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Energy decay and nonexistence of solution for a reaction-diffusion equation with exponential nonlinearity

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Abstract

In this work we consider the energy decay result and nonexistence of global solution for a reaction-diffusion equation with generalized Lewis function and nonlinear exponential growth. There are very few works on the reaction-diffusion equation with exponential growth f as a reaction term by potential well theory. The ingredients used are essentially the Trudinger-Moser inequality.

Keywords: reaction-diffusion equation; stable and unstable set; exponential reaction term; decay rate; global nonexistence

1 Introduction

In this paper, we study the following initial boundary value problem with generalized Lewis function $a(x,t)$ which depends on both spacial variable and time:

$$a(x,t)u_t - \Delta u = f(u), \quad x \in \Omega, t > 0, \tag{1}$$

$$u(x,t) = 0, \quad x \in \partial\Omega, t > 0, \tag{2}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{3}$$

here $f(s)$ is a reaction term with exponential growth at infinity to be specified later, Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^2.

For the reaction-diffusion equation with polynomial growth reaction terms (that is, equation (1) with $a(x,t) = 1$ and $f(u) = |u|^{p-1}u$), there have been many works in the literature; one can find a review of previous results in [1, 2] and references therein, which are not listed in this paper just for concision. Problem (1)-(3) with $a(x,t) > 0$ describes the chemical reaction processes accompanied by diffusion [2]. The author of work [1] proved the existence and asymptotic estimates of global solutions and finite time blow-up of problem (1)-(3) with $a(x,t) > 0$ and the critical Sobolev exponent $p = \frac{n+2}{n-2}$ for $f(u) = u^p$.

In this paper we assume that $f(s)$ is a reaction term with exponential growth like e^{s^2} at infinity. When $a(x,t) = 1$, $f(u) = e^u$, model (1)-(3) was proposed by [3] and [4]. In this case, Fujita [5] studied the asymptotic stability of the solution. Peral and Vazquez [6] and Pulkkinen [7] considered the stability and blow-up of the solution. Tello [8] and Ioku [9] considered the Cauchy problem of heat equation with $f(u) \approx e^{u^2}$ for $|u| \geq 1$.

Recently, Alves and Cavalcanti [10] were concerned with the nonlinear damped wave equation with exponential source. They proved global existence as well as blow-up of solutions in finite time by taking the initial data inside the potential well [11]. Moreover, they also got the optimal and uniform decay rates of the energy for global solutions.

Motivated by the ideas of [1, 10], we concentrate on studying the uniform decay estimate of the energy and finite time blow-up property of problem (1)-(3) with generalized Lewis function $a(x,t)$ and exponential growth f as a reaction term. To the authors' best knowledge, there are very few works in the literature that take into account the reaction-diffusion equation with exponential growth f as a reaction term by potential well theory. The majority of works in the literature make use of the potential well theory when f possesses polynomial growth. See, for instance, the works [12–16] and a long list of references therein. The ingredients used in our proof are essentially the Trudinger-Moser inequality (see [17, 18]). We establish decay rates of the energy by considering ideas from the work of Messaoudi [15]. The case of nonexistence results is also treated, where a finite time blow-up phenomenon is exhibited for finite energy solutions by the standard concavity method adapted for our context.

The remainder of our paper is organized as follows. In Section 2 we present the main assumptions and results, Section 3 and Section 4 are devoted to the proof of the main results.

Throughout this study, we denote by $\|\cdot\|$, $\|\cdot\|_p$, $\|\cdot\|_{H_0^1}$ the usual norms in spaces $L^2(\Omega)$, $L^p(\Omega)$ and $H_0^1(\Omega)$, respectively.

2 Assumptions and preliminaries

In this section, we present the main assumptions and results. We always assume that:

(A1) $a(x,t)$ is a positive differentiable function and is bounded for $t \in [0,+\infty)$, $x \in \Omega$.

(A2) $f: R \to R$ is a C^1 function. The function $f(t)/t$ is increasing in $(0,\infty)$, and for each $\beta > 0$, there exists a positive constant C_β such that

$$\left|f(t)\right| \le C_\beta e^{\beta t^2}, \qquad \left|f'(t)\right| \le C_\beta e^{\beta t^2}. \tag{4}$$

(A3) For each $\varepsilon > 0$, $\beta > 0$ and $p > 1$ fixed, there exists a positive constant $C(\varepsilon,\beta)$ such that

$$\left|f(t)\right| \le \varepsilon|t| + C(\varepsilon,\beta)|t|^{p-1}e^{\beta t^2}, \tag{5}$$

$$\left|F(t)\right| \le \varepsilon|t|^2 + C(\varepsilon,\beta)|t|^{p}e^{\beta t^2}, \tag{6}$$

where $F(t) = \int_0^t f(s)\,ds$.

(A4) There exists a positive constant $\theta > 2$ such that

$$0 < \theta F(t) < f(t)t, \quad t \in R/\{0\}. \tag{7}$$

A typical example of functions satisfies (A2)-(A4) is $f(t) = C|t|^{p-1}te^{Mt^\alpha}$, with given $p > 1$, $M > 0$, $C > 0$, and $\alpha \in (1,2)$.

Now we define some functional as follows:

$$E(t) = E(u) = \frac{1}{2}\|\nabla u\|^2 - \int_\Omega F(u)\,dx, \tag{8}$$

$$I(t) = I(u) = \|\nabla u\|^2 - \int_\Omega uf(u)\,dx, \tag{9}$$

then the 'potential depth' given by

$$d = \inf\Big\{\sup_{\lambda \in R} E(\lambda u), u \in H_0^1/\{0\}\Big\}$$

is a positive constant [10]. Hence, we are able to define stable and unstable sets respectively as follows:

$$W_1 = \big\{u \in H_0^1, E(u) < d, I(u) > 0\big\},$$
$$W_2 = \big\{u \in H_0^1, E(u) < d, I(u) < 0\big\}.$$

We also need the following lemmas.

Lemma 2.1 [17, 18] *Let Ω be a bounded domain in R^2. For all $u \in H_0^1(\Omega)$,*

$$e^{\alpha|u|^2} \in L^1(\Omega) \quad \text{for all } \alpha > 0, \tag{10}$$

and there exist positive constants m_2 such that

$$\sup_{u \in H_0^1(\Omega), \|\nabla u\| \le 1} \int_\Omega e^{\alpha|u|^2}\, dx = m_2 < \infty \quad \text{for all } \alpha \le 4\pi. \tag{11}$$

Lemma 2.2 [19] *Let $\phi(t)$ be a nonincreasing and nonnegative function on $[0, \infty)$, such that*

$$\sup_{s \in [t, t+1]} \phi(s) \le C\big(\phi(t) - \phi(t+1)\big), \quad t > 0, \tag{12}$$

then

$$\phi(t) \le Ce^{-\omega t},$$

where C, ω are positive constants depending on $\phi(0)$ and other known qualities.

Lemma 2.3 [20] *Suppose that a positive, twice-differentiable function $H(t)$ satisfies on $t \ge 0$ the inequality*

$$H''(t)H(t) - (\delta + 1)\big(H'(t)\big)^2 \ge 0, \tag{13}$$

where $\delta > 0$, then there is $t_1 < t_2 = \frac{H(0)}{\delta H'(0)}$ such that $H(t) \to \infty$ as $t \to t_1$.

In order to state and prove our main results, we remind that by the embedding theorem there exists a constant C_0 depending on p and Ω only such that

$$\|u\|_p \le C_0 \|\nabla u\|. \tag{14}$$

By multiplying equation (1) by u_t, integrating over Ω, using integration by parts and $a(x,t) > 0$, we get

$$E'(t) = -\int_\Omega a(x,t) u_t^2(x,t)\, dx \le 0. \tag{15}$$

Our main results read as follows.

Theorem 2.1 *Let* (A1)-(A4) *hold. Assume further that $u_0 \in W_1$ satisfies*

$$\varepsilon_0 C_0^2 + C_{\varepsilon_0} C_0^p m_2^{\frac{1}{2}} \left(\frac{2\theta}{\theta - 2} E(0) \right)^{p-2} < 1 \tag{16}$$

for some sufficiently small $\varepsilon_0 > 0$ and $C_{\varepsilon_0} > 0$. Then there exist positive constants K and k such that the energy $E(t)$ satisfies the decay estimates for large t

$$E(t) \le K e^{-kt}. \tag{17}$$

Theorem 2.2 *Let* (A1)-(A4) *hold. Assume further that $a_t(x,t) \le 0$, $u_0 \in W_2$ and $E(0) < \frac{(\theta-2)d}{\theta} < d$, then the solutions of* (1)-(3) *blow up in finite time.*

3 Proof of decay of the energy

In this section we prove Theorem 2.1. We divide the proof into two lemmas.

Lemma 3.1 *Under the assumptions of Theorem* 2.1, *we have, for all $t \ge 0$, $u(t) \in W_1$.*

Proof Since $I(u_0) \ge 0$, then there exists (by continuity) $T_m < T$ such that

$$I\bigl(u(t)\bigr) \ge 0, \quad \forall t \in [0, T_m].$$

This and (A4) give

$$\begin{aligned} E(t) &= \left(\frac{1}{2} - \frac{1}{\theta} \right) \|\nabla u\|^2 + \frac{1}{\theta} I(u) + \int_\Omega \left(\frac{1}{\theta} u f(u) - F(u) \right) dx \\ &\ge \left(\frac{1}{2} - \frac{1}{\theta} \right) \|\nabla u\|^2, \quad \forall t \in [0, T_m]. \end{aligned} \tag{18}$$

So, by (15) we have

$$\|\nabla u\|^2 \le \frac{2\theta}{\theta - 2} E(t) \le \frac{2\theta}{\theta - 2} E(0) \le \frac{2\theta d}{\theta - 2}, \quad \forall t \in [0, T_m]. \tag{19}$$

We then use (5), the Holder inequality and the embedding theorem to obtain, for each $t \in [0, T_m]$,

$$\begin{aligned} \int_\Omega u f(u)\, dx &\le \int_\Omega \bigl[\varepsilon |u|^2 + C(\varepsilon, \beta) |u|^p e^{\beta u^2} \bigr] dx \\ &\le \varepsilon C_0^2 \|\nabla u\|^2 + C(\varepsilon, \beta) \left(\int_\Omega |u|^{2p}\, dx \right)^{\frac{1}{2}} \left(\int_\Omega e^{2\beta u^2}\, dx \right)^{\frac{1}{2}}. \end{aligned} \tag{20}$$

Once $\|\nabla u\|^2 \le \frac{2\theta d}{\theta - 2}$, we choose β such that $\frac{\theta \beta d}{\theta - 2} < \pi$, then, from Trudinger-Moser inequality (11),

$$\int_\Omega e^{2\beta |u|^2}\, dx \le \int_\Omega e^{2\beta \|\nabla u\|^2 (\frac{u}{\|\nabla u\|})^2}\, dx \le m_2,$$

and therefore, by (16) for $\varepsilon_0 > 0$ and $C_{\varepsilon_0} > 0$, we have

$$\begin{aligned}\int_\Omega uf(u)\,dx &\le \varepsilon_0 C_0^2\|\nabla u\|^2 + C_{\varepsilon_0}C_0^p m_2^{\frac{1}{2}}\|\nabla u\|^p \\ &= \varepsilon_0 C_0^2\|\nabla u\|^2 + C_{\varepsilon_0}C_0^p m_2^{\frac{1}{2}}\left(\frac{2\theta}{\theta-2}E(t)\right)^{p-2}\|\nabla u\|^2 \\ &\le \left[\varepsilon_0 C_0^2 + C_{\varepsilon_0}C_0^p m_2^{\frac{1}{2}}\left(\frac{2\theta}{\theta-2}E(0)\right)^{p-2}\right]\|\nabla u\|^2 < \|\nabla u\|^2. \end{aligned} \tag{21}$$

By virtue of (21) and the definition of $I(t)$, we have

$$I(t) = \|\nabla u\|^2 - \int_\Omega uf(u)\,dx > 0.$$

This shows that $u(t) \in W_1$ for all $t \in [0, T_m]$. By repeating this procedure and the fact that $E(t) \le E(0)$, we obtain

$$\lim_{t\to T_m}\left(\varepsilon_0 C_0^2 + C_{\varepsilon_0}C_0^p m_2^{\frac{1}{2}}\left(\frac{2\theta}{\theta-2}E(t)\right)^{p-2}\right) \le \rho < 1.$$

This is extended to T. □

Lemma 3.2 *Under the assumptions of Theorem* 2.1, *we have*, *for* $\eta = 1 - [\varepsilon_0 C_0^2 + C_{\varepsilon_0}C_0^p m_2^{\frac{1}{2}} \times (\frac{2\theta}{\theta-2}E(0))^{p-2}]$,

$$\eta\|\nabla u\|^2 < I(t). \tag{22}$$

Proof It suffices to rewrite (21) as

$$\begin{aligned}\int_\Omega uf(u)\,dx &\le \left[\varepsilon_0 C_0^2 + C_{\varepsilon_0}C_0^p m_2^{\frac{1}{2}}\left(\frac{2\theta}{\theta-2}E(0)\right)^{p-2}\right]\|\nabla u\|^2 \\ &= (1-\eta)\|\nabla u\|^2 = \|\nabla u\|^2 - \eta\|\nabla u\|^2. \end{aligned} \tag{23}$$

Thus (22) follows from (23). □

Proof of Theorem 2.1 We integrate (15) over $[t, t+1]$ to obtain

$$E(t) - E(t+1) = \int_t^{t+1}\int_\Omega a(x,s)u_t^2(x,s)\,dx\,ds = D^2(t). \tag{24}$$

Now we multiply (1) by u and integrate over $\Omega \times [t, t+1]$ to arrive at

$$\begin{aligned}\int_t^{t+1} I(s)\,ds &= \int_t^{t+1}\left[\|\nabla u\|^2 - \int_\Omega uf(u)\,dx\right]ds \\ &= \int_t^{t+1}\int_\Omega a(x,t)u_t(x,t)u(x,t)\,dx\,ds \\ &\le A\int_t^{t+1}\left\|a^{\frac{1}{2}}u_t(s)\right\|\left\|u(s)\right\|ds, \end{aligned} \tag{25}$$

where $A^2 = \sup_{(x,t)\in\Omega\times[0,+\infty)} |a(x,t)|$. Exploiting (14) and (19), we obtain

$$\begin{aligned}\int_t^{t+1} I(s)\,ds &\leq AC_0\left(\frac{2\theta}{\theta-2}\right)^{\frac{1}{2}}\Bigl(\sup_{s\in[t,t+1]} E^{\frac{1}{2}}(t)\Bigr)\int_t^{t+1}\bigl\|a^{\frac{1}{2}}u_t\bigr\|\,ds\\ &\leq AC_0\left(\frac{2\theta}{\theta-2}\right)^{\frac{1}{2}}\Bigl(\sup_{s\in[t,t+1]} E^{\frac{1}{2}}(t)\Bigr)D(t).\end{aligned} \tag{26}$$

Using (7), (23) and (22), we have

$$\begin{aligned}E(t) &= \frac{\theta-2}{2\theta}\|\nabla u\|^2 + \frac{1}{\theta}I(t) + \int_\Omega\left(\frac{1}{\theta}uf(u) - F(u)\right)dx\\ &\leq \frac{\theta-2}{2\theta}\|\nabla u\|^2 + \frac{1}{\theta}I(t) + \int_\Omega \frac{2}{\theta}uf(u)\,dx\\ &\leq \frac{\theta-2}{2\theta}\|\nabla u\|^2 + \frac{1}{\theta}I(t) + \frac{2}{\theta}(1-\eta)\|\nabla u\|^2\\ &\leq \left[\frac{\theta-2}{2\theta\eta} + \frac{1}{\theta} + \frac{2}{\theta}(1-\eta)\right]I(t).\end{aligned} \tag{27}$$

Integrating both sides of (27) over $[t, t+1]$ and using (26), one can write

$$\int_t^{t+1} E(s)\,ds = \left[\frac{\theta-2}{2\theta\eta} + \frac{1}{\theta} + \frac{2}{\theta}(1-\eta)\right]AC_0\left(\frac{2\theta}{\theta-2}\right)^{\frac{1}{2}}\Bigl(\sup_{s\in[t,t+1]} E^{\frac{1}{2}}(t)\Bigr)D(t). \tag{28}$$

By using (15) again, we have $E(s) \geq E(t+1)$, $\forall s \leq t+1$, hence

$$\int_t^{t+1} E(s)\,ds \geq E(t+1). \tag{29}$$

Inserting (29) in (24) and using (27), we easily have

$$\begin{aligned}E(t) &\leq \int_t^{t+1} E(s)\,ds + \int_t^{t+1}\int_\Omega a(x,s)u_t^2(x,s)\,dx\,ds\\ &\leq \left[\frac{\theta-2}{2\theta\eta} + \frac{1}{\theta} + \frac{2}{\theta}(1-\eta)\right]AC_0\left(\frac{2\theta}{\theta-2}\right)^{\frac{1}{2}}\Bigl(\sup_{s\in[t,t+1]} E^{\frac{1}{2}}(t)\Bigr)D(t) + D^2(t)\\ &\leq C_1\bigl[E^{\frac{1}{2}}(t)D(t) + D^2(t)\bigr]\end{aligned} \tag{30}$$

for C_1 a constant depending on C_0, A, θ, η only. We then use Young's inequality to get from (30) and (24)

$$\sup_{s\in[t,t+1]} E(t) \leq C_2 D^2(t) \leq C_2\bigl(E(t) - E(t+1)\bigr). \tag{31}$$

By (12) in Lemma 2.2 we then get the results. □

4 Proof of the blow-up result

In this section, we shall prove Theorem 2.2 by adapting the concavity method (see Levine [20]). We recall the following lemma in [10].

Lemma 4.1 [10] *Assume that $u_0 \in W_2$ and $E(0) < d$, then it holds that*

$$u(t) \in W_2 \quad \text{for } t \in [0, T_{\max}), \tag{32}$$

$$\|\nabla u\|^2 \geq 2d \quad \text{for } t \in [0, T_{\max}). \tag{33}$$

Proof of Theorem 2.2 Assume by contradiction that the solution is global. Then, for any $T > 0$, we consider the function $H(\cdot) : [0, T] \to R^+$ defined by

$$\begin{aligned} H(t) = {} & \int_0^t \int_\Omega a(x,s)u^2(x,s)\,dx\,ds + \int_0^t \int_\Omega (s-t)a_t(x,s)u^2(x,s)\,dx\,ds \\ & + (T-t)\int_\Omega a(x,0)u_0^2(x)\,dx + \rho(t+t_0)^2, \end{aligned} \tag{34}$$

where t_0, T, ρ are positive constants which will be fixed later. Direct computations show that

$$\begin{aligned} H'(t) &= \int_\Omega a(x,t)u^2(x,t)\,dx - \int_0^t \int_\Omega a_t(x,s)u^2(x,s)\,dx\,ds - \int_\Omega a(x,0)u_0^2\,dx + 2\rho(t+t_0) \\ &= 2\int_0^t \int_\Omega a(x,s)u_t(x,s)u(x,s)\,dx\,ds + 2\rho(t+t_0), \end{aligned} \tag{35}$$

$$H''(t) = 2\int_\Omega a(x,t)u(x,t)u_t(x,s)\,dx + 2\rho. \tag{36}$$

Then, due to equations (1), (7) and (33), we have

$$\begin{aligned} H''(t) &= -2\|\nabla u\|^2 + 2\int_\Omega uf(u)\,dx + 2\rho \\ &\geq -2\|\nabla u\|^2 + 2\theta\int_\Omega F(u)\,dx + 2\rho = (\theta-2)\|\nabla u\|^2 - 2\theta E(t) + 2\rho \\ &= (\theta-2)\|\nabla u\|^2 - 2\theta E(0) + 2\theta\int_0^t \int_\Omega a(x,s)u_t^2(x,s)\,dx\,ds + 2\rho \\ &\geq 2(\theta-2)d - 2\theta E(0) + 2\theta\int_0^t \int_\Omega a(x,s)u_t^2(x,s)\,dx\,ds + 2\rho. \end{aligned} \tag{37}$$

Now we take $0 < \rho < \frac{(\theta-2)d-\theta E(0)}{\theta-1}$ such that $2(\theta-2)d - 2\theta E(0) + 2\rho > 2\theta\rho$ (this ρ can be chosen since $E(0) < \frac{(\theta-2)d}{\theta}$), and then

$$H''(t) \geq 2\theta\rho + 2\theta\int_0^t \int_\Omega a(x,s)u_t^2(x,s)\,dx\,ds. \tag{38}$$

We also note that

$$H(0) = T\int_\Omega a(x,0)u_0^2(x)\,dx + \rho t_0^2 > 0,$$

$$H'(0) = 2\rho t_0 > 0,$$

$$H''(t) \geq 2\theta\rho > 0, \quad t \geq 0.$$

Therefore $H(t)$ and $H'(t)$ are both positive. Since $a_t(x,t) \le 0$ for all $x \in \Omega$ and $t \ge 0$, by the construction of $H(t)$, it is clear that

$$H(t) \ge \int_0^t \int_\Omega a(x,s)u^2(x,s)\,dx\,ds + \rho(t+t_0)^2. \tag{39}$$

Thus, for all $(\xi,\eta) \in R^2$, from (35), (38) and (39) it follows that

$$\begin{aligned} &H(t)\xi^2 + H'(t)\xi\eta + \frac{1}{2\theta}H''(t)\eta^2 \\ &\quad \ge \left(\int_0^t \int_\Omega a(x,s)u(x,s)^2\,dx\,ds + \rho(t+t_0)^2\right)\xi^2 \\ &\qquad + 2\xi\eta\int_0^t \int_\Omega a(x,s)u(x,s)u_t(x,s)\,dx\,ds + 2\rho(t+t_0)\xi\eta \\ &\qquad + \rho\eta^2 + \eta^2\int_0^t \int_\Omega a(x,s)u_t^2(x,s)\,dx\,ds \ge 0, \end{aligned}$$

which implies

$$\big(H'(t)\big)^2 - \frac{2}{\theta}H(t)H''(t) \le 0.$$

That is,

$$H(t)H''(t) - \frac{\theta}{2}\big(H'(t)\big)^2 \ge 0.$$

Then we complete the proof by the standard concavity method (Lemma 2.3) since $\theta > 2$. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors declare that the study was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

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