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Erratum: Existence and uniqueness of anti-periodic solutions for prescribed mean curvature Rayleigh equations

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Abstract
In this paper, we give a complementary proof on the paper 'Existence and uniqueness of anti-periodic solutions for prescribed mean curvature Rayleigh equations'.

Keywords: complementary proof; prescribed mean curvature Rayleigh equations

1 Introduction

In [1], the authors were concerned with the existence and uniqueness of anti-periodic solutions of the following prescribed mean curvature Rayleigh equation:

$$\left(\frac{x'}{\sqrt{1+x'^2}}\right)' + f\bigl(t,x'(t)\bigr) + g\bigl(t,x(t)\bigr) = e(t), \tag{1.1}$$

where $e \in C(R,R)$ is T-periodic, and $f,g \in C(R\times R,R)$ are T-periodic in the first argument, T is a constant.

The paper mentioned above obtained the main result by using Mawhin's continuation theorem in the coincidence degree theory. Unfortunately, the proof of main result Theorem 3.1 (see [1]) has a serious problem: $F_\mu(x) = \mu L(Q_1(t,x_1,x_2))$ where Q_1 depends on $\psi(x_2)$ and $\psi(x) = \frac{x}{\sqrt{1-x^2}}$ which is only defined for $|x| < 1$ and cannot be continuously extended; therefore, F_μ should not be defined on $\overline{\Omega} = \{x \in X : \|x\| < M\}$ since $|x_2(t)| > 1$ can occur, where $\|x\| = \max\{\|x_1\|_\infty, \|x_2\|_\infty\}$ and $M = 1 + \max\{D_1, D_2\}$.

In this paper, we shall give a complementary proof to correct the errors.

2 Complementary proof

Rewrite (1.1) in the equivalent form as follows:

$$\begin{cases} x_1'(t) = \psi(x_2(t)) = \frac{x_2(t)}{\sqrt{1-x_2^2(t)}}, \\ x_2'(t) = -f(t,\psi(x_2(t))) - g(t,x_1(t)) + e(t), \end{cases} \tag{2.1}$$

where $\psi(x) = \frac{x}{\sqrt{1-x^2}}$. In [1], the authors embed (2.1) into a family of equations with one parameter $\lambda \in (0,1]$,

$$\begin{cases} x_1'(t) = \lambda \frac{x_2(t)}{\sqrt{1-x_2^2(t)}} = \lambda\psi(x_2(t)), \\ x_2'(t) = -\lambda f(t, \psi(x_2(t))) - \lambda g(t, x_1(t)) + \lambda e(t). \end{cases} \tag{2.2}$$

They have proved that there exists a constant $D_1 > 0$ such that

$$|x_1'|_2 \le D_1, \quad \text{and} \quad |x_1|_\infty \le D_1, \tag{2.3}$$

and there exists $\eta \in [0, T]$ such that $x_2(\eta) = 0$.

In fact, to use the continuation theorem, it suffices to prove that there exists a positive constant $0 < \varepsilon_0 \ll 1$ such that, for any possible solution $(x_1(t), x_2(t))$ of (2.2), the following condition holds:

$$|x_2(t)| < 1 - \varepsilon_0. \tag{2.4}$$

In what follows, we shall give a complementary proof for the main result in [1] by giving a proof of (2.4).

In [1], the authors assume that

(H_1) $(g(t, x_1) - g(t, x_2))(x_1 - x_2) < 0$, for all $t, x_1, x_2 \in R$ and $x_1 \neq x_2$;

(H_2) there exists $l > 0$ such that

$$|g(t, x_1) - g(t, x_2)| \le l|x_1 - x_2| \quad \text{for all } t, x_1, x_2 \in R;$$

(H_3) there exists β, γ such that

$$\gamma \le \liminf_{|x|\to\infty} \frac{f(t,x)}{x} \le \limsup_{|x|\to\infty} \frac{f(t,x)}{x} \le \beta, \quad \text{uniformly in } t \in R;$$

(H_4) for all $t, x \in R$,

$$f\left(t + \frac{T}{2}, -x\right) = -f(t,x), \qquad g\left(t + \frac{T}{2}, -x\right) = -g(t,x), \qquad e\left(t + \frac{T}{2}\right) = -e(t).$$

Under the conditions mentioned above, we prove that (2.4) holds.

Since $|x_1|_\infty < D_1$ and g, e are continuous, we find that there exists $M_3 > 0$ such that

$$-M_3 < -g\big(t, x_1(t)\big) + e(t) < M_3, \quad \forall t \in R. \tag{2.5}$$

By (H_3), there exists a positive constant $M_4 > 0$ such that

$$f(t,x) \ge \gamma x - M_4, \quad \forall x > 0 \text{ and } \forall t \in R. \tag{2.6}$$

Next, we shall prove that

$$x(t) \le \frac{M_3 + M_4}{\sqrt{(M_3 + M_4)^2 + \gamma^2}}, \quad \forall t \in R.$$

Assume by contradiction that there exist $t_2^* > t_1^* > \eta$ such that

$$x_2(t_1^*) = \frac{M_3 + M_4}{\sqrt{(M_3 + M_4)^2 + \gamma^2}}, \qquad x_2(t_2^*) > \frac{M_3 + M_4}{\sqrt{(M_3 + M_4)^2 + \gamma^2}},$$

and

$$x_2(t) > \frac{M_3 + M_4}{\sqrt{(M_3 + M_4)^2 + \gamma^2}}, \quad \text{for } t \in (t_1^*, t_2^*).$$

Noticing that $\lambda \in (0,1]$, we have, $\forall t \in (t_1^*, t_2^*)$,

$$x_2'(t) = \lambda\bigl(-f\bigl(t, \psi\bigl(x_2(t)\bigr)\bigr) - g\bigl(t, x_1(t)\bigr) + e(t)\bigr) < 0,$$

which is a contradiction.

By (H_3), there exists a positive constant $M_5 > 0$ such that

$$f(t,x) \leq \beta x + M_5, \quad \forall x < 0 \text{ and } \forall t \in R.$$

By using a similar argument, we can prove that

$$x_2(t) \geq -\frac{M_3 + M_5}{\sqrt{(M_3 + M_5)^2 + \beta^2}}, \quad \text{for } t \in R.$$

Therefore, we get from the continuity of $x_2(t)$, for any solution $(x_1(t), x_2(t))$ of (2.2),

$$-\frac{M_3 + M_5}{\sqrt{(M_3 + M_5)^2 + \beta^2}} \leq x_2(t) \leq \frac{M_3 + M_4}{\sqrt{(M_3 + M_4)^2 + \gamma^2}}, \quad \forall t \in R.$$

Consequently, (2.4) holds.

Putting

$$\Omega = \left\{ x = (x,x) \in C_T^{0,\frac{1}{2}}\bigl(R, R^2\bigr) = X : \|x\| < M, \bigl|x_2(t)\bigr| < 1 - \varepsilon_0 \right\},$$

we can use Mawhin's continuation theorem on Ω.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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