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Flow of generalized Burgers fluid between parallel walls induced by rectified sine pulses stress

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Abstract
This paper presents the unsteady magnetohydrodynamic (MHD) flow of a generalized Burgers' fluid between two parallel side walls perpendicular to a plate. The plate applies a shear stress induced by rectified sine pulses to the fluid. The obtained solutions by means of the Laplace and Fourier cosine and sine transforms are presented as a sum of the corresponding Newtonian and non-Newtonian contributions. The effects of the magnetic field, permeability, and the period of the oscillation have been observed on the fluid motion. Moreover, the influence of the side walls on the fluid motion and the distance between the walls for which the velocity of the fluid in the middle of the channel is negligible are presented by graphical illustrations.

MSC: 76A05; 76A10

Keywords: generalized Burgers fluid; rectified sine pulses shear stress; Fourier cosine and sine transforms

1 Introduction

Motion of non-Newtonian fluids on oscillating plates is not only of fundamental theoretical interest but it also occurs in many applied problems, *e.g.*, clay rotation, heart pumping, artificial surfing *etc.* Erdogan [1] obtained a solution as a sum of steady and transient solutions for the flow of a viscous fluid produced by a plane boundary moving in its own plane with a sinusoidal variation of velocity. Exact solutions for unsteady flow of a generalized Burgers fluid due to a rigid plate between two infinite parallel plates, one of which is an oscillating and time-periodic plane Poiseuille flow, was established by Fetecau *et al.* [2]. Zheng *et al.* [3] established an exact solution for the unsteady flow of a generalized Maxwell fluid over a flat plate. The plate was set into oscillating motion induced by hyperbolic sine velocity. Some recent work involving oscillating flows has been presented in many studies [4–7].

MHD flow of fluids and motion of fluids through porous media occur in medicine, engineering problems and geophysics, *e.g.*, cardiology, delivery of medicine to affected areas, regulation of skin, nuclear reactors and geomagnetic dynamo. Khan and Zeeshan [8], and Ghosh and Sana [9] investigated the MHD flow of an Oldroyd-B fluid through a porous space. The motions were generated in the fluid due to the velocity sawtooth pulses of the plate. Ghosh and Sana [10] discussed the unsteady motion of an Oldroyd-

B fluid in a channel bounded by two infinite rigid parallel plates in the presence of an external magnetic field acting normal to the plates. The flow is generated from rest due to rectified sine pulses applied periodically on the upper plate with the lower plate held fixed.

In the above citations, the conditions on the boundary are given in terms of velocity. The stress at the boundary gives important information as regards the nature of dissipation at the boundary. Little work is available in the literature where the oscillating stress is given on the boundary. Vieru *et al.* [11] analyzed the unsteady motion of a second grade fluid between two parallel side walls induced by oscillating shear stress. Li *et al.* [12] presented an analysis for helical flows of a heated generalized Oldroyd-B fluid subject to a linear time-dependent shear stress in a porous medium, where the motion is induced by the longitudinal time-dependent shear stress and the oscillating velocity at the boundary. Jamil *et al.* [13] and Shahid *et al.* [14] determined the starting solutions for the motion of Oldroyd-B fluids induced by quadratic, and cosine and sine oscillating time-dependent shear stress, respectively. Sohail *et al.* [15] presented closed-form expressions for the starting solutions corresponding to the unsteady motion of a Maxwell fluid due to an infinite plate that applies oscillating shear stresses to the fluid. Rubbab *et al.* [16] derived the unsteady natural convection flow of an incompressible viscous fluid near a vertical plate that applies a shear stress which is of exponential order of time.

In spite of all these citations and work in this direction, no attempt is made for oscillations induced by rectified sine pulses stress in a bounded domain. The main objective of the present investigation is to study the MHD oscillatory flow of a generalized Burgers fluid through a porous medium between two parallel walls. The formulation of the governing problem is made using the modified Darcy law of a generalized Burgers fluid. The induced magnetic field is assumed to be small as compared with the applied magnetic field. Analytical expressions for the velocity field and the shear stress are determined by means of the Fourier cosine and sine transforms coupled with Laplace transform. Finally, a comprehensive study of some physical parameters involved is performed to illustrate the influence of these parameters on the velocity.

2 Governing equations

For the generalized Burgers fluid, the Cauchy stress tensor is given by

$$\tau = -p\mathbf{I} + \mathbf{S}, \qquad \mathbf{S} + \lambda_1 \frac{\delta \mathbf{S}}{\delta t} + \lambda_2 \frac{\delta^2 \mathbf{S}}{\delta t^2} = \mu \left(\mathbf{A} + \lambda_3 \frac{\delta \mathbf{A}}{\delta t} + \lambda_4 \frac{\delta^2 \mathbf{A}}{\delta t^2} \right), \tag{1}$$

where $-p\mathbf{I}$ denotes the indeterminate spherical stress, $\mathbf{S}$ is the extra-stress tensor, $\mathbf{A} = \mathbf{L} + \mathbf{L}^T$ is the first Rivlin-Ericksen tensor ($\mathbf{L}$ being the velocity gradient), $\frac{\delta}{\delta t}$ denotes the upper convective derivative, μ is the dynamic viscosity, λ_1 and λ_3 ($< \lambda_1$) are the relaxation and retardation times, λ_2 and λ_4 are the material parameters of the generalized Burgers fluid having the dimension of the square of time, and

$$\frac{\delta^2 \mathbf{S}}{\delta t^2} = \frac{\delta}{\delta t}\left(\frac{\delta \mathbf{S}}{\delta t}\right) = \frac{\delta}{\delta t}\left(\frac{d\mathbf{S}}{dt} - \mathbf{LS} - \mathbf{SL}^T\right). \tag{2}$$

We seek the velocity field **V** and the stress field **S** of the form

$$\mathbf{V} = \mathbf{V}(x,y,t) = w(x,y,t)\hat{\mathbf{k}}, \qquad \mathbf{S} = \mathbf{S}(x,y,t), \tag{3}$$

where $\hat{\mathbf{k}}$ is the unit vector along the z-direction. If the fluid is at rest up to the moment $t = 0$, then

$$\mathbf{V}(x,y,0) = 0, \qquad \mathbf{S}(x,y,0) = \frac{\partial \mathbf{S}(x,y,0)}{\partial t} = 0. \tag{4}$$

Equations (1), (2), and (4) give the trivial stresses $S_{xx} = S_{xy} = S_{yy} = 0$ and the meaningful equations

$$\begin{aligned}
&\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\tau_1(x,y,t) = \mu\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\frac{\partial w(x,y,t)}{\partial x} \\
&\quad \text{for } \tau_1(x,y,0) = 0, && (5) \\
&\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\tau_2(x,y,t) = \mu\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\frac{\partial w(x,y,t)}{\partial y} \\
&\quad \text{for } \tau_2(x,y,0) = 0, && (6)
\end{aligned}$$

where $\tau_1 = \mathbf{S}_{xz}(x,y,t)$ and $\tau_2 = \mathbf{S}_{yz}(x,y,t)$ are the non-zero shear stresses.

The Darcy resistance **R** in a generalized Burgers fluid satisfies the following expression:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\mathbf{R} = -\frac{\mu\phi}{k}\left(1 + \lambda_3 \frac{\partial}{\partial t} + \lambda_4 \frac{\partial^2}{\partial t^2}\right)\mathbf{V}, \tag{7}$$

where ϕ is the porosity and k is the permeability of the medium.

We assume that a uniform magnetic field of strength β_o is applied to the fluid. We also assume that the direction of the magnetic field is perpendicular to the velocity field.

Thus the Lorentz force due to the magnetic field becomes

$$\mathbf{J} \times \mathbf{B} = -\sigma B_0^2 \mathbf{V}, \tag{8}$$

where σ is the electrical conductivity of the fluid.

The balance of linear momentum which governs the MHD flow through the porous medium becomes

$$\rho \frac{d\mathbf{V}}{dt} = \nabla \cdot \tau + \mathbf{J} \times \mathbf{B} + \mathbf{R}, \tag{9}$$

here ρ is the density.

We consider the unsteady flow of an incompressible generalized Burgers fluid over an infinite flat plate between two parallel side walls separated by a distance d, perpendicular to the plate. At time $t = 0$, the plate and the fluid are at rest. At time $t = 0^+$, the plate applies a pulsating shear to the fluid induced by rectified sine pulses.

In view of Eqs. (5)-(9), the governing equation leads to

$$\begin{aligned}&\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial w(x,y,t)}{\partial t}\\&\quad=\nu\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w(x,y,t)\\&\qquad-\Omega\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)w(x,y,t)-\epsilon\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)w(x,y,t),\end{aligned}\tag{10}$$

where $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, $\Omega=\frac{\sigma\beta_0^2}{\rho}$ is the magnetic parameter, and $\epsilon=\frac{\nu\phi}{k}$ is the porosity parameter.

We use the following appropriate initial conditions:

$$w(x,y,0)=\frac{\partial w(x,y,0)}{\partial t}=\frac{\partial^2 w(x,y,0)}{\partial t^2}=0,\quad\text{for } x>0 \text{ and } y\in[0,d],\tag{11}$$

the boundary conditions

$$\begin{aligned}&\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\tau_1(x,y,t)|_{x=0}=\mu\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial w(x,y,t)}{\partial x}\bigg|_{x=0}=Uf(t)\\&\quad\text{for } y\in(0,d) \text{ and } t>0,\end{aligned}\tag{12}$$

$$w(x,0,t)=w(x,d,t)=0\quad\text{for } x,t>0,\tag{13}$$

and the natural conditions

$$w(x,y,t)=\frac{\partial w(x,y,t)}{\partial x}\to 0\quad\text{as } x\to\infty, y\in[0,d], t>0.\tag{14}$$

According to the nature of the applied stress, we assume that the mathematical form of the function $f(t)$ is [8]

$$f(t)=\sin\left(\frac{\pi}{T}t\right)H(t)+2\sum_{p=1}^{\infty}(-1)^p\sin\left(\frac{\pi}{T}(t-pT)\right)H_{pT}(t),\quad p>0 \text{ and } T>0,\tag{15}$$

where $H(\cdot)$ is the Heaviside unit step function of period T and is defined as

$$H_{pT}(t)=0\quad\text{for } t\le pT\quad\text{and}\quad H_{pT}(t)=1\quad\text{for } t>pT.$$

In order to solve the problem, we use the Laplace transform technique and Fourier cosine and sine transforms in this order.

3 Calculation of velocity field

Applying the Laplace transform to Eq. (10), we obtain the following problem:

$$\begin{aligned}&\left(1+\lambda_1 q+\lambda_2 q^2\right)q\bar{w}(x,y,q)\\&\quad=\nu\left(1+\lambda_3 q+\lambda_4 q^2\right)\left(\frac{\partial^2\bar{w}(x,y,q)}{\partial x^2}+\frac{\partial^2\bar{w}(x,y,q)}{\partial y^2}\right)\\&\qquad-\Omega\left(1+\lambda_1 q+\lambda_2 q^2\right)\bar{w}(x,y,q)-\epsilon\left(1+\lambda_3 q+\lambda_4 q^2\right)\bar{w}(x,y,q).\end{aligned}\tag{16}$$

The Laplace transform $\bar{w}(x,y,q)$ of the function $w(x,y,t)$ has to satisfy the conditions

$$\begin{aligned}
&\bigl(1+\lambda_1 q+\lambda_2 q^2\bigr)\bar{\tau}_1(x,y,q)\big|_{x=0}\\
&\quad=\mu\bigl(1+\lambda_3 q+\lambda_4 q^2\bigr)\frac{\partial\bar{w}(x,y,q)}{\partial x}\bigg|_{x=0}\\
&\quad=U\frac{\frac{\pi}{T}}{q^2+\frac{\pi^2}{T^2}}\left(1+2\sum_{p=1}^{\infty}(-1)^p\exp(-pTq)\right)\quad\text{for } y\in[0,d],
\end{aligned}\tag{17}$$

$$\bar{w}(x,0,q)=\bar{w}(x,d,q)=0\quad\text{for } x>0,\tag{18}$$

$$\bar{w}(x,y,q)=\frac{\partial\bar{w}(x,y,q)}{\partial x}\to 0\quad\text{as } x\to\infty, y\in(0,d).\tag{19}$$

Multiplying both sides of Eq. (16) by $\sqrt{\frac{2}{\pi}}\cos(\xi x)\sin(\lambda_n y)$, where $\lambda_n=\frac{n\pi}{d}$, integrating with respect to x and y from 0 to ∞ and 0 to d, respectively, and bearing in mind the conditions (17)-(19), we find that

$$\begin{aligned}
\bar{w}_n(\xi,q)={}&\sqrt{\frac{2}{\pi}}\frac{U}{\rho}\frac{((-1)^n-1)}{\lambda_n}\\
&\times 1/\Bigl(\bigl(\lambda_2 q^3+\bigl(\lambda_1+\nu\lambda_4\bigl(\xi^2+\lambda_n^2\bigr)+\lambda_2\Omega+\lambda_4\epsilon\bigr)q^2\\
&+\bigl(1+\nu\lambda_3\bigl(\xi^2+\lambda_n^2\bigr)+\lambda_1\Omega+\lambda_3\epsilon\bigr)q+\nu\bigl(\xi^2+\lambda_n^2\bigr)+\Omega+\epsilon\bigr)\Bigr)\\
&\times\frac{\frac{\pi}{T}}{(q^2+(\frac{\pi}{T})^2)}\left(1+2\sum_{p=1}^{\infty}(-1)^p\exp(-pTq)\right),
\end{aligned}\tag{20}$$

where

$$\bar{w}_n(\xi,q)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\int_0^{d}\bar{w}(x,y,q)\cos(\xi x)\sin(\lambda_n y)\,dy\,dx;\quad n=1,2,3,\ldots.\tag{21}$$

Equation (20) can also be written as

$$\begin{aligned}
\bar{w}_n(\xi,q)={}&\sqrt{\frac{2}{\pi}}\frac{U}{\rho}\frac{((-1)^n-1)}{\lambda_n}\Biggl[\frac{1}{q+q_{4,n}(\xi)}\\
&-\frac{1}{\lambda_2}\Biggl(\frac{\phi_{1,n}(\xi)}{(q_{1,n}(\xi)-q_{2,n}(\xi))(q_{1,n}(\xi)-q_{3,n}(\xi))(q_{1,n}(\xi)+q_{4,n}(\xi))(q-q_{1,n}(\xi))}\\
&+\frac{\phi_{2,n}(\xi)}{(q_{2,n}(\xi)-q_{1,n}(\xi))(q_{2,n}(\xi)-q_{3,n}(\xi))(q_{2,n}(\xi)+q_{4,n}(\xi))(q-q_{2,n}(\xi))}\\
&+\frac{\phi_{3,n}(\xi)}{(q_{3,n}(\xi)-q_{1,n}(\xi))(q_{3,n}(\xi)-q_{2,n}(\xi))(q_{3,n}(\xi)+q_{4,n}(\xi))(q-q_{3,n}(\xi))}\\
&-\frac{\phi_{4,n}(\xi)}{(q_{1,n}(\xi)+q_{4,n}(\xi))(q_{2,n}(\xi)+q_{4,n}(\xi))(q_{3,n}(\xi)+q_{4,n}(\xi))(q+q_{4,n}(\xi))}\Biggr)\Biggr]\\
&\times\frac{\frac{\pi}{T}}{(q^2+(\frac{\pi}{T})^2)}\left(1+2\sum_{p=1}^{\infty}(-1)^p\exp(-pTq)\right),
\end{aligned}\tag{22}$$

where

$$\begin{aligned}
q_{4,n}(\xi) &= \nu\big(\xi^2+\lambda_n^2\big)+\Omega+\epsilon,\\
q_{i,n}(\xi) &= s_{i,n}(\xi)-\frac{\lambda_1+\nu\lambda_4(\xi^2+\lambda_n^2)+\lambda_2\Omega+\lambda_4\epsilon}{3\lambda_2^2},\\
\phi_{i,n}(\xi) &= \lambda_2 q_{i,n}^3(\xi)+\big(\lambda_1+\nu\lambda_4\big(\xi^2+\lambda_n^2\big)+\lambda_2\Omega+\lambda_4\epsilon\big)q_{i,n}^2(\xi)\\
&\quad+\big(\nu\lambda_3\big(\xi^2+\lambda_n^2\big)+\lambda_1\Omega+\lambda_3\epsilon\big)q_{i,n}(\xi),\quad i=1,2,3,\quad\text{and}\\
\phi_{4,n}(\xi) &= -\lambda_2 q_{4,n}^3(\xi)+\big(\lambda_1+\nu\lambda_4\big(\xi^2+\lambda_n^2\big)+\lambda_2\Omega+\lambda_4\epsilon\big)q_{i,n}^2(\xi)\\
&\quad-\big(\nu\lambda_3\big(\xi^2+\lambda_n^2\big)+\lambda_1\Omega+\lambda_3\epsilon\big)q_{i,n}(\xi).
\end{aligned}\tag{23}$$

In the above relations

$$\begin{aligned}
s_{1,n}(\xi) &= \left(-\frac{\beta_{1,n}(\xi)}{2}+\sqrt{\frac{(\beta_{1,n}(\xi))^2}{4}+\frac{(\alpha_1(\xi))^3}{27}}\right)^{1/3}\\
&\quad+\left(-\frac{\beta_{1,n}(\xi)}{2}-\sqrt{\frac{(\beta_{1,n}(\xi))^2}{4}+\frac{(\alpha_{1,n}(\xi))^3}{27}}\right)^{1/3},
\end{aligned}\tag{24}$$

$$\begin{aligned}
s_{2,n}(\xi) &= Z\left(-\frac{\beta_{1,n}(\xi)}{2}+\sqrt{\frac{(\beta_{1,n}(\xi))^2}{4}+\frac{(\alpha_{1,n}(\xi))^3}{27}}\right)^{1/3}\\
&\quad+Z^2\left(-\frac{\beta_{1,n}(\xi)}{2}-\sqrt{\frac{(\beta_{1,n}(\xi))^2}{4}+\frac{(\alpha_{1,n}(\xi))^3}{27}}\right)^{1/3},
\end{aligned}\tag{25}$$

$$\begin{aligned}
s_{3,n}(\xi) &= Z^2\left(-\frac{\beta_{1,n}(\xi)}{2}+\sqrt{\frac{(\beta_{1,n}(\xi))^2}{4}+\frac{(\alpha_{1,n}(\xi))^3}{27}}\right)^{1/3}\\
&\quad+Z\left(-\frac{\beta_{1,n}(\xi)}{2}-\sqrt{\frac{(\beta_{1,n}(\xi))^2}{4}+\frac{(\alpha_{1,n}(\xi))^3}{27}}\right)^{1/3},
\end{aligned}\tag{26}$$

where

$$\alpha_{1,n}(\xi) = \frac{1+\nu\lambda_3(\xi^2+\lambda_n^2)+\lambda_1\Omega+\lambda_3\epsilon}{\lambda_2}-\frac{(\lambda_1+\nu\lambda_4(\xi^2+\lambda_n^2)+\lambda_2\Omega+\lambda_4\epsilon)^2}{3\lambda_2^2},\tag{27}$$

$$\begin{aligned}
\beta_{1,n}(\xi) &= \frac{\nu(\xi^2+\lambda_n^2)+\Omega+\epsilon}{\lambda_2}+2\frac{(1+\nu\lambda_3(\xi^2+\lambda_n^2)+\lambda_1\Omega+\lambda_3\epsilon)^3}{27\lambda_2^3}\\
&\quad-\frac{(\lambda_1+\nu\lambda_4(\xi^2+\lambda_n^2)+\lambda_2\Omega+\lambda_4\epsilon)(1+\nu\lambda_3(\xi^2+\lambda_n^2)+\lambda_1\Omega+\lambda_3\epsilon)}{3\lambda_2^2},
\end{aligned}\tag{28}$$

and

$$Z=\frac{-1+i\sqrt{3}}{2}.\tag{29}$$

To solve Eq. (22), let us take

$$H_n(q)=\frac{\frac{\pi}{T}}{(q^2+(\frac{\pi}{T})^2)(q-q_{1,n}(\xi))}+2\sum_{p=1}^{\infty}(-1)^p\frac{\frac{\pi}{T}}{(q^2+(\frac{\pi}{T})^2)(q-q_{1,n}(\xi))}\exp(-pTq),\tag{30}$$

and

$$G_n(q) = \frac{\frac{\pi}{T}}{(q^2 + (\frac{\pi}{T})^2)(q - q_{1,n}(\xi))}, \tag{31}$$

we can prove that

$$\begin{aligned} g_n(t) = L^{-1}\big(G_n(q)\big) &= \pi T \frac{\exp(q_{1,n}(\xi)t)}{\pi^2 + T^2 q_{1,n}^2(\xi)} \\ &\quad - \frac{1}{\pi^2 + T^2 q_{1,n}^2(\xi)} \left(T^2 q_{1,n}(\xi) \sin\left(\frac{\pi t}{T}\right) + \pi T \cos\left(\frac{\pi t}{T}\right)\right). \end{aligned} \tag{32}$$

The inverse Laplace transform of Eq. (30), using Eq. (32), is given by

$$\begin{aligned} H_n(\xi, t) &= \pi T \frac{\exp(q_{1,n}(\xi)t)}{\pi^2 + T^2 q_1^2(\xi)} H(t) \\ &\quad - \frac{1}{\pi^2 + T^2 q_{1,n}^2(\xi)} \left(T^2 q_1(\xi) \sin\left(\frac{\pi t}{T}\right) + \pi T \cos\left(\frac{\pi t}{T}\right)\right) H(t) + 2 \sum_{p=1}^{\infty} (-1)^p \\ &\quad \times \left[\pi T \frac{\exp(q_{1,n}(\xi)(t - pT))}{\pi^2 + T^2 q_{1,n}^2(\xi)} - \frac{1}{\pi^2 + T^2 q_{1,n}^2(\xi)} \left(T^2 q_{1,n}(\xi) \sin\left(\frac{\pi (t - pT)}{T}\right) \right.\right. \\ &\quad \left.\left. + \pi T \cos\left(\frac{\pi (t - pT)}{T}\right)\right)\right] H_{pT}(t). \end{aligned} \tag{33}$$

Inversion of Eq. (22) by means of the Laplace formula and Fourier cosine and sine transforms and using Eq. (33) result in

$$\begin{aligned} w(x, y, t) &= \frac{2}{d} \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi}} \sin(\lambda_n y) \int_0^{\infty} B_n(\xi) \cos(\xi x) \left[\pi T \left\{ \frac{\exp[-q_{4,n}(\xi)t]}{\pi^2 + T^2 q_{4,n}^2(\xi)} \right.\right. \\ &\quad \left. \times \left(H(t) + 2 \sum_{p=1}^{\infty} (-1)^p \exp\big(-q_{4,n}(\xi)(t - pT)\big) H_{pT}(t) \right) \right\} \\ &\quad \left. + \frac{T}{\pi^2 + T^2 q_{4,n}^2(\xi)} \big(T q_{4,n}(\xi) F_1(t) - \pi F_2(t) \big) \right] d\xi \\ &\quad - \frac{2}{d} \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi}} \sin(\lambda_n y) \int_0^{\infty} A_n(\xi) \cos(\xi x) \left[\pi T \left\{ \psi_{1,n}(\xi) \frac{\exp[q_{1,n}(\xi)t]}{\pi^2 + T^2 q_{1,n}^2(\xi)} \right.\right. \\ &\quad \times \left(H(t) + 2 \sum_{p=1}^{\infty} (-1)^p \exp\big((t - pT) q_{1,n}(\xi)\big) H_{pT}(t) \right) \\ &\quad + \psi_{2,n}(\xi) \frac{\exp[q_{2,n}(\xi)t]}{\pi^2 + T^2 q_{2,n}^2(\xi)} \left(H(t) + 2 \sum_{p=1}^{\infty} (-1)^p \exp\big((t - pT) q_{2,n}(\xi)\big) H_{pT}(t) \right) \\ &\quad + \psi_{3,n}(\xi) \frac{\exp[q_{3,n}(\xi)t]}{\pi^2 + T^2 q_{3,n}^2(\xi)} \left(H(t) + 2 \sum_{p=1}^{\infty} (-1)^p \exp\big((t - pT) q_{3,n}(\xi)\big) H_{pT}(t) \right) \\ &\quad - \psi_{4,n}(\xi) \frac{\exp[-q_{4,n}(\xi)t]}{\pi^2 + T^2 q_{4,n}^2(\xi)} \end{aligned}$$

$$
\begin{aligned}
&\times \left(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\bigl(-q_{4,n}(\xi)(t-pT)\bigr)H_{pT}(t) \right) \Biggr\} \\
&- F_1(t)T^2 \left(\frac{q_{1,n}(\xi)\psi_{1,n}(\xi)}{\pi^2 + T^2 q_{1,n}^2(\xi)} + \frac{q_{2,n}(\xi)\psi_{2,n}(\xi)}{\pi^2 + T^2 q_{2,n}^2(\xi)} \right. \\
&\left. + \frac{q_{3,n}(\xi)\psi_{3,n}(\xi)}{\pi^2 + T^2 q_{3,n}^2(\xi)} - \frac{q_{4,n}(\xi)\psi_{4,n}(\xi)}{\pi^2 + T^2 q_{4,n}^2(\xi)} \right) \\
&- F_2(t)\pi T \left(\frac{\psi_{1,n}(\xi)}{\pi^2 + T^2 q_{1,n}^2(\xi)} + \frac{\psi_{2,n}(\xi)}{\pi^2 + T^2 q_{2,n}^2(\xi)} \right. \\
&\left. + \frac{\psi_{3,n}(\xi)}{\pi^2 + T^2 q_{3,n}^2(\xi)} + \frac{\psi_{4,n}(\xi)}{\pi^2 + T^2 q_{4,n}^2(\xi)} \right) \Biggr] \, d\xi,
\end{aligned}
\tag{34}
$$

where

$$
\begin{aligned}
\psi_{1,n}(\xi) &= \frac{\phi_{1,n}(\xi)}{(q_{1,n}(\xi) - q_{2,n}(\xi))(q_{1,n}(\xi) - q_{3,n}(\xi))(q_{1,n}(\xi) + q_{4,n}(\xi))}, \\
\psi_{2,n}(\xi) &= \frac{\phi_{2,n}(\xi)}{(q_{2,n}(\xi) - q_{1,n}(\xi))(q_{2,n}(\xi) - q_{3,n}(\xi))(q_{2,n}(\xi) + q_{4,n}(\xi))}, \\
\psi_{3,n}(\xi) &= \frac{\phi_{3,n}(\xi)}{(q_{3,n}(\xi) - q_{1,n}(\xi))(q_{3,n}(\xi) - q_{2,n}(\xi))(q_{3,n}(\xi) + q_{4,n}(\xi))}, \\
\psi_{4,n}(\xi) &= \frac{\phi_{4,n}(\xi)}{(q_{1,n}(\xi) + q_{4,n}(\xi))(q_{2,n}(\xi) + q_{4,n}(\xi))(q_{3,n}(\xi) + q_{4,n}(\xi))},
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
F_1(t) &= \sin\left(\frac{\pi t}{T}\right)H(t) + 2\sum_{p=1}^{\infty}(-1)^p \sin\left(\frac{\pi}{T}(t-pT)\right)H_{pT}(t), \\
B_n(\xi) &= \sqrt{\frac{2}{\pi}}\frac{U}{\rho}\frac{((-1)^n - 1)}{\lambda_n},
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
F_2(t) &= \cos\left(\frac{\pi t}{T}\right)H(t) + 2\sum_{p=1}^{\infty}(-1)^p \cos\left(\frac{\pi}{T}(t-pT)\right)H_{pT}(t), \\
A_n(\xi) &= \sqrt{\frac{2}{\pi}}\frac{U}{\rho}\frac{((-1)^n - 1)}{\lambda_n \lambda_2}.
\end{aligned}
\tag{37}
$$

The first part of Eq. (34) gives the corresponding solution for a Newtonian fluid, while the second part gives the corresponding non-Newtonian contribution.

The transient part of velocity for $\operatorname{Re}(q_{1,n}(\xi)), \operatorname{Re}(q_{2,n}(\xi)), \operatorname{Re}(q_{3,n}(\xi)) < 0, \operatorname{Re}(q_{4,n}(\xi)) > 0$ is

$$
\begin{aligned}
&w_t(x,y,t) \\
&\quad = \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty} B_n(\xi)\cos(\xi x) \\
&\qquad \times \left[\frac{\pi T}{\pi^2 + T^2 q_{4,n}^2(\xi)} \left(H(t) + 2\sum_{p=1}^{\infty}(-1)^p H_{pT}(t) \right) \right] \\
&\qquad - \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty} A_n(\xi)\cos(\xi x)
\end{aligned}
$$

$$\begin{aligned}
&\times \Biggl[\pi T\biggl\{\frac{\psi_{1,n}(\xi)}{\pi^2+T^2q_{1,n}^2(\xi)}+\frac{\psi_{2,n}(\xi)}{\pi^2+T^2q_{2,n}^2(\xi)} \\
&+\frac{\psi_{3,n}(\xi)}{\pi^2+T^2q_{3,n}^2(\xi)}-\frac{\psi_{4,n}(\xi)}{\pi^2+T^2q_{4,n}^2(\xi)}\biggr\}\Biggl(H(t)+2\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\Biggr)\Biggr]\,d\xi, \qquad (38)
\end{aligned}$$

while the steady state part is given by

$$\begin{aligned}
w_s(x,y,t) &= \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty}B_n(\xi)\cos(\xi x) \\
&\quad\times\biggl[\frac{T}{\pi^2+T^2q_{4,n}^2(\xi)}\bigl(Tq_{4,n}(\xi)F_1(t)-\pi F_2(t)\bigr)\biggr]\,d\xi \\
&\quad+\frac{2T}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty}A_n(\xi)\cos(\xi x) \\
&\quad\times\biggl[\frac{\psi_{1,n}(\xi)}{\pi^2+T^2q_{1,n}^2(\xi)}\bigl(TF_1(t)q_{1,n}(\xi)+\pi F_2(t)\bigr) \\
&\quad+\frac{\psi_{2,n}(\xi)}{\pi^2+T^2q_{2,n}^2(\xi)}\bigl(TF_1(t)q_2(\xi)+\pi F_2(t)\bigr) \\
&\quad+\frac{\psi_{3,n}(\xi)}{\pi^2+T^2q_{3,n}^2(\xi)}\bigl(TF_1(t)q_{3,n}(\xi)+\pi F_2(t)\bigr) \\
&\quad-\frac{\psi_{4,n}(\xi)}{\pi^2+T^2q_{4,n}^2(\xi)}\bigl(TF_1(t)q_{4,n}(\xi)-\pi F_2(t)\bigr)\biggr]\,d\xi. \qquad (39)
\end{aligned}$$

4 Calculation of tangential stresses

To obtain the expressions for the shear stresses $\tau_1(x,y,t)$ and $\tau_2(x,y,t)$, applying the Laplace transform to Eqs. (5) and (6), we have the expressions

$$\begin{aligned}
\bar{\tau}_1(x,y,q) &= \mu\frac{(1+\lambda_3 q+\lambda_4 q^2)}{(1+\lambda_1 q+\lambda_2 q^2)}\frac{\partial\bar{w}(x,y,q)}{\partial x}, \\
\bar{\tau}_2(x,y,q) &= \mu\frac{(1+\lambda_3 q+\lambda_4 q^2)}{(1+\lambda_1 q+\lambda_2 q^2)}\frac{\partial\bar{w}(x,y,q)}{\partial y}.
\end{aligned} \qquad (40)$$

From Eq. (20), with inverse Fourier cosine and sine transforms, we have

$$\begin{aligned}
\bar{w}(x,y,q) &= \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}A_n(\xi)\sin(\lambda_n y)\int_0^{\infty}\cos\xi x\frac{1}{(q-q_{1,n}(\xi))(q-q_{2,n}(\xi))(q-q_{3,n}(\xi))} \\
&\quad\times\frac{\frac{\pi}{T}}{(q^2+(\frac{\pi}{T})^2)}\Biggl(1+2\sum_{p=1}^{\infty}(-1)^p\exp(-pTq)\Biggr)\,d\xi. \qquad (41)
\end{aligned}$$

Using Eq. (41) in the set of Eq. (40), we have

$$\begin{aligned}
\bar{\tau}_1(x,y,q) &= -\frac{2\mu}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}A_n(\xi)\sin(\lambda_n y) \\
&\quad\times\int_0^{\infty}\frac{\xi\sin\xi x(1+\lambda_3 q+\lambda_4 q^2)}{(1+\lambda_1 q+\lambda_2 q^2)(q-q_{1,n}(\xi))(q-q_{2,n}(\xi))(q-q_{3,n}(\xi))}
\end{aligned}$$

$$\times \frac{\frac{\pi}{T}}{(q^2 + (\frac{\pi}{T})^2)} \left(1 + 2\sum_{p=1}^{\infty} (-1)^p \exp(-pTq) \right) d\xi, \tag{42}$$

$$\begin{aligned} \bar{\tau}_2(x,y,q) = {}& \frac{2\mu}{d} \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi}} \lambda_n A_n(\xi) \cos(\lambda_n y) \\ & \times \int_0^{\infty} \frac{\cos \xi x (1 + \lambda_3 q + \lambda_4 q^2)}{(1 + \lambda_3 q + \lambda_4 q^2)(q - q_{1,n}(\xi))(q - q_{2,n}(\xi))(q - q_{3,n}(\xi))} \\ & \times \frac{\frac{\pi}{T}}{(q^2 + (\frac{\pi}{T})^2)} \left(1 + 2\sum_{p=1}^{\infty} (-1)^p \exp(-pTq) \right) d\xi. \end{aligned} \tag{43}$$

Let us take

$$\bar{A}(q) = \frac{1 + \lambda_3 q + \lambda_4 q^2}{1 + \lambda_1 q + \lambda_2 q^2}, \tag{44}$$

which can also be written in the form

$$\bar{A}(q) = a_2 + a_3 \frac{q + a_1}{(q + a_1)^2 - b_1^2} + a_4 \frac{b_1}{(q + a_1)^2 - b_1^2}, \tag{45}$$

where

$$\begin{aligned} & a_1 = \frac{\lambda_1}{2\lambda_2}, \qquad a_2 = \frac{\lambda_4}{\lambda_2}, \qquad a_3 = \frac{\lambda_2\lambda_3 - \lambda_1\lambda_4}{\lambda_2^2}, \\ & a_4 = \frac{2\lambda_2(\lambda_2 - \lambda_4) - \lambda_1(\lambda_2\lambda_3 - \lambda_1\lambda_4)}{\lambda_2^2\sqrt{\lambda_1^2 - 4\lambda_2}}, \qquad b_1 = \frac{\sqrt{\lambda_1^2 - 4\lambda_2}}{2\lambda_2}, \end{aligned} \tag{46}$$

where $\lambda_1^2 - 4\lambda_2 > 0$.

Applying the inverse Laplace transform to Eq. (45), we obtain

$$A(t) = a_2 + a_3 \cosh(b_1 t) \exp(-a_1 t) + a_4 \sinh(b_1 t) \exp(-a_1 t). \tag{47}$$

Let

$$\tau(M,N,t) = (A * B)(t) = \int_0^t A(t - q) B(M,N,q)\, dq. \tag{48}$$

Employing the methodology as for the velocity field, the inverse Laplace transform of Eqs. (42) and (43) results in

$$\begin{aligned} \tau_1(x,y,t) = {}& -\frac{2\mu}{d} \sum_{n=1}^{\infty} \sqrt{\frac{2}{\pi}} A_n(\xi) \sin(\lambda_n y) \int_0^{\infty} \xi \sin \xi x \int_0^t \big(a_2 + \big(a_3 \cosh\big(b_1(t-q)\big) \\ & + a_4 \sinh\big(b_1(t-q)\big)\big) \exp\big(-a_1(t-q)\big)\big) \\ & \times \Bigg[\pi T \Bigg\{ \frac{\eta_{1,n}(\xi) \exp[q_{1,n}(\xi) q]}{\pi^2 + T^2 q_{1,n}^2(\xi)} \left(H(q) + 2\sum_{p=1}^{\infty} (-1)^p \exp\big(-pTq_{1,n}(\xi)\big) H_{pT}(q) \right) \\ & + \frac{\eta_{2,n}(\xi) \exp[q_{2,n}(\xi) q]}{\pi^2 + T^2 q_{2,n}^2(\xi)} \end{aligned}$$

$$
\begin{aligned}
&\times\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{2,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{3,n}(\xi)\exp[q_{3,n}(\xi)q]}{\pi^2+T^2q_{3,n}^2(\xi)}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{3,n}(\xi)\bigr)H_{pT}(q)\right)\Biggr\}\\
&-F_1(q)T^2\left(\frac{q_{1,n}(\xi)\eta_{1,n}(\xi)}{\pi^2+T^2q_{1,n}^2(\xi)}+\frac{q_{2,n}(\xi)\eta_{2,n}(\xi)}{\pi^2+T^2q_{2,n}^2(\xi)}+\frac{q_3(\xi)\eta_{3,n}(\xi)}{\pi^2+T^2q_3^2(\xi)}\right)\\
&-F_2(q)\pi T\left(\frac{\eta_{1,n}(\xi)}{\pi^2+T^2q_{1,n}^2(\xi)}+\frac{\eta_{2,n}(\xi)}{\pi^2+T^2q_{2,n}^2(\xi)}\right.\\
&\left.+\frac{\eta_{3,n}(\xi)}{\pi^2+T^2q_{3,n}^2(\xi)}\right)\Biggr]\,dq\,d\xi,
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
\tau_2(x,y,t)={}&\frac{2\mu}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\lambda_nA_n(\xi)\cos(\lambda_ny)\int_0^{\infty}\cos\xi x\int_0^t\Bigl(a_2+\bigl(a_3\cosh\bigl(b_1(t-q)\bigr)\\
&+a_4\sinh\bigl(b_1(t-q)\bigr)\bigr)\exp\bigl(-a_1(t-q)\bigr)\Bigr)\\
&\times\Biggl[\pi T\Biggl\{\frac{\eta_{1,n}(\xi)\exp[q_{1,n}(\xi)q]}{\pi^2+T^2q_{1,n}^2(\xi)}\\
&\times\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{1,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{2,n}(\xi)\exp[q_{2,n}(\xi)q]}{\pi^2+T^2q_{2,n}^2(\xi)}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{2,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{3,n}(\xi)\exp[q_{3,n}(\xi)q]}{\pi^2+T^2q_{3,n}^2(\xi)}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{3,n}(\xi)\bigr)H_{pT}(q)\right)\Biggr\}\\
&-F_1(q)T^2\left(\frac{q_{1,n}(\xi)\eta_{1,n}(\xi)}{\pi^2+T^2q_{1,n}^2(\xi)}+\frac{q_{2,n}(\xi)\eta_{2,n}(\xi)}{\pi^2+T^2q_{2,n}^2(\xi)}+\frac{q_3(\xi)\eta_{3,n}(\xi)}{\pi^2+T^2q_3^2(\xi)}\right)\\
&-F_2(q)\pi T\left(\frac{\eta_{1,n}(\xi)}{\pi^2+T^2q_{1,n}^2(\xi)}+\frac{\eta_{2,n}(\xi)}{\pi^2+T^2q_{2,n}^2(\xi)}\right.\\
&\left.+\frac{\eta_{3,n}(\xi)}{\pi^2+T^2q_{3,n}^2(\xi)}\right)\Biggr]\,dq\,d\xi,
\end{aligned}
\tag{50}
$$

where

$$
\begin{aligned}
\eta_{1,n}(\xi)&=\frac{1}{(q_{1,n}(\xi)-q_{2,n}(\xi))(q_{1,n}(\xi)-q_{3,n}(\xi))},\\
\eta_{2,n}(\xi)&=\frac{1}{(q_{2,n}(\xi)-q_{1,n}(\xi))(q_{2,n}(\xi)-q_{3,n}(\xi))},\\
\eta_{3,n}(\xi)&=\frac{1}{(q_{3,n}(\xi)-q_{1,n}(\xi))(q_{3,n}(\xi)-q_{2,n}(\xi))},
\end{aligned}
$$

the shear stresses for the generalized Burgers fluid.

5 Limiting cases

5.1 Burgers fluid

Letting $\lambda_4 = 0$, we obtain the velocity field and the associated shear stresses corresponding to a Burgers fluid performing the same motion.

5.2 Oldroyd-B fluid

Letting $\lambda_2 = \lambda_4 = 0$ into Eq. (20), and following the same way as before, the velocity and tangential stresses expressions for Oldroyd-B fluid take the form

$$
\begin{aligned}
w_O(x,y,t) = {}& \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty} B_n(\xi)\cos(\xi x)\Bigg[\pi T\Bigg\{\frac{\exp[-q_{4,n}(\xi)t]}{\pi^2 + T^2 q_{4,n}^2(\xi)} \\
& \times\Bigg(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\big(-q_{4,n}(\xi)(t-pT)\big)H_{pT}(t)\Bigg)\Bigg\} \\
& + \frac{T}{\pi^2 + T^2 q_{4,n}^2(\xi)}\big(Tq_{4,n}(\xi)F_1(t) - \pi F_2(t)\big)\Bigg]\,d\xi \\
& - \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty} B_n(\xi)\cos(\xi x) \\
& \times\Bigg[\pi T\Bigg\{\psi_{5,n}(\xi)\frac{\exp[q_{5,n}(\xi)t]}{\pi^2 + T^2 q_{5,n}^2(\xi)} \\
& \times\Bigg(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\big((t-pT)q_{5,n}(\xi)\big)H_{pT}(t)\Bigg) \\
& + \psi_{6,n}(\xi)\frac{\exp[q_{6,n}(\xi)t]}{\pi^2 + T^2 q_{6,n}^2(\xi)}\Bigg(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\big((t-pT)q_{6,n}(\xi)\big)H_{pT}(t)\Bigg) \\
& - \psi_{7,n}(\xi)\frac{\exp[-q_{4,n}(\xi)t]}{\pi^2 + T^2 q_{4,n}^2(\xi)} \\
& \times\Bigg(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\big(-q_{4,n}(\xi)(t-pT)\big)H_{pT}(t)\Bigg)\Bigg\} \\
& - F_1(t)T^2\Bigg(\frac{q_{5,n}(\xi)\psi_{5,n}(\xi)}{\pi^2 + T^2 q_{5,n}^2(\xi)} + \frac{q_{6,n}(\xi)\psi_{6,n}(\xi)}{\pi^2 + T^2 q_{6,n}^2(\xi)} - \frac{q_{4,n}(\xi)\psi_{7,n}(\xi)}{\pi^2 + T^2 q_{4,n}^2(\xi)}\Bigg) \\
& - F_2(t)\pi T\Bigg(\frac{\psi_{5,n}(\xi)}{\pi^2 + T^2 q_{5,n}^2(\xi)} \\
& + \frac{\psi_{6,n}(\xi)}{\pi^2 + T^2 q_{6,n}^2(\xi)} + \frac{\psi_{7,n}(\xi)}{\pi^2 + T^2 q_{4,n}^2(\xi)}\Bigg)\Bigg]\,d\xi,
\end{aligned}
\tag{51}
$$

$$
\begin{aligned}
\tau_{1O}(x,y,t) = {}& -\frac{2\mu}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}A_n(\xi)\sin(\lambda_n y)\int_0^{\infty}\xi\sin\xi x \\
& \times\Bigg[\pi T\Bigg\{\frac{\eta_{5,n}(\xi)\exp[q_{5,n}(\xi)q]}{\pi^2 + T^2 q_{5,n}^2(\xi)} \\
& \times\Bigg(H(q) + 2\sum_{p=1}^{\infty}(-1)^p \exp\big(-pTq_{5,n}(\xi)\big)H_{pT}(q)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\eta_{6,n}(\xi)\exp[q_{6,n}(\xi)q]}{\pi^2+T^2q_{6,n}^2(\xi)}\\
&\times\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{6,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{4,n}(\xi)\exp[-\frac{q}{\lambda_1}]}{\pi^2+(\frac{T}{\lambda_1})^2}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\left(\frac{pT}{\lambda_1}\right)H_{pT}(q)\right)\Biggr\}\\
&-F_1(q)T^2\left(\frac{q_{5,n}(\xi)\eta_{5,n}(\xi)}{\pi^2+T^2q_{5,n}^2(\xi)}+\frac{q_{6,n}(\xi)\eta_{6,n}(\xi)}{\pi^2+T^2q_{6,n}^2(\xi)}-\frac{1}{\lambda_1}\frac{\eta_{4,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\\
&-F_2(q)\pi T\left(\frac{\eta_{5,n}(\xi)}{\pi^2+T^2q_{5,n}^2(\xi)}+\frac{\eta_{6,n}(\xi)}{\pi^2+T^2q_{6,n}^2(\xi)}+\frac{\eta_{4,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\Biggr]\,d\xi,
\end{aligned}
\tag{52}
$$

$$
\begin{aligned}
\tau_{2O}(x,y,t)={}&\frac{2\mu}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\lambda_nA_n(\xi)\cos(\lambda_ny)\int_0^{\infty}\cos\xi x\\
&\times\Biggl[\pi T\Biggl\{\frac{\eta_{5,n}(\xi)\exp[q_{5,n}(\xi)q]}{\pi^2+T^2q_{5,n}^2(\xi)}\\
&\times\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{5,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{6,n}(\xi)\exp[q_{6,n}(\xi)q]}{\pi^2+T^2q_{6,n}^2(\xi)}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{6,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{4,n}(\xi)\exp[-\frac{q}{\lambda_1}]}{\pi^2+(\frac{T}{\lambda_1})^2}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\left(\frac{pT}{\lambda_1}\right)H_{pT}(q)\right)\Biggr\}\\
&-F_1(q)T^2\left(\frac{q_{5,n}(\xi)\eta_{5,n}(\xi)}{\pi^2+T^2q_{5,n}^2(\xi)}+\frac{q_{6,n}(\xi)\eta_{6,n}(\xi)}{\pi^2+T^2q_{6,n}^2(\xi)}-\frac{1}{\lambda_1}\frac{\eta_{4,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\\
&-F_2(q)\pi T\left(\frac{\eta_{5,n}(\xi)}{\pi^2+T^2q_{5,n}^2(\xi)}+\frac{\eta_{6,n}(\xi)}{\pi^2+T^2q_{6,n}^2(\xi)}+\frac{\eta_{4,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\Biggr]\,d\xi,
\end{aligned}
\tag{53}
$$

where

$$
\begin{aligned}
\psi_{5,n}(\xi)&=\frac{\lambda_1q_{5,n}^2(\xi)+(\nu\lambda_3(\xi^2+\lambda_n^2)+\lambda_1\Omega+\lambda_3\epsilon)q_{5,n}(\xi)}{(q_{5,n}(\xi)-q_{6,n}(\xi))(q_{5,n}(\xi)+q_{4,n}(\xi))},\\
\psi_{6,n}(\xi)&=\frac{\lambda_1q_{6,n}^2(\xi)+(\nu\lambda_3(\xi^2+\lambda_n^2)+\lambda_1\Omega+\lambda_3\epsilon)q_{6,n}(\xi)}{(q_{6,n}(\xi)-q_{5,n}(\xi))(q_{6,n}(\xi)+q_{4,n}(\xi))},\\
\psi_{7,n}(\xi)&=\frac{\lambda_1q_{4,n}^2(\xi)-(\nu\lambda_3(\xi^2+\lambda_n^2)+\lambda_1\Omega+\lambda_3\epsilon)q_{4,n}(\xi)}{(q_{4,n}(\xi)+q_{5,n}(\xi))(q_{4,n}(\xi)+q_{6,n}(\xi))},
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
&q_{5,n}(\xi),q_{6,n}(\xi)\\
&\quad=\Bigl(-\bigl(1+\nu\lambda_3\bigl(\xi^2+\lambda_n^2\bigr)+\lambda_1\Omega+\lambda_3\epsilon\bigr)\\
&\qquad\pm\sqrt{\bigl(1+\nu\lambda_3\bigl(\xi^2+\lambda_n^2\bigr)+\lambda_1\Omega+\lambda_3\epsilon\bigr)^2-4\lambda_1\bigl(\nu\lambda_3\bigl(\xi^2+\lambda_n^2\bigr)+\lambda_1\Omega+\lambda_3\epsilon\bigr)}\Bigr)\Big/(2\lambda_1),
\end{aligned}
\tag{55}
$$

$$\eta_{4,n}(\xi) = \frac{\lambda_1 - \lambda_3}{(1 + \lambda_1 q_{5,n}(\xi))(1 + \lambda_1 q_{6,n}(\xi))},$$
$$\eta_{5,n}(\xi) = \frac{1 + \lambda_3 q_{5,n}(\xi)}{(1 + \lambda_1 q_{5,n}(\xi))(q_{5,n}(\xi) - q_{6,n}(\xi))}, \tag{56}$$
$$\eta_{6,n}(\xi) = \frac{1 + \lambda_3 q_{6,n}(\xi)}{(1 + \lambda_1 q_{6,n}(\xi))(q_{6,n}(\xi) - q_{5,n}(\xi))}. \tag{57}$$

5.3 Maxwell fluid

Letting $\lambda_3 = 0$ in the set of equations for Oldroyd-B fluid, we obtain the corresponding expressions for the Maxwell fluid:

$$\begin{aligned} w_M(x,y,t) &= \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty} B_n(\xi)\cos(\xi x)\Biggl[\pi T\Biggl\{\frac{\exp[-q_{4,n}(\xi)t]}{\pi^2 + T^2 q_{4,n}^2(\xi)} \\ &\quad \times \Biggl(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\bigl(-q_{4,n}(\xi)(t - pT)\bigr)H_{pT}(t)\Biggr)\Biggr\} \\ &\quad + \frac{T}{\pi^2 + T^2 q_{4,n}^2(\xi)}\bigl(Tq_{4,n}(\xi)F_1(t) - \pi F_2(t)\bigr)\Biggr] d\xi \\ &\quad - \frac{2}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\sin(\lambda_n y)\int_0^{\infty} B_n(\xi)\cos(\xi x)\Biggl[\pi T\Biggl\{\psi_{8,n}(\xi)\frac{\exp[q_{7,n}(\xi)t]}{\pi^2 + T^2 q_{7,n}^2(\xi)} \\ &\quad \times \Biggl(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\bigl((t - pT)q_{7,n}(\xi)\bigr)H_{pT}(t)\Biggr) \\ &\quad + \psi_{9,n}(\xi)\frac{\exp[q_{8,n}(\xi)t]}{\pi^2 + T^2 q_{8,n}^2(\xi)}\Biggl(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\bigl((t - pT)q_{8,n}(\xi)\bigr)H_{pT}(t)\Biggr) \\ &\quad - \psi_{10,n}(\xi)\frac{\exp[-q_{4,n}(\xi)t]}{\pi^2 + T^2 q_{4,n}^2(\xi)} \\ &\quad \times \Biggl(H(t) + 2\sum_{p=1}^{\infty}(-1)^p \exp\bigl(-q_{4,n}(\xi)(t - pT)\bigr)H_{pT}(t)\Biggr)\Biggr\} \\ &\quad - F_1(t)T^2\Biggl(\frac{q_{7,n}(\xi)\psi_{8,n}(\xi)}{\pi^2 + T^2 q_{7,n}^2(\xi)} + \frac{q_{8,n}(\xi)\psi_{9,n}(\xi)}{\pi^2 + T^2 q_{8,n}^2(\xi)} - \frac{q_{4,n}(\xi)\psi_{10,n}(\xi)}{\pi^2 + T^2 q_{4,n}^2(\xi)}\Biggr) \\ &\quad - F_2(t)\pi T\Biggl(\frac{\psi_{8,n}(\xi)}{\pi^2 + T^2 q_{7,n}^2(\xi)} + \frac{\psi_{9,n}(\xi)}{\pi^2 + T^2 q_{8,n}^2(\xi)} \\ &\quad + \frac{\psi_{10,n}(\xi)}{\pi^2 + T^2 q_{4,n}^2(\xi)}\Biggr)\Biggr] d\xi, \end{aligned} \tag{58}$$

$$\begin{aligned} \tau_{1M}(x,y,t) &= -\frac{2\mu}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}A_n(\xi)\sin(\lambda_n y)\int_0^{\infty}\xi\sin\xi x \\ &\quad \times \Biggl[\pi T\Biggl\{\frac{\eta_{8,n}(\xi)\exp[q_{7,n}(\xi)q]}{\pi^2 + T^2 q_{7,n}^2(\xi)} \\ &\quad \times \Biggl(H(q) + 2\sum_{p=1}^{\infty}(-1)^p \exp\bigl(-pTq_{7,n}(\xi)\bigr)H_{pT}(q)\Biggr) \end{aligned}$$

$$
\begin{aligned}
&+\frac{\eta_{9,n}(\xi)\exp[q_{8,n}(\xi)q]}{\pi^2+T^2q_{8,n}^2(\xi)}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{8,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{7,n}(\xi)\exp[-\frac{q}{\lambda_1}]}{\pi^2+(\frac{T}{\lambda_1})^2}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\left(\frac{pT}{\lambda_1}\right)H_{pT}(q)\right)\Biggr\}\\
&-F_1(q)T^2\left(\frac{q_{7,n}(\xi)\eta_{8,n}(\xi)}{\pi^2+T^2q_{7,n}^2(\xi)}+\frac{q_{8,n}(\xi)\eta_{9,n}(\xi)}{\pi^2+T^2q_{8,n}^2(\xi)}-\frac{1}{\lambda_1}\frac{\eta_{7,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\\
&-F_2(q)\pi T\left(\frac{\eta_{9,n}(\xi)}{\pi^2+T^2q_{7,n}^2(\xi)}+\frac{\eta_{8,n}(\xi)}{\pi^2+T^2q_{8,n}^2(\xi)}+\frac{\eta_{7,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\Biggr]\,d\xi, \qquad (59)
\end{aligned}
$$

$$
\begin{aligned}
\tau_{2M}(x,y,t)={}&\frac{2\mu}{d}\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\lambda_nA_n(\xi)\cos(\lambda_ny)\int_0^{\infty}\cos\xi x\\
&\times\Biggl[\pi T\Biggl\{\frac{\eta_{8,n}(\xi)\exp[q_{7,n}(\xi)q]}{\pi^2+T^2q_{7,n}^2(\xi)}\\
&\times\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{7,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{9,n}(\xi)\exp[q_{8,n}(\xi)q]}{\pi^2+T^2q_{8,n}^2(\xi)}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\bigl(-pTq_{8,n}(\xi)\bigr)H_{pT}(q)\right)\\
&+\frac{\eta_{7,n}(\xi)\exp[-\frac{q}{\lambda_1}]}{\pi^2+(\frac{T}{\lambda_1})^2}\left(H(q)+2\sum_{p=1}^{\infty}(-1)^p\exp\left(\frac{pT}{\lambda_1}\right)H_{pT}(q)\right)\Biggr\}\\
&-F_1(q)T^2\left(\frac{q_{7,n}(\xi)\eta_{8,n}(\xi)}{\pi^2+T^2q_{7,n}^2(\xi)}+\frac{q_{8,n}(\xi)\eta_{9,n}(\xi)}{\pi^2+T^2q_{8,n}^2(\xi)}-\frac{1}{\lambda_1}\frac{\eta_{7,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\\
&-F_2(q)\pi T\left(\frac{\eta_{9,n}(\xi)}{\pi^2+T^2q_{7,n}^2(\xi)}+\frac{\eta_{8,n}(\xi)}{\pi^2+T^2q_{8,n}^2(\xi)}+\frac{\eta_{7,n}(\xi)}{\pi^2+(\frac{T}{\lambda_1})^2}\right)\Biggr]\,d\xi, \qquad (60)
\end{aligned}
$$

where

$$
\begin{aligned}
\psi_{8,n}(\xi)&=\frac{\lambda_1q_{7,n}^2(\xi)+\lambda_1\Omega q_{7,n}(\xi)}{(q_{7,n}(\xi)-q_{8,n}(\xi))(q_{7,n}(\xi)+q_{4,n}(\xi))},\\
\psi_{9,n}(\xi)&=\frac{\lambda_1q_{8,n}^2(\xi)+\lambda_1\Omega q_{8,n}(\xi)}{(q_{8,n}(\xi)-q_{7,n}(\xi))(q_{8,n}(\xi)+q_{4,n}(\xi))}, \qquad (61)\\
\psi_{10,n}(\xi)&=\frac{\lambda_1q_{4,n}^2(\xi)-\lambda_1\Omega q_{4,n}(\xi)}{(q_{4,n}(\xi)+q_{7,n}(\xi))(q_{4,n}(\xi)+q_{8,n}(\xi))},
\end{aligned}
$$

$$
q_{7,n}(\xi),q_{8,n}(\xi)=\frac{-(1+\lambda_1\Omega)\pm\sqrt{(1+\lambda_1\Omega)^2-4\lambda_1^2\Omega}}{2\lambda_1}, \qquad (62)
$$

$$
\begin{aligned}
\eta_{7,n}(\xi)&=\frac{\lambda_1}{(1+\lambda_1q_{7,n}(\xi))(1+\lambda_1q_{8,n}(\xi))},\\
\eta_{8,n}(\xi)&=\frac{1}{(1+\lambda_1q_{7,n}(\xi))(q_{7,n}(\xi)-q_{8,n}(\xi))}, \qquad (63)
\end{aligned}
$$

$$
\eta_{9,n}(\xi)=\frac{1}{(1+\lambda_1q_{8,n}(\xi))(q_{8,n}(\xi)-q_{7,n}(\xi))}. \qquad (64)
$$

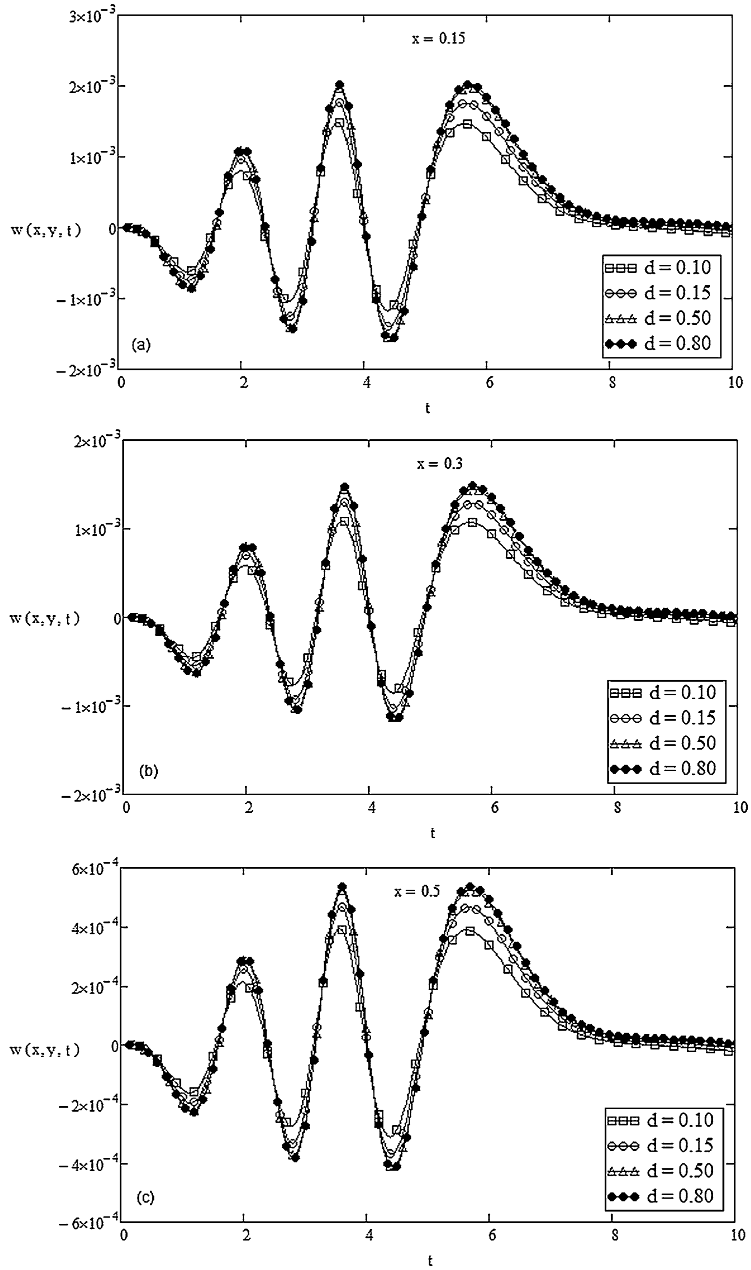

Figure 1 Velocity profile for generalized Burgers fluid for different values of *x* and *d*. Other parameters and values are taken as $\rho = 975$, $x = 0.01$, $y = \frac{d}{2}$, $\mu = 3.9$, $T = \frac{\pi}{4}$, $U = 15$, $\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_3 = 0.5$, $\lambda_4 = 2$, $\epsilon = 1.7$, and $\Omega = 3$.

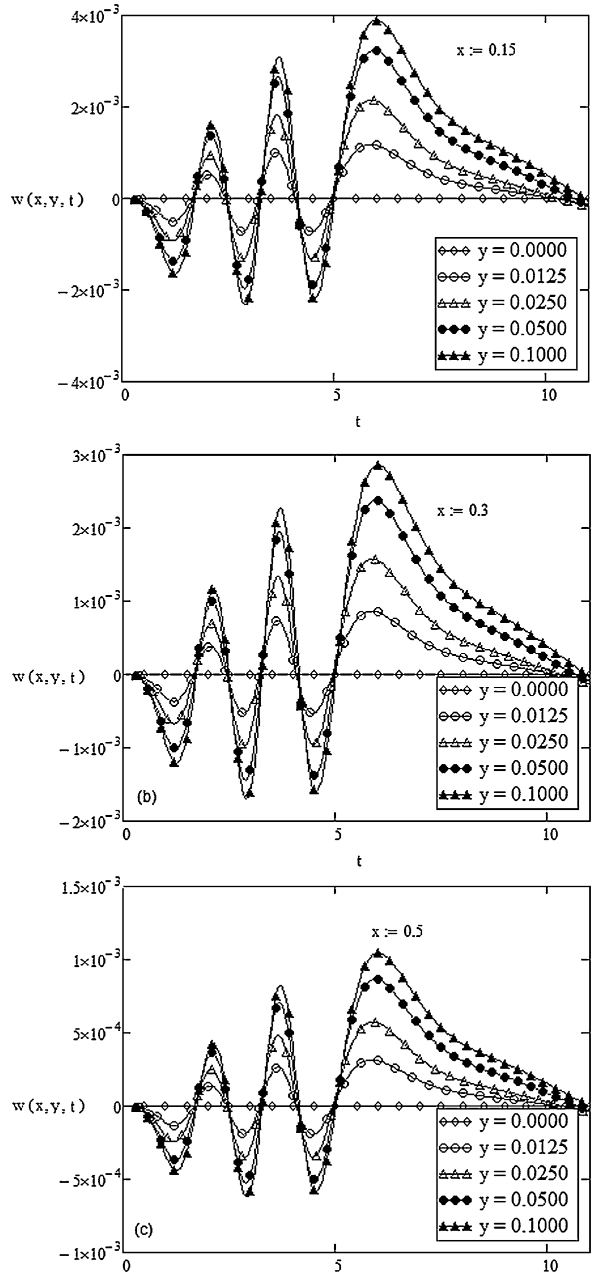

Figure 2 Velocity profile for a generalized Burgers fluid for different values of x and y. Other parameters and values are taken as $\rho = 975$, $d = 0.2$, $\mu = 3.9$, $T = \frac{\pi}{4}$, $U = 15$, $\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_3 = 0.5$, $\lambda_4 = 2$, $\epsilon = 0.3$, and $\Omega = 0.5$.

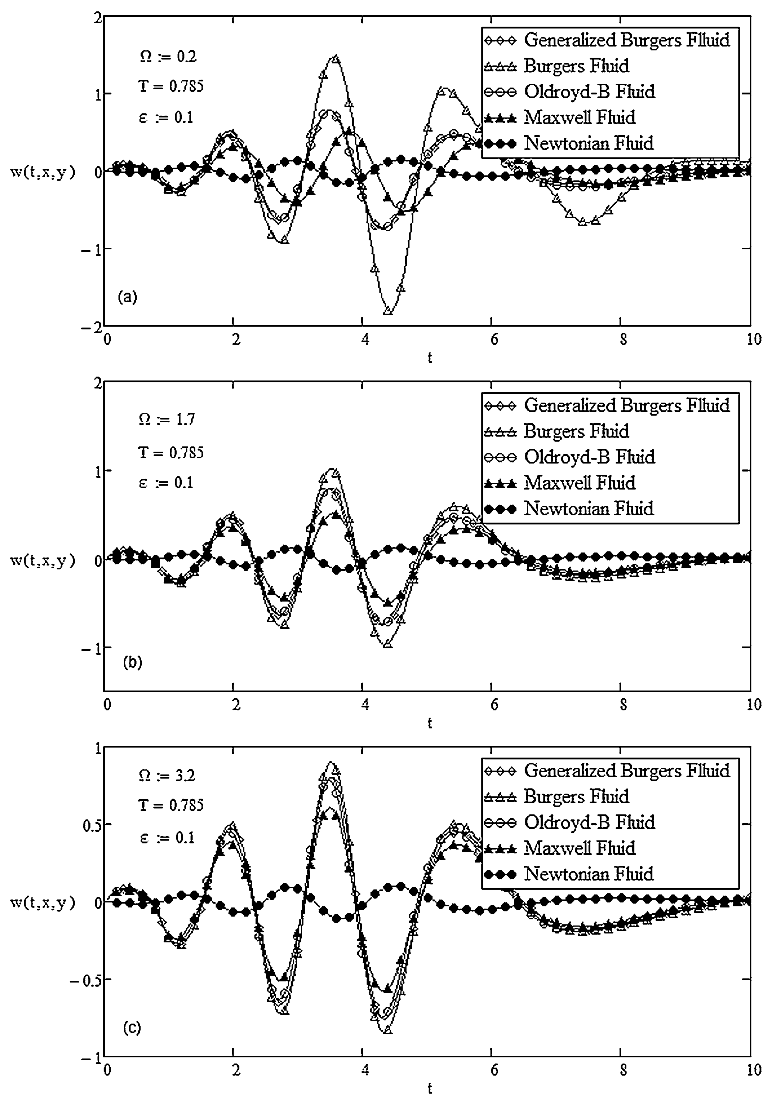

Figure 3 Diagrams of the velocity for different values of the parameter Ω. Comparison between five models.

6 Results and discussion

The present problem is concerned with unsteady motion of the generalized Burgers fluid generated from rest induced by rectified sine pluses shear stress. The Laplace transform technique and Fourier cosine and sine transforms have been used as mathematical tools in this order. The obtained expression for the velocity field has been written as the sum of Newtonian and non-Newtonian contributions.

By using the numerical calculations and graphical illustrations, the following physical aspects of the fluid behavior have been analyzed:

(a) Influence of side walls on the velocity field.

In order to study the influence of the side walls on the fluid velocity we prepared the diagrams contained in Figures 1 and 2. These diagrams present the velocity field $w(x, y, t)$ for three values of the distance x at the bottom plate. It is seen that if $d = 0.1$ (small distance between walls), the amplitude of oscillation of velocity in the middle of the channel is

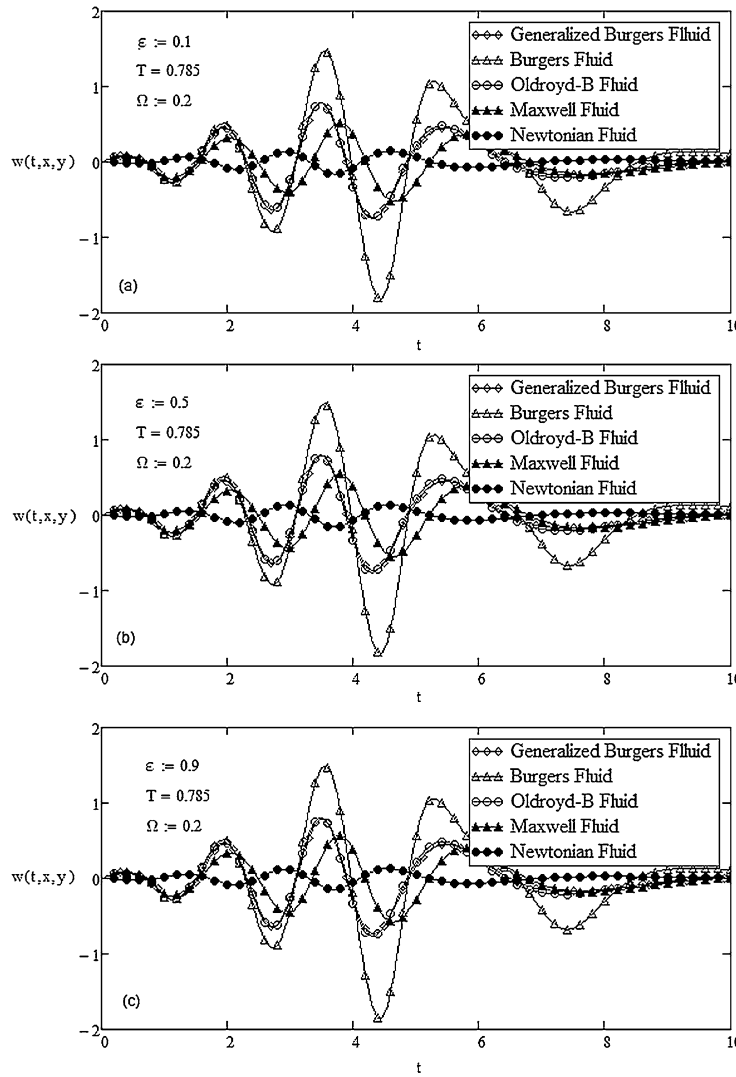

Figure 4 Diagrams of the velocity for different values of the parameter ϵ. Comparison between five models.

small. By increasing the value of d, the amplitude of oscillation of velocity increases. But it exists up to a value of 0.5 (in our case) after which the velocity remains the same (velocity profiles are almost identical) if d increases. Therefore, in the considered case, after $d = 0.5$ the influence of the side walls on the velocity is negligible. In Figure 2 have been sketched the velocity profiles for different positions in the channel, starting from the side walls till the middle of the channel, *i.e.* for different values of the variable y, for a fixed distance between the side walls. The amplitude of oscillations decreases far from the plate. It is also observed that, if the bottom plate is set into oscillation, the velocity increases with respect to the y-coordinate, from zero to a maximum in the middle of the channel.

(b) Comparative study of various models.

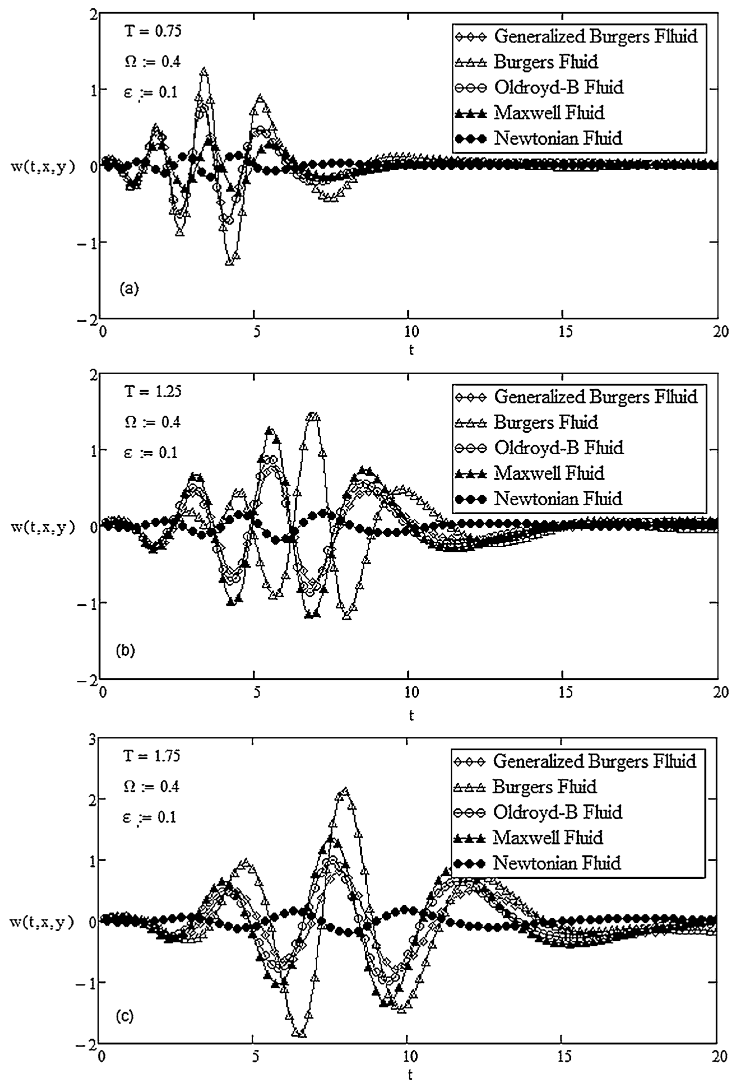

Figure 5 Diagrams of the velocity for different values of the parameter *T*. Comparison between five models.

Figures 3, 4, and 5 are sketched in order to compare the various fluid types. Also, the influence of the magnetic field, permeability and the period of oscillation of sawtooth pulses on the velocity field can be observed from these diagrams. In these figures, we have considered the following values of the parameters: $U = 15$, $\lambda_1 = 2.9$, $\lambda_2 = 4$, $\lambda_3 = 0.35$, $\lambda_4 = 1.5$, $x = 0.05$, $d = 0.1$, $y = d/2$, $\nu = 0.004$, $\mu = 3.9$.

Figure 3 shows the diagrams of velocity $w(x,y,t)$, *versus* t, for the porosity parameter $\epsilon = 0.1$, the pulse period $T = \frac{\pi}{4}$, and for three values of the magnetic parameter, $\Omega = 0.2, 1.7, 3.2$. It is observed that there is a time interval in which the velocity has an oscillating behavior for all kinds of fluids. After this moment, the velocity tends to a common value (the differences between velocities of different fluids are insignificant). For low values of the magnetic field strength, the amplitudes of the velocity oscillations are smaller for the generalized Burgers, Oldroyd-B, and Newtonian fluids, and much larger for Maxwell and Burgers fluids (see diagrams for $\Omega = 0.2$). If the magnetic field is stronger, the ve-

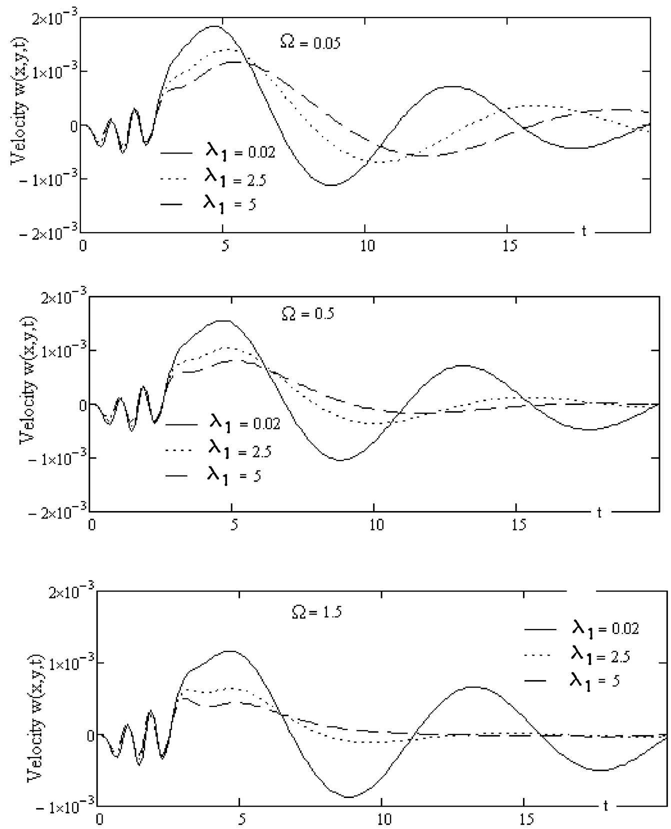

Figure 6 The influence of the parameter λ_1 on the velocity field for x = 0.25, d = 0.50, y = 0.25 and $\lambda_2 = 2$, $\lambda_3 = 0.5$, $\lambda_4 = 2$.

locity amplitudes of Maxwell and Burgers fluids decrease while, the velocity amplitudes of generalized Burgers and Newtonian fluids increase (case $\Omega = 3.2$). It is important to note that velocities of the fluids tend to a common value in shorter time if the magnetic field is stronger. Figure 4 is plotted for $\Omega = 0.2$, $T = \frac{\pi}{4}$ and for three values of the porosity parameter $\epsilon = 0.1, 0.5, 0.9$. In this case, amplitudes of the velocity oscillations of the generalized Burgers, Oldroyd-B, and Newtonian fluids are lower than those corresponding to the Burgers and Maxwell fluids.

Increasing permeability leads to increase the velocity amplitudes. The effect of the pulse period T on the velocity field is shown in Figure 5 for $\Omega = 0.4$, $\epsilon = 0.1$ and three values of the parameter $T = 0.75, 1.25, 1.75$. The Burgers fluid oscillates with larger amplitude, while other fluids have oscillations with amplitude close as order of magnitude. For low values of the parameter T, velocities tend to a common value in a shorter time than in the case of large values of the parameter T.

(c) Influence of parameters λ_1, λ_2, λ_3, λ_4 on the velocity field.

In order to study the behavior of the fluid for various values of the material parameters λ_1, λ_2, λ_3, λ_4, the diagrams of the velocity field from Figures 6, 7, 8 and 9 were plotted. In these figures we used the numerical values $U = 25$, $x = 0.25$, $d = 0.5$, $y = \frac{\pi}{2}$, $T = \frac{\pi}{8}$, $\rho = 830$, $\mu = 3.78$, $\epsilon = 2$, and three values for the magnetic parameter, namely $\Omega = 0.05, 0.5, 1.5$. Therefore, the influence of the magnetic parameter on the velocity field is also analyzed and, similar conclusions to those from the case b) were obtained. In Figure 6 the param-

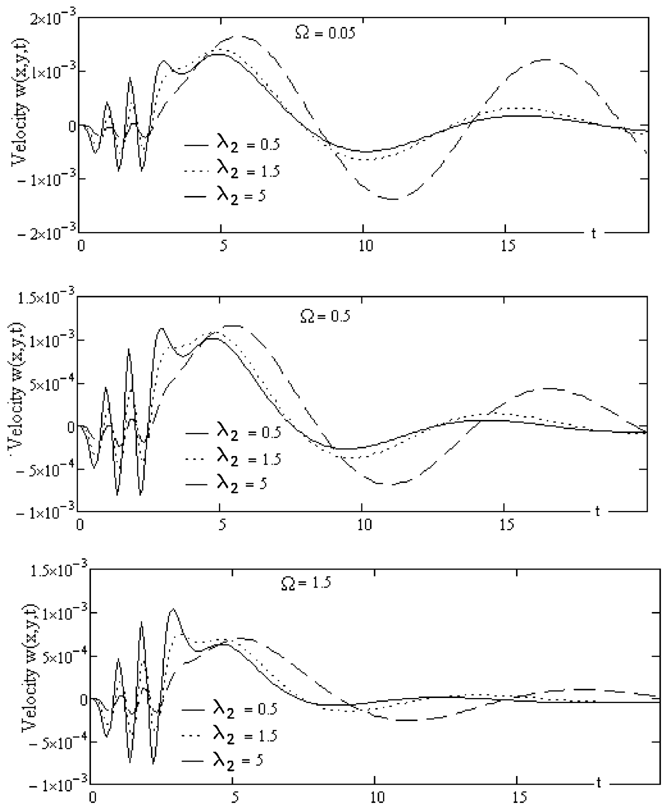

Figure 7 The influence of the parameter λ_2 on the velocity field for $x = 0.25$, $d = 0.50$, $y = 0.25$ and $\lambda_1 = 2$, $\lambda_3 = 0.5$, $\lambda_4 = 2$.

eter λ_1 is variable and parameters λ_2, λ_3, λ_4 are constant. It can be seen that, if the values of the parameter λ_1 increase, the fluid flows more slowly. Also, be noted that for the same value of the parameter λ_1, the increasing of magnetic parameter values result in decreasing velocity of fluid flow (the velocity amplitudes decrease if the values of the magnetic parameter increase). Figure 7 corresponds to the variation of the parameter λ_2. Unlike the previous case when λ_1 is variable, in this case the velocity amplitudes are higher in the early period of the movement. Another difference appears in the behavior of the fluid, namely, velocity amplitudes increase if the parameter λ_2 increases. In this case, for a constant value of the parameter λ_2, the increasing of the magnetic field strength leads to attenuated fluid motion. The diagrams of Figure 8 correspond to the variable parameter λ_3. It is clear that the fluid behavior is similar to the case of the variation of the parameter λ_1. For increasing values of the parameter λ_3, the fluid velocity amplitudes decrease and, also, if λ_3 remains constant and the values of the magnetic parameter increase, then the fluid flows more slowly. The influence of the parameter λ_4 on the fluid velocity is shown in Figure 9. The fluid behavior is similar like in the cases of variations of parameters λ_1 and λ_3.

7 Conclusions

The purpose of this work is to provide exact solutions for the velocity field as well as shear stresses corresponding to the oscillating flow of a generalized Burgers fluid between two parallel side walls over a plate. The oscillation is induced by rectified sine

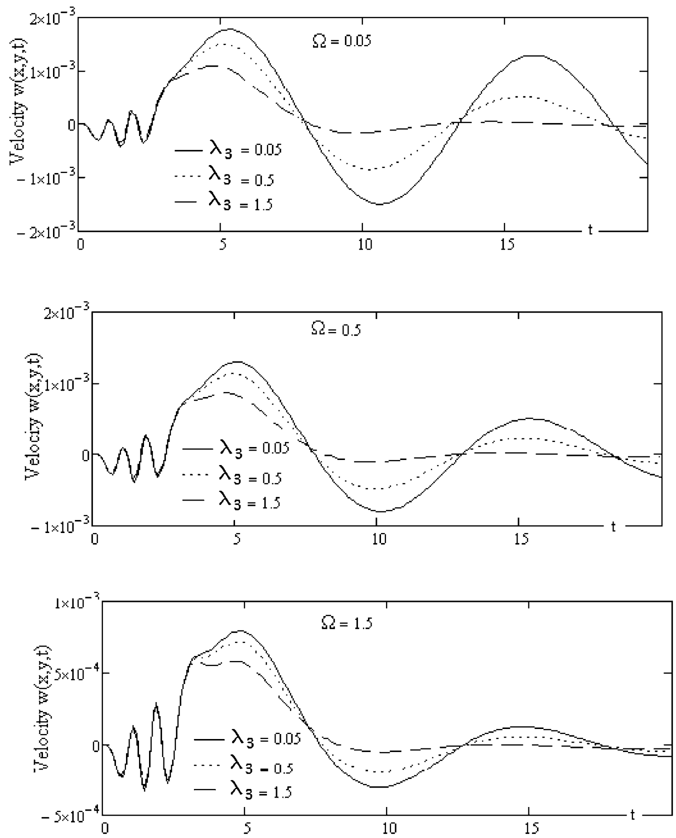

Figure 8 The influence of the parameter λ_3 on the velocity field for $x = 0.25$, $d = 0.50$, $y = 0.25$ and $\lambda_1 = 2$, $\lambda_2 = 2$, $\lambda_4 = 2$.

pulses shear stress applied to the bottom plane. These solutions, presented as a sum of the Newtonian and non-Newtonian contributions, are obtained by using Fourier cosine and sine transforms, and the Laplace transform. The main findings are summarized as follows:

- The amplitude of oscillation of velocity in the middle of the channel is small. With the increase of distance between the walls, the amplitude of oscillation of velocity increases.
- Amplitude of oscillations decreases far from the plate. As the bottom plate is set into oscillation, with respect to y-coordinate, the velocity increases from zero to a maximum in the middle of the channel.
- Velocities of all the fluids tend to a common value in shorter time if the magnetic field is stronger.
- Increasing permeability leads to the increasing magnitudes of amplitudes of velocity.
- For low values of the time period T, velocities tend to a common value in a shorter time as compared to large values of the parameter T.
- As the values of the parameters λ_1, λ_3, and λ_4 increase, the fluid flows more slowly whereas behavior of λ_2 is opposite.

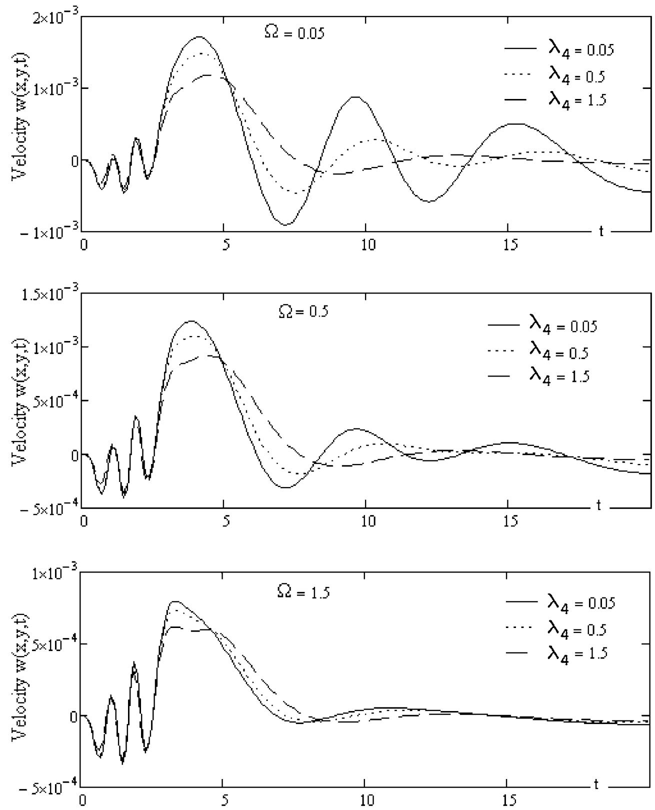

Figure 9 The influence of the parameter λ_2 on the velocity field for $x = 0.25$, $d = 0.50$, $y = 0.25$ and $\lambda_1 = 2$, $\lambda_2 = 2$, $\lambda_3 = 0.5$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors' contribution for this article is as follows: QS 40%, MN 30%, MI 15% and UA 15%. All authors read and approved the final manuscript.

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References
1. Erdogan, ME: A note on an unsteady flow of a viscous fluid due to an oscillating plane wall. Int. J. Non-Linear Mech. **35**, 1-6 (2000)
2. Fetecau, C, Hayat, T, Fetecau, C: Steady-state solutions for some simple flows of generalized Burgers fluids. Int. J. Non-Linear Mech. **41**, 880-887 (2006)
3. Zheng, L, Wang, KN, Gao, YT: Unsteady flow and heat transfer of a generalized Maxwell fluid due to a hyperbolic sine accelerating plate. Comput. Math. Appl. **61**, 2209-2212 (2011)
4. Fetecau, C, Hayat, T, Khan, M, Fetecau, C: A note on longitudinal oscillations of a generalized Burgers fluid in cylindrical domains. J. Non-Newton. Fluid Mech. **165**, 350-351 (2010)
5. Fetecau, C, Hayat, T, Fetecau, C: Starting solutions for oscillating motions of Oldroyd-B fluids. J. Non-Newton. Fluid Mech. **153**, 191-201 (2008)
6. Anjum, A, Ayub, M, Khan, M: Starting solutions for oscillating motions of an Oldroyd-B fluid over a plane wall. Commun. Nonlinear Sci. Numer. Simul. **17**, 472-482 (2012)
7. Bose, D, Basu, U: Unsteady incompressible flow of a generalised Oldroyed-B fluid between two infinite parallel plates. World J. Mech. **3**, 146-151 (2013)

8. Khan, M, Zeeshan: MHD flow of an Oldroyd-B fluid through a porous space induced by sawtooth pulses. Chin. Phys. Lett. **28**, 84701-84704 (2011)
9. Ghosh, AK, Sana, P: On hydromagnetic flow of an Oldroyd-B fluid near a pulsating plate. Acta Astronaut. **64**, 272-280 (2009)
10. Ghosh, AK, Sana, P: On hydromagnetic channel flow of an Oldroyd-B fluid induced by rectified sine pulses. Comput. Appl. Math. **28**, 365-395 (2009)
11. Vieru, D, Fetecau, C, Rana, M: Starting solutions for the flow of second grade fluids in a rectangular channel due to an oscillating shear stress. In: AIP Conf. Proc., vol. 1450, pp. 45-54 (2012). doi:10.1063/1.4724116
12. Li, C, Zheng, L, Zhang, Y, Ma, L, Zhang, X: Helical flows of a heated generalized Oldroyd-B fluid subject to a time-dependent shear stress in porous medium. Commun. Nonlinear Sci. Numer. Simul. **17**, 5026-5041 (2012)
13. Jamil, M, Fetecau, C, Imran, M: Unsteady helical flows of Oldroyd-B fluids. Commun. Nonlinear Sci. Numer. Simul. **16**, 1378-1386 (2011)
14. Shahid, N, Rana, M, Siddique, I: Exact solution for motion of an Oldroyd-B fluid over an infinite flat plate that applies an oscillating shear stress to the fluid. Bound. Value Probl. (2012). doi:10.1186/1687-2770-2012-48
15. Sohail, A, Vieru, D, Imran, MA: Influence of side walls on the oscillating motion of a Maxwell fluid over an infinite plate. Mechanika **19**(3), 269-276 (2013)
16. Rubbab, Q, Vieru, D, Fetecau, C, Fetecau, C: Natural convection flow near a vertical plate that applies a shear stress to a viscous fluid. PLoS ONE **8**(11), e78352 (2013). doi:10.1371/journal.pone.0078352

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