\mathrm{d}x\,\mathrm{d}t\\ &\quad= \int_0^t\int_\Omega b|u|^{m+q}\,\mathrm{d}x - a_0\int_0^t\int_\Omega u^{m+1}\,\mathrm{d}x.\end{aligned} \tag{3.6}$$

Applying $|u|_\infty \leq M$, $m-1-(\alpha-1)p^- \leq 0$ and Lemma 2.1, we get that

$$\begin{aligned}(m-1)\int_0^t\int_\Omega u^{m-1}|\nabla u|^{p(x,\tau)}\,\mathrm{d}x\,\mathrm{d}\tau &\geq C_0\int_0^t\int_\Omega \bigl|\nabla u^\alpha\bigr|^{p(x,\tau)}\,\mathrm{d}x\,\mathrm{d}\tau\\ &\geq C_0\min\bigl\{\bigl|\nabla u^\alpha\bigr|^{p^-}_{p(\cdot,t)}\cdot\bigl|\nabla u^\alpha\bigr|^{p^+}_{p(\cdot,t)}\bigr\}\\ &\geq C_0\min\bigl\{M^{p^+-p^-},1\bigr\}\bigl|\nabla u^\alpha\bigr|^{p^+}_{p^-}\\ &\geq C_1\bigl|\nabla u^\alpha\bigr|^{p^+}_{p^-}.\end{aligned}$$

Applying the embedding theorem $W^{1,p^-}\hookrightarrow L^{\frac{Np^-}{N-p^-}}$, we have

$$\begin{aligned}&\left(\int_\Omega u^{\frac{\alpha Np^-}{N-p^-}}\,\mathrm{d}x\right)^{\frac{N-p^-}{Np^-}} \leq C_0\left(\int_\Omega \bigl|\nabla u^\alpha\bigr|^{p^-}\,\mathrm{d}x\right)^{\frac{1}{p^-}},\\ &\int_0^t\int_\Omega b\bigl|u^{q+m}\bigr|\,\mathrm{d}x \leq \left(\int_0^t\int_\Omega b^{\frac{m+1}{1-q}}\,\mathrm{d}x\,\mathrm{d}t\right)^{\frac{1-q}{m+1}}\cdot\left(\int_0^t\int_\Omega |u|^{m+1}\,\mathrm{d}x\,\mathrm{d}t\right)^{\frac{q+1}{m+q}}.\end{aligned} \tag{3.7}$$

Similarly as Case 1, letting $\alpha = \frac{(m+1)(N-p^-)}{Np^-}$, we have

$$F'(t) + F(t)^{\frac{p^+(N-p^-)}{Np^-}} + a_0 F(t) \le C_1 F(t)^{\frac{q+m}{1+m}},$$

with $F(t) = \int_0^T \int_\Omega |u|^{m+1}\,\mathrm{d}x\,\mathrm{d}t$.

Case 3. $\frac{2N-1}{N+1} < p^- < p^+ < \frac{Np^-}{Np^- + p^- - N} < 2$.

We choose u^m as a test-function. We have

$$m = 1 + \frac{p^+(N-p^-) - Np^-(p^+ - 1)}{p^-}.$$

The rest of the argument is the same as Case 2, we omit it here. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Three authors collaborated in all the steps concerning the research and achievements presented in the final manuscript.

Acknowledgements
The work was supported the Natural Science Foundation of Jilin Province (Grant No. 201215044). We are very grateful to the anonymous referees for their valuable suggestions that improved the article.

Received: 13 March 2013 Accepted: 25 October 2013 Published: 27 Nov 2013

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10.1186/1687-2770-2013-259
Cite this article as: Sun et al.: **Extinction in finite time of solutions to the nonlinear diffusion equations involving $p(x,t)$-Laplacian operator.** *Boundary Value Problems* 2013, 2013:259