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Global attractor for the generalized hyperelastic-rod equation

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Abstract

In this paper, we investigate the dynamical behavior of the initial boundary value problem for a class of generalized hyperelastic-rod equations. Under certain conditions, the existence of a global solution in H^3 is proved by using some prior estimates and the Galerkin method. Moreover, the existence of an absorbing set and a global attractor in H^2 is obtained.

Keywords: generalized hyperelastic-rod equation; global solution; global attractor

1 Introduction

Camassa and Holm [1] first proposed a completely integrable dispersive shallow water equation as follows:

$$u_t - u_{xxt} + 3uu_x + ku_x = 2u_xu_{xx} + uu_{xxx}. \tag{1.1}$$

The C-H equation (1.1) was obtained by using an asymptotic expansion directly in the Hamiltonian for the Euler equations in the shallow water regime and possessed a bi-Hamiltonian structure and an infinite number of conservation laws in involution. Research on the C-H equation becomes a hot field due to its good properties [2–4] since it was proposed in 1993. Some equations also have similar characters to the C-H equation, which are called C-H family equations. Because of the wide applications in applied sciences such as physics, the C-H family equations have attracted much attention in recent years.

In 1998, Dai [5] derived the following hyperelastic-rod wave equation for finite-length and finite-amplitude waves in 1998 when doing research on hyperelastic compressible material:

$$v_\tau + \sigma_1 vv_\xi + \sigma_2 v_{\xi\xi\tau} + \sigma_3(2v_\xi v_{\xi\xi} + vv_{\xi\xi\xi}) = 0, \tag{1.2}$$

where $v(\xi,\tau)$ represents the radial stretch relative to a pre-stressed state. The three coefficients σ_1, σ_2, and σ_3 are constants determined by the pre-stress and the material parameters, $\sigma_1 \neq 0$, $\sigma_2 < 0$, $\sigma_3 \leq 0$.

If $\tau = \frac{3\sqrt{-\sigma_2}}{\sigma_1}t$ and $\xi = \sqrt{-\sigma_2}x$, then the following equation can be obtained by (1.2):

$$u_t - u_{xxt} + 3uu_x = \gamma(2u_xu_{xx} + uu_{xxx}), \qquad \gamma = \frac{3\sigma_3}{\sigma_1\sigma_2}. \tag{1.3}$$

The constant γ is called the pre-stressed coefficient of the material rod.

There have been many research results as regards the hyperelastic-rod equation (1.3) [6–12], such as traveling-wave solutions, blow-up of solutions, well-posedness of solutions, the existence of weak solutions, the global solutions of Cauchy problem, the periodic boundary value problem, *etc.*

In 2005, Coclite *et al.* [13, 14] studied the following extension of (1.3):

$$u_t - u_{xxt} + g(u)u_x = \gamma(2u_x u_{xx} + uu_{xxx}), \qquad g(0) = 0. \tag{1.4}$$

The existence of a global weak solution to (1.4) for any initial function u_0 belonging to $H^1(R)$ was obtained. They showed stability of the solution when a regularizing term vanishes based on a vanishing viscosity argument and presented a 'weak equals strong' uniqueness result.

It is easy to observe that if $\gamma = 0$ and $g(u) = 2ku + a$, (1.4) becomes the BBM equation (1.5) [15, 16],

$$u_t - u_{xxt} + au_x + k\left(u^2\right)_x = 0. \tag{1.5}$$

Here $\gamma = 1$ and $g(u) = 3u + k$, (1.4) is transformed into the C-H equation (1.1).

If $\gamma = 1$ and $g(u) = (b+1)u$, (1.4) can be changed to the D-P equation (1.6) [17–20],

$$u_t - u_{xxt} + (b+1)uu_x = 2u_x u_{xx} + uu_{xxx}. \tag{1.6}$$

Actually, the KdV equation [21], the C-H equation, the hyperelastic-rod wave equation *etc.* are all considered as special cases of the generalized hyperelastic-rod equation. So many researchers focused on this class of equations [22–24]. Among them, Holden and Raynaud [22] studied the following generalized hyperelastic-rod equation:

$$u_t - u_{xxt} + f(u)_x - f(u)_{xxx} + \left(g(u) + \frac{1}{2}f''(u)(u_x)^2\right)_x = 0. \tag{1.7}$$

They considered the Cauchy problem of (1.7) and proved the existence of global and conservative solutions. It was shown that the equation was well-posed for initial data in $H^1(R)$ if one included a Radon measure corresponding to the energy of the system with the initial data.

However, there are few works with respect to the global asymptotical behaviors of solutions and the existence of global attractors, which are important for the study of the dynamical properties of general nonlinear dissipative dynamical systems [25–27]. Motivated by the references cited above, the goal of the present paper is to investigate the initial boundary problem of the following equation:

$$\begin{cases} u_t - u_{xxt} + [G(u)]_x^{(4)} = 2u_x u_{xx} + uu_{xxx}, & t > 0, x \in \Omega, \\ u(0,x) = u_0(x), & x \in \Omega, \end{cases} \tag{1.8}$$

where $\Omega = [0, L]$. We will study the dynamics behavior of (1.8) and discuss the existence of the global solution and the global attractor under the periodic boundary condition when $G(u)$ satisfies the particular conditions.

The rest of this paper is organized as follows: Section 2 describes the main definitions used in this paper. The existence of the global solution is discussed in Section 3. The existence of the absorbing set is detailed in Section 4. Section 5 shows the existence of the global attractor.

2 Preliminaries

In this work, $(\cdot,\cdot)$ stands for the inner product in the usual sense and $\|\cdot\|$ represents the norm determined by the inner product, $\|u\|_{H^m(\Omega)} = \|D^m u\|_{L^2(\Omega)}$. Apparently, this norm is equal to the natural norm in $H^m(\Omega)$. The following signs are adopted in this paper to express the norms of different spaces: $\|u\|_{L^2(\Omega)} \stackrel{\Delta}{=} |u|$, $\|Du\|_{L^2(\Omega)} \stackrel{\Delta}{=} \|u\|$, $\|D^m u\|_{L^2(\Omega)} \stackrel{\Delta}{=} |D^m u|$.

The notion of bilinear operator is introduced, $B(u,v) = u\nabla v$, where ∇ is called a first order differential operator. Then we can get $b(u,v,\omega) = (B(u,v),\omega) = \int_\Omega (u\nabla v)\omega\,dx$.

The generalized hyperelastic-rod equation we studied is one-dimensional, and the operator ∇ acting on $u(x,t)$ is not identically vanishing, so $b(u,v,\omega) = 0$ cannot be found. However, the following formulas can be derived by the periodic boundary condition and formula of integration by parts:

$$\begin{aligned}
\bigl(B(u,v),\omega\bigr) &= -\bigl(B(u,\omega),v\bigr) - \bigl(B(\omega,u),v\bigr),\\
\bigl(B(v,u),\omega\bigr) &= -\bigl(B(\omega,v),u\bigr) - \bigl(B(v,\omega),u\bigr),
\end{aligned}$$

furthermore, $(B(u,v),u) = -2(B(u,u),v)$, $(B(u,v),u) = -2(B(v,u),u)$, so we get $(B(u,u),v) = (B(v,u),u)$ and $(B(u,u),u) = 0$.

Suppose $A = -\Delta$ is a second order differential operator, $v = u + Au$, then A is a self-adjoint operator, which possesses the eigenvalues like $(k_1^2 + k_2^2)(\frac{2\pi}{L})^2$, where $k_1, k_2 \in N_0$ and $k_1^2 + k_2^2 \neq 0$. λ_1 represents the smallest eigenvalue of A.

Based on the above statements, the initial boundary value problem of (1.8) under the periodic boundary condition can be rewritten as follows:

$$\frac{dv}{dt} + \bigl[G(u)\bigr]_x^{(4)} + B(u,v) + 2B(v,u) - 3B(u,u) = 0, \tag{2.1}$$

$$u(x,0) = u_0, \tag{2.2}$$

$$u(0,t) = u(L,t). \tag{2.3}$$

In this work, we assume that $H = \{u \mid u \in L^2(\Omega) \text{ and } u(0,t) = u(L,t)\}$, $\mathrm{V} = \{u \mid u' \in L^2(\Omega) \text{ and } u(0,t) = u(L,t)\}$, $G_u'(u) \ge g_0 > 0$ and $|G_u^{(k)}(u)| \le C|u|^{5-k}$, $k = 1,2,3,4$, C is a constant.

3 The existence of global solution

Theorem 1 *If* $u_0 \in \mathrm{V}$, $G_u'(u) \ge g_0 > 0$, *and* $|G_u^{(k)}(u)| \le C|u|^{5-k}$, $k = 1,2,3,4$, *then* (2.1)-(2.3) *possess the global solution* $u = u(\cdot,u_0) \in C([0,\infty); H^3(R)) \cap C^1([0,\infty); H^2(R))$.

Proof The Galerkin method is adopted to prove this theorem. Assume that $\{\phi_i\}_{j=1}^\infty$ is an orthogonal basis of H constituted by the eigenvectors of the operator A, $H_m = \operatorname{span}\{\phi_1, \phi_2, \ldots, \phi_m\}$, P_m is the orthogonal projection from H to H_m. Through the Galerkin method, we can obtain the following ordinary differential equations by (2.1), (2.2):

$$\frac{dv_m}{dt} + \bigl[G(u_m)\bigr]_x^{(4)} + P_m B(u_m, v_m) + 2P_m B(v_m, u_m) - 3P_m B(u_m, u_m) = 0, \tag{3.1}$$

$$u_m(0) = P_m u(0), \tag{3.2}$$

where $v_m = u_m + Au_m$. Considering the expressions of $B(u_m, v_m)$, $B(v_m, u_m)$, $B(u_m, u_m)$, according to the qualitative theories of ordinary differential equations, (3.1)-(3.2) have a unique solution u_m in $(0, T_m)$. In order to prove the existence of a global solution, we need to do some prior estimates as regards u_m.

Taking the inner product of (3.1) with u_m in Ω, we have

$$\begin{aligned}
&\left(\frac{dv_m}{dt}, u_m\right) + \left(\left[G(u_m)\right]_x^{(4)}, u_m\right) + P_m\big(B(u_m, v_m), u_m\big) \\
&\quad + 2P_m\big(B(v_m, u_m), u_m\big) - 3P_m\big(B(u_m, u_m), u_m\big) = 0.
\end{aligned}$$

By using integration by parts and the periodic boundary conditions, we get

$$\begin{aligned}
&\left(\frac{dv_m}{dt}, u_m\right) = \frac{1}{2}\frac{d}{dt}\left(\int_\Omega \left(u_m^2 + u_{mx}^2\right) dx\right) = \frac{1}{2}\frac{d}{dt}\left(|u_m|^2 + \|u_m\|^2\right), \\
&P_m\big(B(u_m, v_m), u_m\big) + 2P_m\big(B(v_m, u_m), u_m\big) - 3P_m\big(B(u_m, u_m), u_m\big) = 0, \\
&\left(\left[G(u_m)\right]_x^{(4)}, u_m\right) = \int_\Omega \left[G(u_m)\right]_x^{(4)} u_m\, dx = -\int_\Omega G'_{u_m}(u_m) u_{mx} u_{mxxx}\, dx.
\end{aligned}$$

Moreover, in terms of $\int_\Omega u_{mx} u_{mxxx}\, dx = -\int_\Omega u_{mxx}^2\, dx \le 0$ and $G'_u(u) \ge g_0 > 0$, we have

$$\begin{aligned}
&\left(\left[G(u_m)\right]_x^{(4)}, u_m\right) \ge -g_0 \int_\Omega u_{mx} u_{mxxx}\, dx, \\
&\frac{1}{2}\frac{d}{dt}\left(|u_m|^2 + \|u_m\|^2\right) - g_0 \int_R u_{mx} u_{mxxx}\, dx \le 0.
\end{aligned}$$

Employing $\int_\Omega u_{mx} u_{mxxx}\, dx = -\int_\Omega u_{mxx}^2\, dx$ again, the following formula can be obtained:

$$\frac{d}{dt}\left(|u_m|^2 + \|u_m\|^2\right) + 2g_0 |Au_m|^2 \le 0.$$

By the Poincaré inequality, $|Au_m|^2 > \lambda_1 \|u_m\|^2$, we have

$$\frac{d}{dt}\left(|u_m|^2 + \|u_m\|^2\right) + g_0\lambda_1 \|u_m\|^2 + g_0 |Au_m|^2 \le 0.$$

Let $g_1 = \min\{g_0\lambda_1, g_0\}$, then

$$\frac{d}{dt}\left(|u_m|^2 + \|u_m\|^2\right) + g_1\left(\|u_m\|^2 + |Au_m|^2\right) \le 0. \tag{3.3}$$

Using the Poincaré inequality again, $\|u_m\|^2 > \lambda_1 |u_m|^2$ and $|Au_m|^2 > \lambda_1 \|u_m\|^2$, (3.3) can be changed to

$$\frac{d}{dt}\left(|u_m|^2 + \|u_m\|^2\right) + g_1\lambda_1\left(|u_m|^2 + \|u_m\|^2\right) \le 0.$$

So we can obtain

$$|u_m|^2 + \|u_m\|^2 \le \left(\left|u_m(0)\right|^2 + \left\|u_m(0)\right\|^2\right) \exp\{-g_1\lambda_1 t\} \le \left|u_m(0)\right|^2 + \left\|u_m(0)\right\|^2 \triangleq r_1.$$

Integrating (3.3) over the interval $[t, t+r]$,

$$g_1 \int_t^{t+r} \left(\|u_m(s)\|^2 + |Au_m(s)|^2 \right) ds \le r_1. \tag{3.4}$$

Taking the inner product of (3.1) with Au_m in Ω, we have

$$\begin{aligned}&\left(\frac{dv_m}{dt}, Au_m\right) + \left([G(u_m)]_x^{(4)}, Au_m\right) + P_m\left(B(u_m, v_m), Au_m\right)\\&\quad + 2P_m\left(B(v_m, u_m), Au_m\right) - 3P_m\left(B(u_m, u_m), Au_m\right) = 0.\end{aligned}$$

By using integration by parts and the periodic boundary conditions, we get

$$\begin{aligned}&\left(\frac{dv_m}{dt}, Au_m\right) = \frac{1}{2}\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right),\\&\left([G(u_m)]_x^{(4)}, Au_m\right) = \int_\Omega [G(u_m)]_x^{(4)} Au_m\, dx = \int_\Omega G'_{u_m}(u_m) u_{mx} u_{mxxxxxx}\, dx.\end{aligned}$$

Moreover, $\int_\Omega u_{mx}u_{mxxxxxx}\, dx = \int_\Omega u_{mxxx}^2\, dx \ge 0$ and $G'_u(u) \ge g_0 > 0$. So

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right) + g_0 \int_R u_{mx}u_{mxxxxxx}\, dx + P_m\left(B(u_m, v_m), Au_m\right)\\&\quad + 2P_m\left(B(v_m, u_m), Au_m\right) - 3P_m\left(B(u_m, u_m), Au_m\right) \le 0.\end{aligned}$$

Employing $\int_\Omega u_{mx}u_{mxxxxxx}\, dx = \int_\Omega u_{mxxx}^2\, dx$ again, we obtain

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right) + g_0|\nabla Au_m|^2 + P_m\left(B(u_m, v_m), Au_m\right)\\&\quad + 2P_m\left(B(v_m, u_m), Au_m\right) - 3P_m\left(B(u_m, u_m), Au_m\right) \le 0.\end{aligned} \tag{3.5}$$

By computing, we have

$$\begin{aligned}&P_m\left(B(u_m, v_m), Au_m\right) + 2P_m\left(B(v_m, u_m), Au_m\right) - 3P_m\left(B(u_m, u_m), Au_m\right)\\&\quad = 2P_m\left(B(Au_m, u_m), Au_m\right) + P_m\left(B(u_m, Au_m), Au_m\right).\end{aligned}$$

According to the Agmon inequality when $n = 1$, $\|\varphi\|_{L^\infty} \le c\|\varphi\|_{L^2}^{\frac{1}{2}}\|\varphi\|_{H^1}^{\frac{1}{2}}$, where c is a constant which only depends on Ω. Furthermore, we can get

$$\begin{aligned}&P_m\left(B(Au_m, u_m), Au_m\right) \le \|\nabla u_m\|_{L^\infty}|Au_m|^2 \le c_1\|u_m\|^{\frac{1}{2}}|Au_m|^{\frac{5}{2}},\\&P_m\left(B(u_m, Au_m), Au_m\right) \le \frac{1}{2}\|\nabla u_m\|_{L^\infty}|Au_m|^2 \le \frac{c_2}{2}\|u_m\|^{\frac{1}{2}}|Au_m|^{\frac{5}{2}}.\end{aligned}$$

So the following inequality can be gotten by (3.5):

$$\frac{1}{2}\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right) + g_0|\nabla Au_m|^2 \le c_1\|u_m\|^{\frac{1}{2}}|Au_m|^{\frac{5}{2}} + \frac{c_2}{2}\|u_m\|^{\frac{1}{2}}|Au_m|^{\frac{5}{2}}.$$

By the Poincaré inequality, $|\nabla Au_m|^2 > \lambda_1|Au_m|^2$, together with $g_1 = \min\{g_0\lambda_1, g_0\}$, we have

$$\frac{1}{2}\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right) + \frac{g_1}{2}\left(|Au_m|^2 + |\nabla Au_m|^2\right) \le c_1\|u_m\|^{\frac{1}{2}}|Au_m|^{\frac{5}{2}} + \frac{c_2}{2}\|u_m\|^{\frac{1}{2}}|Au_m|^{\frac{5}{2}}.$$

By the Young inequality, the following inequality can be obtained:

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right) + \frac{g_1}{2}\left(|Au_m|^2 + |\nabla Au_m|^2\right)\\ &\quad\le \frac{1}{2}g_1\lambda_1\left(\|u_m\|^2 + |Au_m|^2\right) + c_3\|u_m\||Au_m|\left(\|u_m\|^2 + |Au_m|^2\right),\end{aligned} \tag{3.6}$$

where

$$c_3 = \frac{[\min\{c_1, \frac{c_2}{2}\}]^2}{2g_1\lambda_1},$$

and, by using the Poincaré inequality, we have

$$\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right) \le 2c_3\|u_m\||Au_m|\left(\|u_m\|^2 + |Au_m|^2\right).$$

Using the Young inequality again, we can further get

$$\frac{d}{dt}\left(\|u_m\|^2 + |Au_m|^2\right) \le c_3\left(\|u_m\|^2 + |Au_m|^2\right)^2. \tag{3.7}$$

Denoting $y = \|u_m(s)\|^2 + |Au_m(s)|^2$, $g = c_3(\|u_m(s)\|^2 + |Au_m(s)|^2)$. According to (3.4),

$$\int_t^{t+r} y\,ds \le \frac{r_1}{g_1}, \qquad \int_t^{t+r} g\,ds \le \frac{c_3 r_1}{g_1}.$$

Based on the uniform Grownwall inequality, we have

$$\|u_m\|^2 + |Au_m|^2 \le \frac{r_1}{rg_1}\exp\left\{\frac{c_3 r_1}{g_1}\right\} \triangleq r_2, \quad t > t_0 + r, \tag{3.8}$$

where r, r_1, and c_3 are nonnegative constants.

Integrating (3.6) over the interval $[t, t+r]$ to obtain

$$\begin{aligned}&\frac{1}{2}g_1\int_t^{t+r}\left(|Au_m|^2 + |\nabla Au_m|^2\right)ds\\ &\quad\le \int_t^{t+r}\left(\frac{1}{2}g_1\lambda_1\left(\|u_m\|^2 + |Au_m|^2\right) + \frac{c_3}{2}\left(\|u_m\|^2 + |Au_m|^2\right)^2\right)ds + \left(\|u_m\|^2 + |Au_m|^2\right)\\ &\quad\le \frac{1}{2}\left(g_1\lambda_1 r_2 + c_3 r_2^2\right)r + r_2 \triangleq r_3.\end{aligned} \tag{3.9}$$

Taking the inner product of (3.1) with A^2u_m in Ω, together with integration by parts, the periodic boundary conditions, and $G_u'(u) \ge g_0 > 0$, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(|Au_m|^2 + |\nabla Au_m|^2\right) + g_0\left|A^2u_m\right|^2 + P_m\left(B(u_m, v_m), A^2u_m\right)\\ &\quad + 2P_m\left(B(v_m, u_m), A^2u_m\right) - 3P_m\left(B(u_m, u_m), A^2u_m\right) \le 0.\end{aligned}$$

Through the Young inequality, the Hölder inequality and the Poincaré inequality, we deduce that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(|Au_m|^2+|\nabla Au_m|^2\right)+\frac{g_1}{2}\left(|\nabla Au_m|^2+\left|A^2u_m\right|^2\right)\\&\quad\le\frac{1}{2}g_1\lambda_1\left(|Au_m|^2+|\nabla Au_m|^2\right)+c_4\|u_m\||Au_m|\left(|Au_m|^2+|\nabla Au_m|^2\right).\end{aligned}$$

According to the Poincaré inequality and the Young inequality again, we have

$$\begin{aligned}\frac{d}{dt}\left(|Au_m|^2+|\nabla Au_m|^2\right)&\le 2c_4\|u_m\||Au_m|\left(|Au_m|^2+|\nabla Au_m|^2\right)\\&\le c_4\left(\|u_m\|^2+|Au_m|^2\right)\left(|Au_m|^2+|\nabla Au_m|^2\right).\end{aligned}$$

From (3.4) and (3.9), we get

$$\begin{aligned}&c_4\int_t^{t+r}\left(\left\|u_m(s)\right\|^2+\left|Au_m(s)\right|^2\right)ds\le\frac{r_1c_4}{g_1},\\&\int_t^{t+r}\left(|Au_m|^2+|\nabla Au_m|^2\right)ds\le\frac{2r_3}{g_1}.\end{aligned}$$

Based on the uniform Grownwall inequality, we have

$$|Au_m|^2+|\nabla Au_m|^2\le\frac{2r_3}{rg_1}\exp\left\{\frac{r_1c_4}{g_1}\right\}\triangleq r_4,\quad t>t_0.\tag{3.10}$$

Overall, $|u_m|^2\le r_1$, $\|u_m\|^2\le r_2$, $|Au_m|^2\le r_3$, $|\nabla Au_m|^2\le r_4$, that is, $|v_m|^2\le r_1+r_3$, $\|v_m\|^2\le r_2+r_4$.

According to the qualitative theories of ordinary differential equations, (3.1)-(3.2) have a global solution u_m.

From the above discussion, we have

$$\begin{aligned}&\left|P_mB(u_m,v_m)\right|\le|u_m|\|v_m\|\le\left(r_1(r_2+r_4)\right)^{\frac{1}{2}}\triangleq r_5,\\&\left|P_mB(v_m,u_m)\right|\le|v_m|\|u_m\|\le\left(r_2(r_1+r_3)\right)^{\frac{1}{2}}\triangleq r_6,\\&\left|P_mB(u_m,u_m)\right|\le|u_m|\|u_m\|\le(r_1r_2)^{\frac{1}{2}}\triangleq r_7.\end{aligned}$$

Then (3.1) can be rewritten as

$$\frac{dv_m}{dt}=3P_mB(u_m,u_m)-P_mB(u_m,v_m)-2P_m(v_m,u_m)-\left[G(u_m)\right]_x^{(4)}.$$

Because of $|G_u^{(k)}(u)|\le C|u|^{5-k}$, $k=1,2,3,4$,

$$\begin{aligned}\left|\frac{dv_m}{dt}\right|&\le 3\left|P_mB(u_m,u_m)\right|+\left|P_mB(u_m,v_m)\right|+2\left|P_mB(v_m,u_m)\right|+\left|P_mB\left(\left[G(u_m)_x'''\right]_u',u_m\right)\right|\\&\le 3r_7+r_5+2r_6+h(C,r_1,r_2,r_3,r_4)\|u_m\|\le 3r_7+r_5+2r_6+hr_2^{\frac{1}{2}}\triangleq k,\end{aligned}\tag{3.11}$$

where h is a constant which depends on C, r_1, r_2, r_3, r_4.

According to the Aubin compactness theorem, we conclude that there is a convergent subsequence $u_{m'}$, so that $u_{m'} \to u$, or equivalently $v_{m'} \to v$. Suppose that $u_{m'}$ and $v_{m'}$ are replaced by u_m and v_m, then we need to prove that u, v satisfy (2.1).

Selecting $\omega \in D(A)$ randomly, $|\omega|$ is bounded as we see from the above discussion. By the ordinary differential equation (3.1), we have

$$\begin{aligned}&\big(v_m(t),\omega\big)+\int_{t_0}^{t}\big(G\big(u_m(s)\big),A^2\omega\big)\,ds+\int_{t_0}^{t}\big(B\big(u_m(s),v_m(s)\big),P_m\omega\big)\,ds\\&\quad+2\int_{t_0}^{t}\big(B\big(v_m(s),u_m(s)\big),P_m\omega\big)\,ds-3\int_{t_0}^{t}\big(B\big(u_m(s),u_m(s)\big),P_m\omega\big)\,ds=\big(v_m(t_0),\omega\big).\end{aligned}$$

Obviously, $\lim_{m\to+\infty}|P_m\omega-\omega|=0$, $\lim_{m\to+\infty}|P_mA^2\omega-A^2\omega|=0$, according to the convergence,

$$\begin{aligned}&\lim_{m\to+\infty}\int_{t_0}^{t}\big(G\big(u_m(s)\big),A^2\omega\big)\,ds=\int_{t_0}^{t}\big(G\big(u(s)\big),A^2\omega\big)\,ds,\\&\left|\int_{t_0}^{t}\big(B\big(u_m(s),v_m(s)\big),P_m\omega\big)\,ds-\int_{t_0}^{t}\big(B\big(u(s),v(s)\big),\omega\big)\,ds\right|\\&\quad\le\left|\int_{t_0}^{t}\big(B\big(u_m(s),v_m(s)\big),P_m\omega-\omega\big)\,ds\right|+\left|\int_{t_0}^{t}\big(B\big(u_m(s)-u(s),v_m(s)\big),\omega\big)\,ds\right|\\&\qquad+\left|\int_{t_0}^{t}\big(B\big(u(s),v_m(s)-v(s)\big),\omega\big)\,ds\right|,\end{aligned}$$

where

$$I_m^{(1)}=\left|\int_{t_0}^{t}\big(B\big(u_m(s),v_m(s)\big),P_m\omega-\omega\big)\,ds\right|\le\int_{t_0}^{t}\big|B\big(u_m(s),v_m(s)\big)\big||P_m\omega-\omega|\,ds.$$

Considering the boundness of $|B(u_m(s),v_m(s))|$, so $I_m^{(1)}\to 0$,

$$\begin{aligned}I_m^{(2)}&=\left|\int_{t_0}^{t}\big(B\big(u_m(s)-u(s),v_m(s)\big),\omega\big)\,ds\right|\le\int_{t_0}^{t}\big|B\big(u_m(s)-u(s),v_m(s)\big)\big||\omega|\,ds\\&\le\int_{t_0}^{t}\big|u_m(s)-u(s)\big|\big\|v_m(s)\big\||\omega|\,ds\to 0,\\I_m^{(3)}&=\left|\int_{t_0}^{t}\big(B\big(u(s),v_m(s)-v(s)\big),\omega\big)\,ds\right|\le\int_{t_0}^{t}\big|B\big(u(s),v_m(s)-v(s)\big)\big||\omega|\,ds\\&\le\int_{t_0}^{t}\big|u(s)\big|\big\|v_m(s)-v(s)\big\||\omega|\,ds\to 0,\\&\left|\int_{t_0}^{t}\big(B\big(v_m(s),u_m(s)\big),P_m\omega\big)\,ds-\int_{t_0}^{t}\big(B\big(v(s),u(s)\big),\omega\big)\,ds\right|\\&\le\left|\int_{t_0}^{t}\big(B\big(v_m(s),u_m(s)\big),P_m\omega-\omega\big)\,ds\right|+\left|\int_{t_0}^{t}\big(B\big(v_m(s)-v(s),u_m(s)\big),\omega\big)\,ds\right|\\&\quad+\left|\int_{t_0}^{t}\big(B\big(v(s),u_m(s)-u(s)\big),\omega\big)\,ds\right|\\&=I_m^{(4)}+I_m^{(5)}+I_m^{(6)},\end{aligned}$$

where

$$
\begin{aligned}
I_m^{(4)} &= \left|\int_{t_0}^t \bigl(B\bigl(v_m(s), u_m(s)\bigr), P_m\omega - \omega\bigr)\,ds\right| \le \int_{t_0}^t \bigl|B\bigl(v_m(s), u_m(s)\bigr)\bigr||P_m\omega - \omega|\,ds \to 0, \\
I_m^{(5)} &= \left|\int_{t_0}^t \bigl(B\bigl(v_m(s) - v(s), u_m(s)\bigr), \omega\bigr)\,ds\right| \le \int_{t_0}^t \bigl|B\bigl(v_m(s) - v(s), u_m(s)\bigr)\bigr||\omega|\,ds \\
&\le \int_{t_0}^t \bigl|v_m(s) - v(s)\bigr|\bigl\|u_m(s)\bigr\||\omega|\,ds \to 0, \\
I_m^{(6)} &= \left|\int_{t_0}^t \bigl(B\bigl(v(s), u_m(s) - u(s)\bigr), \omega\bigr)\,ds\right| \le \int_{t_0}^t \bigl|B\bigl(v(s), u_m(s) - u(s)\bigr)\bigr||\omega|\,ds \\
&\le \int_{t_0}^t \bigl|v(s)\bigr|\bigl\|u_m(s) - u(s)\bigr\||\omega|\,ds \to 0,
\end{aligned}
$$

$$
\begin{aligned}
&\left|\int_{t_0}^t \bigl(B\bigl(u_m(s), u_m(s)\bigr), P_m\omega\bigr)\,ds - \int_{t_0}^t \bigl(B\bigl(u(s), u(s)\bigr), \omega\bigr)\,ds\right| \\
&\quad\le \left|\int_{t_0}^t \bigl(B\bigl(u_m(s), u_m(s)\bigr), P_m\omega - \omega\bigr)\,ds\right| + \left|\int_{t_0}^t \bigl(B\bigl(u_m(s) - u(s), u_m(s)\bigr), \omega\bigr)\,ds\right| \\
&\qquad + \left|\int_{t_0}^t \bigl(B\bigl(u(s), u_m(s) - u(s)\bigr), \omega\bigr)\,ds\right| \\
&\quad= I_m^{(7)} + I_m^{(8)} + I_m^{(9)} \to 0.
\end{aligned}
$$

From the above discussion, we can deduce that u, v satisfy the following equation:

$$
\begin{aligned}
&\bigl(v(t), \omega\bigr) + \int_{t_0}^t \bigl(G\bigl(u(s)\bigr), A^2\omega\bigr)\,ds + \int_{t_0}^t \bigl(B\bigl(u(s), v(s)\bigr), \omega\bigr)\,ds \\
&\quad + 2\int_{t_0}^t \bigl(B\bigl(v(s), u(s)\bigr), \omega\bigr)\,ds - 3\int_{t_0}^t \bigl(B\bigl(u(s), u(s)\bigr), \omega\bigr)\,ds = \bigl(v(t_0), \omega\bigr).
\end{aligned}
$$

Above all, u is the solution of (2.1)-(2.3), that is, their global solution exists. □

4 The existence of the absorbing set

Theorem 2 *If* $u_0 \in \mathrm{V}$, *the semi-group of the solution to* (2.1)-(2.3), *i.e.* $S(t) : H^2(\Omega) \to H^2(\Omega)$, $u(t) = S(t)u_0$, *has an absorbing set.*

Proof Taking the inner product of (2.1) with u in Ω we obtain

$$\left(\frac{dv}{dt}, u\right) + \bigl(A^2G(u), u\bigr) + 2\bigl(B(v,u), u\bigr) + \bigl(B(u,v), u\bigr) - 3\bigl(B(u,u), u\bigr) = 0.$$

Because of $G'_u(u) \ge g_0$, $g_0 > 0$, we have

$$\frac{1}{2}\frac{d}{dt}\bigl(|u|^2 + \|u\|^2\bigr) + g_0|Au|^2 \le 0.$$

By the Poincaré inequality, $|Au|^2 > \lambda_1\|u\|^2$, we get

$$\frac{d}{dt}\bigl(|u|^2 + \|u\|^2\bigr) + g_0\lambda_1\|u\|^2 + g_0|Au|^2 \le 0.$$

Let $g_1 = \min\{g_0\lambda_1, g_0\}$, then

$$\frac{d}{dt}\bigl(|u|^2 + \|u\|^2\bigr) + g_1\bigl(\|u\|^2 + |Au|^2\bigr) \le 0. \tag{4.1}$$

Using the Poincaré inequality, $\|u\|^2 > \lambda_1|u|^2$ and $|Au|^2 > \lambda_1\|u\|^2$, (4.1) is changed to

$$\frac{d}{dt}\bigl(|u|^2 + \|u\|^2\bigr) + g_1\lambda_1\bigl(|u|^2 + \|u\|^2\bigr) \le 0.$$

By the Grownwall inequality, we obtain

$$|u|^2 + \|u\|^2 \le \bigl(\bigl|u(0)\bigr|^2 + \bigl\|u(0)\bigr\|^2\bigr)\exp\{-g_1\lambda_1 t\}. \tag{4.2}$$

It is easy to see that $|u(x,t)|$ and $\|u(x,t)\|$ are uniformly bounded from (4.2). In other words, the semi-group $S(t)$ is uniformly bounded in $L^2(\Omega)$ and $H^1(\Omega)$.

Integrating (4.1) over the interval $[t, t+r]$, we have

$$\lim_{s\to+\infty}\int_t^{t+r}\bigl(\bigl\|u(x,s)\bigr\|^2 + \bigl|Au(x,s)\bigr|^2\bigr)\,ds \le \frac{1}{g_1}\bigl(|u_0|^2 + \|u_0\|^2\bigr),$$
$$\int_t^{t+s}\bigl(\bigl\|u(x,s)\bigr\|^2 + \bigl|Au(x,s)\bigr|^2\bigr)\,ds \le \frac{1}{g_1}\bigl(|u_0|^2 + \|u_0\|^2\bigr) \le \frac{\rho_0}{g_1}.$$

If $B(0,\rho)$ is an open ball in $L^2(\Omega)$ and $H^1(\Omega)$ whose radius is ρ, it is easy to calculate that $S(t)u_0 \in B(0,\rho)$ when $t \ge t_0$, $t_0 = \max(-\frac{1}{g_1\lambda_1}\ln\frac{\rho}{\rho_0}, 0)$.

We will make a uniform estimate of (2.1)-(2.3) in $H^2(\Omega)$.

Taking the inner product of (2.1) with Au in Ω, and denoting $F(u,Au) = (B(u,v),Au) + 2(B(v,u),Au) - 3(B(u,u),Au)$, we have

$$\frac{1}{2}\frac{d}{dt}\bigl(\|u\|^2 + |Au|^2\bigr) + g_0|\nabla Au|^2 + F(u,Au) \le 0. \tag{4.3}$$

By computing, $F(u,Au) = 2(B(Au,u),Au) + (B(u,Au),Au)$, through the Agmon inequality, we get

$$\bigl|\bigl(B(Au,u),Au\bigr)\bigr| \le \|\nabla u\|_{L^\infty(\Omega)}\|Au\|^2_{L^2(\Omega)} \le c_5\|u\|^{\frac{1}{2}}|Au|^{\frac{5}{2}},$$
$$\bigl|\bigl(B(u,Au),Au\bigr)\bigr| \le \frac{1}{2}\|\nabla u\|_{L^\infty(\Omega)}\|Au\|^2_{L^2(\Omega)} \le \frac{c_6}{2}\|u\|^{\frac{1}{2}}|Au|^{\frac{5}{2}}.$$

So we have

$$\begin{aligned}\bigl|F(u,Au)\bigr| &\le 2\bigl|\bigl(B(Au,u),Au\bigr)\bigr| + \bigl|\bigl(B(u,Au),Au\bigr)\bigr| \le c_7\|u\|^{\frac{1}{2}}|Au|^{\frac{1}{2}}\bigl(\|u\|^2 + |Au|^2\bigr)\\ &\le \frac{1}{2}g_1\lambda_1\bigl(\|u\|^2 + |Au|^2\bigr) + c_8\|u\||Au|\bigl(\|u\|^2 + |Au|^2\bigr),\end{aligned}$$

where

$$c_7 = \max\left\{2c_5, \frac{c_6}{2}\right\}, \qquad c_8 = \frac{c_7^2}{2g_1\lambda_1}.$$

By the Poincaré inequality, $|\nabla Au|^2 > \lambda_1|Au|^2$, $g_1 = \min\{g_0\lambda_1, g_0\}$, and (4.3) it can be deduced that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\bigl(\|u\|^2 + |Au|^2\bigr) + \frac{g_1}{2}\bigl(|Au|^2 + |\nabla Au|^2\bigr)\\ &\quad\le \frac{1}{2}g_1\lambda_1\bigl(\|u\|^2 + |Au|^2\bigr) + c_8\|u\||Au|\bigl(\|u\|^2 + |Au|^2\bigr).\end{aligned} \tag{4.4}$$

Employing the Poincaré inequality again, we obtain

$$\frac{d}{dt}\bigl(\|u\|^2 + |Au|^2\bigr) \le 2c_8\|u\||Au|\bigl(\|u\|^2 + |Au|^2\bigr).$$

Using the Young inequality, the following inequality can be gotten:

$$\frac{d}{dt}\bigl(\|u\|^2 + |Au|^2\bigr) \le c_8\bigl(\|u\|^2 + |Au|^2\bigr)^2.$$

By denoting $y = \|u\|^2 + |Au|^2$, $g = c_8(\|u\|^2 + |Au|^2)$,

$$\int_t^{t+r} y(s)\,ds \le \frac{1}{g_1}\bigl(|u_0|^2 + \|u_0\|^2\bigr) = \alpha_1, \qquad \int_t^{t+r} g(s)\,ds \le \frac{c_8}{g_1}\bigl(|u_0|^2 + \|u_0\|^2\bigr) = c_8\alpha_1.$$

According to the uniform Grownwall inequality, we get

$$\|u\|^2 + |Au|^2 \le \frac{\alpha_1}{r}\exp\{c_8\alpha_1\}, \quad t > t_0 + r,$$

where r, α_1, c_8 are nonnegative constants. Let $\rho_1 = \frac{\alpha_1}{r}\exp\{c_8\alpha_1\}$, and then $|Au|^2 \le \rho_1$. In other words, $B(0,\rho_1)$ is the attracting set of $S(t)$ in $H^2(\Omega)$. This completes the proof of Theorem 2. □

5 The existence of global attractor

Theorem 3 *If* $u_0 \in \mathrm{V}$, *the semi-group of the solution* $S(t)$ *to* (2.1)-(2.3) *has a global attractor in* $H^2(\Omega)$.

Proof Based on the proof of Theorem 2, we only need to prove that $S(t)$ is a completely continuous operator, thus the existence of global attractor can be proved.

Taking the inner product of (2.1) with $t^2\Delta Au$ in Ω, furthermore, according to integration by parts and the Green formula, we have

$$\begin{aligned}&\left(\frac{dv}{dt}, t^2\Delta Au\right) + \bigl(A^2G(u), t^2\Delta Au\bigr) + \bigl(B(u,v), t^2\Delta Au\bigr)\\ &\quad + 2\bigl(B(v,u), t^2\Delta Au\bigr) - 3\bigl(B(u,u), t^2\Delta Au\bigr) = 0,\\ &\left(\frac{dv}{dt}, t^2\Delta Au\right) = -\frac{1}{2}\frac{d}{dt}\bigl(|tAu|^2 + |t\nabla Au|^2\bigr) + \bigl(\bigl|t^{\frac{1}{2}}Au\bigr|^2 + \bigl|t^{\frac{1}{2}}\nabla Au\bigr|^2\bigr).\end{aligned} \tag{5.1}$$

By the assumption of $G'_u(u) \ge g_0$, $g_0 > 0$, we can get

$$\bigl(A^2G(u), t^2\Delta Au\bigr) \le -g_0|t\Delta Au|^2,$$

$$\begin{aligned}
&\bigl(B(u,v),t^2\Delta Au\bigr)+2\bigl(B(v,u),t^2\Delta Au\bigr)-3\bigl(B(u,u),t^2\Delta Au\bigr)\\
&\quad=\bigl(B(u,Au),t^2\Delta Au\bigr)+2\bigl(B(Au,u),t^2\Delta Au\bigr)=-\frac{5}{2}\int_\Omega t^2u_xu_{xxx}^2\,dx
\end{aligned}$$

and through the Agmon inequality and the Poincaré inequality, we obtain

$$\begin{aligned}
&\bigl|\bigl(B(u,v),t^2\Delta Au\bigr)+2\bigl(B(v,u),t^2\Delta Au\bigr)-3\bigl(B(u,u),t^2\Delta Au\bigr)\bigr|\\
&\quad=\left|\frac{5}{2}\int_\Omega t^2u_xu_{xxx}^2\,dx\right|\le\frac{5}{2}\left|\int_\Omega t^2u_xu_{xxx}^2\,dx+\int_\Omega t^2u_xu_{xx}^2\,dx\right|\\
&\quad\le\frac{5}{2}c_9\|u\|^{\frac{1}{2}}|Au|^{\frac{1}{2}}\bigl(|tAu|^2+|t\nabla Au|^2\bigr).
\end{aligned}$$

By (5.1), we can get the following inequality:

$$\begin{aligned}
&\frac{d}{dt}\bigl(|tAu|^2+|t\nabla Au|^2\bigr)+2g_0|t\Delta Au|^2\\
&\quad\le 5c_9\|u\|^{\frac{1}{2}}|Au|^{\frac{1}{2}}\bigl(|tAu|^2+|t\nabla Au|^2\bigr)+2\bigl(\bigl|t^{\frac{1}{2}}Au\bigr|^2+\bigl|t^{\frac{1}{2}}\nabla Au\bigr|^2\bigr).
\end{aligned}$$

Based on the Poincaré inequality: $|t\Delta Au|^2>\lambda_1|t\nabla Au|^2$, $|t\nabla Au|^2>\lambda_1|tAu|^2$, $g_1=\min\{g_0\lambda_1, g_0\}$, and the Young inequality, we have

$$\begin{aligned}
&\frac{d}{dt}\bigl(|tAu|^2+|t\nabla Au|^2\bigr)+g_1\lambda_1\bigl(|tAu|^2+|t\nabla Au|^2\bigr)\\
&\quad\le 5c_9\|u\|^{\frac{1}{2}}|Au|^{\frac{1}{2}}\bigl(|tAu|^2+|t\nabla Au|^2\bigr)+2\bigl(\bigl|t^{\frac{1}{2}}Au\bigr|^2+\bigl|t^{\frac{1}{2}}\nabla Au\bigr|^2\bigr)\\
&\quad\le g_1\lambda_1\bigl(|tAu|^2+|t\nabla Au|^2\bigr)+c_{10}\|u\||Au|\bigl(|tAu|^2+|t\nabla Au|^2\bigr)\\
&\qquad+c_{11}\bigl(|Au|^2+|\nabla Au|^2\bigr),
\end{aligned}\tag{5.2}$$

where

$$c_{10}=\frac{25c_9^2}{4g_1\lambda_1},\qquad c_{11}=\frac{8}{g_1\lambda_1}.$$

By (4.4), the following inequality can be obtained:

$$\begin{aligned}
&\frac{d}{dt}\bigl(\|u\|^2+|Au|^2\bigr)+g_1\bigl(|Au|^2+|\nabla Au|^2\bigr)\le g_1\lambda_1\bigl(\|u\|^2+|Au|^2\bigr)+c_8\bigl(\|u\|^2+|Au|^2\bigr)^2,\\
&\quad t\ge t_0.
\end{aligned}$$

Integrating the above inequality over the interval $[t,t+r]$, we get

$$\int_t^{t+r}\bigl(\bigl|Au(x,s)\bigr|^2+\bigl|\nabla Au(x,s)\bigr|^2\bigr)\,ds\le\left(\lambda_1\rho_1+\frac{c_8\rho_1^2}{g_1}\right)r+\frac{\rho_1}{g_1}.$$

Equation (5.2) can be rewritten as follows:

$$\frac{d}{dt}\bigl(|tAu|^2+|t\nabla Au|^2\bigr)\le c_{10}\|u\||Au|\bigl(|tAu|^2+|t\nabla Au|^2\bigr)+c_{11}\bigl(|Au|^2+|\nabla Au|^2\bigr).$$

By denoting $(\lambda_1\rho_1 + \frac{c_8\rho_1^2}{g_1})r + \frac{\rho_1}{g_1} = \alpha_2(\lambda_1, \rho_1, g_1)$, we have

$$\begin{aligned}\int_t^{t+r} c_{10}\|u(x,s)\|\,|Au(x,s)|\,ds &\le \frac{c_{10}}{2}\int_t^{t+r}\left(\|u(x,s)\|^2 + |Au(x,s)|^2\right)ds\\ &\le \frac{c_{10}\rho_0}{2g_1} \triangleq \alpha_3(\rho_0, g_1),\end{aligned}$$

$$\int_t^{t+r}\left(|sAu(x,s)|^2 + |s\nabla Au(x,s)|^2\right) \le (t+r)^2\alpha_2 \triangleq \alpha_4(\lambda_1, \rho_1, g_1).$$

By the uniform Gronwall inequality, we have

$$|tAu|^2 + |t\nabla Au|^2 \le \left(\frac{\alpha_4}{r} + c_{11}\alpha_2\right)\exp(\alpha_3).$$

Let $(\frac{\alpha_4}{r} + c_{11}\alpha_2)\exp(\alpha_3) = E(\lambda_1, \rho_1, g_1, t)$, then we can obtain $|\nabla Au| < \frac{E(\lambda_1,\rho_1,g_1,t)}{t}$.

Therefore, we can conclude that $S(t)$ is equicontinuous. From the Ascoli-Arzela theorem, $S(t)$ is a completely continuous operator. Thus, we have proved that $S(t)$ has a global attractor in $H^2(\Omega)$. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
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References

1. Camassa, R, Holm, D: An integrable shallow water equation with peaked solitons. Phys. Rev. Lett. **71**, 1661-1664 (1993)
2. Constantin, A, Escher, J: Global existence and blow-up for a shallow water equation. Ann. Sc. Norm. Super. Pisa **26**, 303-328 (1998)
3. Ding, D, Tian, L: The study on solution of Camassa-Holm equation with weak dissipation. Commun. Pure Appl. Anal. **5**, 483-493 (2006)
4. Lai, S, Li, N, Wu, Y: The existence of global weak solutions for a weakly dissipative Camassa-Holm equation in $H^1(R)$. Bound. Value Probl. **2013**, 26 (2013)
5. Dai, H: Model equations for nonlinear dispersive waves in a compressible Mooney-Rivlin rod. Acta Mech. **127**, 193-207 (1998)
6. Dai, H: Exact travelling-wave solutions of an integrable equation arising in hyperelastic rod. Wave Motion **28**, 367-381 (1998)
7. Dai, H, Huo, Y: Solitary waves in an inhomogeneous rod composed of a general hyperelastic material. Wave Motion **35**, 55-69 (2002)
8. Zhou, Y: Blow-up of solutions to a nonlinear dispersive rod equation. Calc. Var. Partial Differ. Equ. **25**, 63-77 (2005)
9. Zhou, Y: Blow-up phenomenon for a periodic rod equation. Phys. Lett. A **353**, 479-486 (2006)
10. Zhou, Y: Local well-posedness and blow-up criteria of solutions for a rod equation. Math. Nachr. **278**, 1726-1739 (2005)
11. Karapetyan, D: Non-uniform dependence and well-posedness for the hyperelastic rod equation. J. Differ. Equ. **249**, 796-826 (2010)

12. Mustafa, O: Global conservative solutions of the hyperelastic rod equation. Int. Math. Res. Not. (2007). doi:10.1093/imrn/rnm040
13. Coclite, G, Holden, H, Karlsen, K: Global weak solutions to a generalized hyperelastic-rod wave equation. SIAM J. Math. Anal. **37**, 1044-1069 (2005)
14. Coclite, G, Holden, H, Karlsen, K: Well-posedness for a parabolic-elliptic system. Discrete Contin. Dyn. Syst. **13**, 659-682 (2005)
15. Benjamin, T, Bona, J, Mahony, J: Model equations for long waves in nonlinear dispersive systems. Philos. Trans. R. Soc. Lond. Ser. A **272**, 47-78 (1972)
16. Bona, J, Tzvetkov, N: Sharp well-posedness results for the BBM equation. Discrete Contin. Dyn. Syst. **23**, 1241-1252 (2009)
17. Lenells, J: Traveling wave solutions of the Degasperis-Procesi equation. J. Math. Anal. Appl. **306**, 72-82 (2005)
18. Henry, D: Infinite propagation speed for the Degasperis-Procesi equation. J. Math. Anal. Appl. **311**, 755-759 (2005)
19. Escher, J, Liu, Y, Yin, Z: Global weak solution and blow-up structure for the Degasperis-Procesi equation. J. Funct. Anal. **241**, 457-485 (2006)
20. Tian, L, Fan, J: The attractor on viscosity Degasperis-Procesi equation. Nonlinear Anal., Real World Appl. **9**, 1461-1473 (2008)
21. Johnson, R: Camassa-Holm, Korteweg-de Vries and related models for water waves. J. Fluid Mech. **455**, 63-82 (2002)
22. Holden, H, Raynaud, X: Global conservative solutions of the generalized hyperelastic-rod wave equation. J. Differ. Equ. **233**, 448-484 (2007)
23. Cohen, D, Raynaud, X: Geometric finite difference schemes for the generalized hyperelastic-rod wave equation. J. Comput. Appl. Math. **235**, 1925-1940 (2011)
24. Lai, S, Wu, Y: The study of global weak solutions for a generalized hyperelastic-rod wave equation. Nonlinear Anal. **80**, 96-108 (2013)
25. Chen, C, Shi, L, Wang, H: Existence of global attractors in L^p for m-Laplacian parabolic equation in R^N. Bound. Value Probl. **2009**, 563767 (2009)
26. Ding, D, Tian, L: The attractor in dissipative Camassa-Holm equation. Acta Math. Appl. Sin. **27**, 536-545 (2004)
27. Zeng, W, Lu, X, Liu, Q: Blow-up profile for a degenerate parabolic equation with a weighted localized term. Bound. Value Probl. **2013**, 269 (2013)

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