Boundary Value Problems
a SpringerOpen Journal

RESEARCH Open Access

On the oscillation of odd order advanced differential equations

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Dedicated to Professor Ivan T Kiguradze.

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Abstract

The aim of this paper is to study the asymptotic properties and oscillation of the *n*th order advanced differential equations

$$\left(r(t)\left[x^{(n-1)}(t)\right]^{\gamma}\right)' + q(t)x^{\gamma}\left[\tau(t)\right] = 0.$$

The results obtained are based on the Riccati transformation.

MSC: 34K11; 34C10

Keywords: *n*th order differential equations; Riccati transformation; oscillation

1 Introduction

In this paper, we shall study the asymptotic and oscillation behavior of the solutions of the higher order advanced differential equations

$$\left(r(t)\left[x^{(n-1)}(t)\right]^{\gamma}\right)' + q(t)x^{\gamma}\left[\tau(t)\right] = 0. \tag{1.1}$$

Throughout the paper, we assume $q, \tau \in C([t_0, \infty))$, $r \in C^1([t_0, \infty))$ and

(H_1) n is odd, γ is the ratio of two positive odd integers,
(H_2) $r(t) > 0$, $r'(t) \geq 0$, $q(t) > 0$, $\tau(t) \geq t$.

Whenever, it is assumed

$$R(t) = \int_{t_0}^{t} r^{-1/\gamma}(s)\,\mathrm{d}s \to \infty \quad \text{as } t \to \infty. \tag{1.2}$$

By a solution of Eq. (1.1), we mean a function $x(t) \in C^{n-1}([T_x, \infty))$, $T_x \geq t_0$, which has the property $r(t)(x^{(n-1)}(t))^{\gamma} \in C^1([T_x, \infty))$ and satisfies Eq. (1.1) on $[T_x, \infty)$. We consider only those solutions $x(t)$ of (1.1) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for all $T \geq T_x$. We assume that (1.1) possesses such a solution. A solution of (1.1) is called oscillatory if it has arbitrarily large zeros on $[T_x, \infty)$, and otherwise it is called to be nonoscillatory.

The problem of the oscillation of differential equations has been widely studied by many authors who have provided many techniques especially for lower order delay differential equations. Dong in [1] improved and extended the Riccati transformation to obtain new oscillatory criteria for the second order delay differential equations

$$\big[r(t)\big[x'(t)\big]^{\gamma}\big]' + q(t)x^{\gamma}\big[\tau(t)\big] = 0.$$

Grace *et al.* in [2] and the present authors in [3–6] used the comparison technique for the third order delay differential equation

$$\big[r(t)\big[x''(t)\big]^{\gamma}\big]' + q(t)x^{\gamma}\big[\tau(t)\big] = 0$$

that was compared with the oscillation of certain first order differential equation.

On the other hand, there are comparatively less methods established for the advanced differential equations. The aim of the paper is to fill this gap in the oscillation theory.

Remark 1 All functional inequalities considered in this paper are assumed to hold eventually, that is, they are satisfied for all t large enough.

2 Main results

Our results essentially use the following estimate which is due to Philos and Staikos (see [7] and [8]).

Lemma A *Let $z \in C^j([t_0,\infty))$. Assume that $z^{(j)}$ is of fixed sign and not identically zero on a subray of $[t_0,\infty)$. If, moreover, $z(t) > 0$, $z^{(j-1)}(t)z^{(j)}(t) \le 0$, and $\lim_{t\to\infty} z(t) \ne 0$, then for every $k \in (0,1)$ there exists $t_k \ge t_0$ such that*

$$z(t) \ge \frac{k}{(j-1)!} t^{j-1} \big|z^{(j-1)}(t)\big| \tag{2.1}$$

holds on $[t_k,\infty)$.

The following useful result will be used later in the proofs of our main results.

Lemma 1 *Assume $x(t) > 0$, $x'(t) > 0$, $x''(t) > 0$, eventually. Then, for arbitrary $k_0 \in (0,1)$,*

$$x\big[\tau(t)\big] \ge k_0 \frac{\tau(t)}{t} x(t), \tag{2.2}$$

eventually.

Proof It follows from the monotonicity of $x'(t)$ that

$$x\big[\tau(t)\big] - x(t) = \int_t^{\tau(t)} x'(s)\,\mathrm{d}s \ge x'(t)\big(\tau(t) - t\big).$$

That is,

$$\frac{x[\tau(t)]}{x(t)} \ge 1 + \frac{x'(t)}{x(t)}\big(\tau(t) - t\big). \tag{2.3}$$

On the other hand, since $x(t) \to \infty$ as $t \to \infty$, then for any $k_0 \in (0,1)$ there exists t_1 large enough such that

$$k_0 x(t) \le x(t) - x(t_1) = \int_{t_1}^{t} x'(s)\,\mathrm{d}s \le x'(t)(t - t_1) \le x'(t)t,$$

or equivalently,

$$\frac{x'(t)}{x(t)} \ge \frac{k_0}{t}. \tag{2.4}$$

Using (2.4) in (2.3), we obtain

$$\frac{x[\tau(t)]}{x(t)} \ge 1 + \frac{k_0}{t}\big(\tau(t) - t\big) \ge k_0 \frac{\tau(t)}{t}.$$

The proof is complete. □

The positive solutions of (1.1) have the following structure.

Lemma 2 *If $x(t)$ is a positive solution of* (1.1), *then $r(t)[x^{(n-1)}(t)]^{\gamma}$ is decreasing, all derivatives $x^{(i)}(t)$, $1 \le i \le n-1$, are of constant signs, and $x(t)$ satisfies either*

$$x'(t) > 0, \qquad x''(t) > 0, \qquad x^{(n-1)}(t) > 0, \qquad x^{(n)}(t) < 0 \tag{2.5}$$

or

$$(-1)^i x^{(i)}(t) > 0, \quad i = 1, 2, \ldots, n. \tag{2.6}$$

Proof Since $x(t)$ is a positive solution of (1.1), then it follows from (1.1) that

$$\big(r(t)\big[x^{(n-1)}(t)\big]^{\gamma}\big)' = -q(t)x^{\gamma}\big[\tau(t)\big] < 0.$$

Thus, $r(t)[x^{(n-1)}(t)]^{\gamma}$ is decreasing, which implies that either $x^{(n-1)}(t) > 0$ or $x^{(n-1)}(t) < 0$. But the case $x^{(n-1)}(t) < 0$ implies $r(t)[x^{(n-1)}(t)]^{\gamma} < -M < 0$. An integration from t_1 to t yields

$$x^{(n-2)}(t) < x^{(n-2)}(t_1) - M^{1/\gamma} \int_{t_1}^{t} r^{-1/\gamma}(s)\,\mathrm{d}s,$$

but in view of (1.2) $x^{(n-2)}(t) \to -\infty$ for $t \to \infty$. Repeating this procedure, we obtain that $x(t) \to -\infty$ and this is a contradiction, and we conclude that $x^{(n-1)}(t) > 0$. Moreover, $x^{(n-1)}(t) > 0$ implies that either $x^{(n-2)}(t) > 0$ or $x^{(n-2)}(t) < 0$, but the first case leads to $x^{(i)}(t) > 0$ for $0 \le i \le n-2$. Repeating these considerations, we verify that $x(t)$ satisfies either (2.5) or (2.6).

On the other hand, since $x^{(n-1)}(t) > 0$, then using $r'(t) > 0$ in

$$0 > \big(r(t)\big[x^{(n-1)}(t)\big]^{\gamma}\big)' = r'(t)\big[x^{(n-1)}(t)\big]^{\gamma} + r(t)\gamma\big[x^{(n-1)}(t)\big]^{\gamma-1}x^{(n)}(t),$$

we conclude that $x^{(n)}(t) < 0$. The proof is complete. □

Now, we offer some criteria for certain asymptotic behavior of all nonoscillatory solutions. For our further references, we set

$$Q(t) = \int_t^\infty q(s)\left(\frac{\tau(s)}{s}\right)^\gamma \mathrm{d}s$$

and

$$P(t) = \frac{1}{r^{1/\gamma}(t)}\left[\int_t^\infty q(s)\,\mathrm{d}s\right]^{1/\gamma}.$$

Theorem 1 *Assume that*

$$\liminf_{t\to\infty} \frac{1}{Q(t)} \int_t^\infty \frac{s^{n-2}Q^{1+1/\gamma}(s)}{r^{1/\gamma}(s)}\,\mathrm{d}s > \frac{(n-2)!}{(\gamma+1)^{1+1/\gamma}} \tag{2.7}$$

and

$$\int_{t_0}^\infty s^{n-2}P(s)\,\mathrm{d}s = \infty, \tag{2.8}$$

then every nonoscillatory solution $x(t)$ *of* (1.1) *satisfies* $\lim_{t\to\infty} x(t) = 0$.

Proof Assume that $x(t)$ is an eventually positive solution of (1.1). First assume that $x(t)$ satisfies (2.5). By (2.7), it is easy to see that there exists some $k \in (0,1)$ such that

$$\liminf_{t\to\infty} \frac{k^{1+1/\gamma}}{Q(t)} \int_t^\infty \frac{s^{n-2}Q^{1+1/\gamma}(s)}{r^{1/\gamma}(s)}\,\mathrm{d}s > \frac{(n-2)!}{(\gamma+1)^{1+1/\gamma}}. \tag{2.9}$$

We put $k_0 = k^{1/\gamma}$, then setting (2.2) into (1.1), we get

$$\bigl(r(t)\bigl[x^{(n-1)}(t)\bigr]^\gamma\bigr)' + kq(t)\frac{\tau^\gamma(t)}{t^\gamma}x^\gamma(t) \le 0.$$

We define

$$w(t) = \frac{r(t)[x^{(n-1)}(t)]^\gamma}{x^\gamma(t)} > 0. \tag{2.10}$$

Differentiating $w(t)$, one gets

$$\begin{aligned} w'(t) &= \frac{[r(t)[x^{(n-1)}(t)]^\gamma]'}{x^\gamma(t)} - \gamma\frac{r(t)[x^{(n-1)}(t)]^\gamma}{x^\gamma(t)}\frac{x'(t)}{x(t)} \\ &\le -\frac{kq(t)\tau^\gamma(t)}{t^\gamma} - \gamma w(t)\frac{x'(t)}{x(t)}. \end{aligned} \tag{2.11}$$

On the other hand, Lemma A implies

$$x'(t) \ge \frac{k}{(n-2)!}t^{n-2}x^{(n-1)}(t).$$

Setting the last inequality into (2.11), we obtain

$$w'(t) \le -k\left[q(t)\left(\frac{\tau(t)}{t}\right)^{\gamma} + \gamma w^{1+1/\gamma}(t)\frac{t^{n-2}}{(n-2)!r^{1/\gamma}(t)}\right].$$

Integrating the last inequality from t to ∞, we have

$$w(t) \ge k\left[Q(t) + \frac{\gamma}{(n-2)!}\int_t^{\infty} w^{1+1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s\right] \tag{2.12}$$

or

$$\frac{w(t)}{kQ(t)} \ge 1 + \frac{\gamma k^{1+1/\gamma}}{(n-2)!Q(t)}\int_t^{\infty}\frac{s^{n-2}}{r^{1/\gamma}(s)}Q^{1+1/\gamma}(s)\left(\frac{w(s)}{kQ(s)}\right)^{1+1/\gamma}\mathrm{d}s,$$

eventually, let us say $t \ge t_1$. Since $w(t) > kQ(t)$, then

$$\inf_{t\ge t_1}\frac{w(t)}{kQ(t)} = \lambda \ge 1.$$

Thus,

$$\frac{w(t)}{kQ(t)} \ge 1 + \frac{\gamma (k\lambda)^{1+1/\gamma}}{(n-2)!Q(t)}\int_t^{\infty}\frac{s^{n-2}}{r^{1/\gamma}(s)}Q^{1+1/\gamma}(s)\,\mathrm{d}s. \tag{2.13}$$

From (2.9), we see that there exists some positive η such that

$$\frac{k^{1+1/\gamma}}{(n-2)!Q(t)}\int_t^{\infty}\frac{s^{n-2}}{r^{1/\gamma}(s)}Q^{1+1/\gamma}(s)\,\mathrm{d}s > \eta > (\gamma+1)^{-\frac{\gamma+1}{\gamma}}. \tag{2.14}$$

Combining (2.13) together with (2.14), we have

$$\frac{w(t)}{kQ(t)} \ge 1 + \gamma\lambda^{1+1/\gamma}\eta.$$

Therefore,

$$\lambda \ge 1 + \gamma\lambda^{1+1/\gamma}\eta > 1 + \gamma\lambda^{1+1/\gamma}(\gamma+1)^{-\frac{\gamma+1}{\gamma}}$$

or equivalently,

$$0 > \frac{1}{\gamma+1} + \frac{\gamma}{\gamma+1}\left(\frac{\lambda}{\gamma+1}\right)^{1+1/\gamma} - \frac{\lambda}{\gamma+1}.$$

This contradicts the fact that the function

$$f(\alpha) = \frac{1}{\gamma+1} + \frac{\gamma}{\gamma+1}\alpha^{1+1/\gamma} - \alpha$$

is nonnegative for all $\alpha > 0$, and we conclude that $x(t)$ cannot satisfy (2.5).

Now we assume that $x(t)$ satisfies (2.6). Then there exists a finite $\lim_{t\to\infty} x(t) = \ell$. We claim that $\ell = 0$. Assume that $\ell > 0$. Integrating (1.1) from t to ∞, we obtain

$$r(t)\big(x^{(n-1)}(t)\big)^{\gamma} \ge \int_t^{\infty} q(s)x^{\gamma}\big[\tau(s)\big]\,\mathrm{d}s \ge \ell^{\gamma}\int_t^{\infty} q(s)\,\mathrm{d}s,$$

which implies

$$x^{(n-1)}(t) \geq \ell P(t).$$

Integrating the last inequality twice from t to ∞, we get

$$x^{(n-3)}(t) \geq \ell \int_t^\infty \int_u^\infty P(s)\,\mathrm{d}s\,\mathrm{d}u = \ell \int_t^\infty P(s)(s-t)\,\mathrm{d}s.$$

Repeating this procedure, we arrive at

$$-x'(t) \geq \frac{\ell}{(n-3)!} \int_t^\infty (s-t)^{n-3} P(s)\,\mathrm{d}s.$$

Now, integrating from t_1 to ∞, we see that

$$x(t_1) \geq \frac{\ell}{(n-2)!} \int_{t_1}^\infty (s-t_1)^{n-2} P(s)\,\mathrm{d}s \geq \frac{\ell}{2^{n-2}(n-2)!} \int_{2t_1}^\infty s^{n-2} P(s)\,\mathrm{d}s,$$

which contradicts (2.8), and so we have verified that $\lim_{t\to\infty} x(t) = 0$. □

Example 1 Consider the odd order ($n \geq 3$) nonlinear differential equation

$$\bigl(t\bigl(x^{(n-1)}(t)\bigr)^3\bigr)' + \frac{\beta}{t^{3n-3}} x^3(\lambda t) = 0, \quad \beta > 0, \lambda > 1. \tag{2.15}$$

Here $q(t) = \beta/t^{3n-3}$ and $\tau(t) = \lambda t$, so that

$$Q(t) = \int_t^\infty q(s)\left(\frac{\tau(s)}{s}\right)^3 \mathrm{d}s = \frac{\lambda^3 \beta}{(3n-4)t^{3n-4}},$$

$$P(t) = \frac{1}{r^{1/3}(t)} \left[\int_t^\infty q(s)\,\mathrm{d}s\right]^{1/3} = \left(\frac{\beta}{3n-4}\right)^{1/3} \frac{1}{t^{n-1}}.$$

Consequently,

$$\int_{t_0}^\infty s^{n-2} P(s)\,\mathrm{d}s = \left(\frac{\beta}{3n-4}\right)^{1/3} \int_{t_0}^\infty \frac{1}{s}\,\mathrm{d}s = \infty,$$

i.e., (2.8) holds; moreover, (2.7) reduces to

$$\lambda \beta^{1/3} > \left(\frac{3n-4}{4}\right)^{4/3} (n-2)!,$$

which, by Theorem 1, guarantees that all nonoscillatory solutions of (2.15) tend to zero at infinity.

Let $\{A_m(t)\}_{m=0}^\infty$ be a sequence of continuous functions defined as follows,

$$A_0(t) = kQ(t), \quad k \in (0,1) \text{ fixed}$$

and

$$A_{m+1}(t) = A_0(t) + \frac{k\gamma}{(n-2)!}\int_t^\infty A_m^{1+1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s, \quad m = 0,1,\ldots. \tag{2.16}$$

Then we have the following result.

Theorem 2 *Assume that* (2.8) *holds and there exists some* $A_m(t)$ *such that*

$$\int_{t_0}^\infty q(t)\left(\frac{\tau(t)}{t}\right)^\gamma \exp\left(\frac{k\gamma}{(n-2)!}\int_{t_0}^t A_m^{1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s\right)\mathrm{d}t = \infty \tag{2.17}$$

for some $k \in (0,1)$. *Then every nonoscillatory solution* $x(t)$ *of* (1.1) *satisfies* $\lim_{t\to\infty} x(t) = 0$.

Proof Assume that $x(t)$ is an eventually positive solution of (1.1). By Lemma 2, $x(t)$ satisfies either (2.5) or (2.6). It follows from the proof of Theorem 1 that if $x(t)$ satisfies (2.6), then (2.8) insures that it tends to zero at infinity.

Assume that $x(t)$ satisfies (2.5). It follows from the proof of Theorem 1 that (2.12) holds for every $k \in (0,1)$.

By induction, using (2.12), it is easy to see that the sequence $\{A_m(t)\}_{m=0}^\infty$ is nondecreasing and $w(t) \ge A_m(t)$. Thus the sequence $\{A_m(t)\}_{m=0}^\infty$ converges to $A(t)$. By the Lebesgue monotone convergence theorem and letting $m \to \infty$ in (2.16), we get

$$A(t) = A_0(t) + \frac{k\gamma}{(n-2)!}\int_t^\infty A^{1+1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s,$$

which in view of $A(t) \ge A_m(t)$ implies

$$\begin{aligned} A'(t) &= -kq(t)\left(\frac{\tau(t)}{t}\right)^\gamma - \frac{k\gamma}{(n-2)!}A^{1+1/\gamma}(t)\frac{t^{n-2}}{r^{1/\gamma}(t)} \\ &\le -kq(t)\left(\frac{\tau(t)}{t}\right)^\gamma - \frac{k\gamma}{(n-2)!}A(t)A_m^{1/\gamma}(t)\frac{t^{n-2}}{r^{1/\gamma}(t)}, \end{aligned}$$

eventually, let us say $t \ge t_1$. Therefore,

$$\begin{aligned} &\left[A(t)\exp\left(\frac{k\gamma}{(n-2)!}\int_{t_1}^t A_m^{1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s\right)\right]' \\ &\quad\le -kq(t)\left(\frac{\tau(t)}{t}\right)^\gamma \exp\left(\frac{k\gamma}{(n-2)!}\int_{t_1}^t A_m^{1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s\right). \end{aligned}$$

An integration from t_1 to t yields

$$\begin{aligned} 0 &\le A(t)\exp\left(\frac{k\gamma}{(n-2)!}\int_{t_1}^t A_m^{1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s\right) \\ &\le A(t_1) - k\int_{t_1}^t q(u)\left(\frac{\tau(u)}{u}\right)^\gamma \exp\left(\frac{k\gamma}{(n-2)!}\int_{t_1}^u A_m^{1/\gamma}(s)\frac{s^{n-2}}{r^{1/\gamma}(s)}\,\mathrm{d}s\right)\mathrm{d}u. \end{aligned}$$

Letting $t \to \infty$, we obtain a contradiction. The proof is complete. □

Theorem 3 *Assume that* (2.8) *holds and there exist some* $k \in (0,1)$ *and* $A_m(t)$ *such that*

$$k \limsup_{t\to\infty} \frac{t^{(n-1)\gamma}}{r(t)} A_m(t) > \big((n-1)!\big)^\gamma. \tag{2.18}$$

Then every nonoscillatory solution $x(t)$ *of* (1.1) *satisfies* $\lim_{t\to\infty} x(t) = 0$.

Proof Assume that $x(t)$ is an eventually positive solution of (1.1) satisfying (2.5). It follows from Lemma A that

$$x(t) \ge \frac{k^{1/\gamma}}{(n-1)!} t^{n-1} x^{(n-1)}(t),$$

eventually, where $k \in (0,1)$ is the same as in $A_m(t)$. Then

$$\frac{1}{w(t)} = \frac{1}{r(t)} \left(\frac{x(t)}{x^{(n-1)}(t)}\right)^\gamma \ge \frac{1}{r(t)} \frac{k}{((n-1)!)^\gamma} t^{(n-1)\gamma},$$

or equivalently,

$$\big((n-1)!\big)^\gamma \ge k \frac{t^{(n-1)\gamma}}{r(t)} w(t) \ge k \frac{t^{(n-1)\gamma}}{r(t)} A_m(t),$$

which contradicts (2.18). □

Letting $m = 0$ in Theorem 3, we have the following result.

Corollary 1 *Assume that* (2.8) *holds and*

$$\limsup_{t\to\infty} \frac{t^{(n-1)\gamma}}{r(t)} \int_t^\infty q(s) \left(\frac{\tau(s)}{s}\right)^\gamma \mathrm{d}s > \big((n-1)!\big)^\gamma. \tag{2.19}$$

Then every nonoscillatory solution $x(t)$ *of* (1.1) *satisfies* $\lim_{t\to\infty} x(t) = 0$.

Proof It follows from (2.19) that there exists some $k \in (0,1)$ such that

$$k^2 \limsup_{t\to\infty} \frac{t^{(n-1)\gamma}}{r(t)} \int_t^\infty q(s) \left(\frac{\tau(s)}{s}\right)^\gamma \mathrm{d}s > \big((n-1)!\big)^\gamma,$$

which is equivalent to

$$k \limsup_{t\to\infty} \frac{t^{(n-1)\gamma}}{r(t)} A_0(t) > \big((n-1)!\big)^3.$$

The assertion now follows from Theorem 3. □

Example 2 Consider the third order nonlinear differential equation

$$\left(t^2 \big(x^{(n-1)}(t)\big)^3\right)' + \frac{\beta}{t^{3n-4}} x^3(\lambda t) = 0, \quad \beta > 0, \lambda \ge 1, t \ge 1. \tag{2.20}$$

A simple calculation leads to

$$Q(t) = \frac{\lambda^3\beta}{(3n-5)t^{3n-5}}, \qquad P(t) = \left(\frac{\beta}{3n-5}\right)^{1/3}\frac{1}{t^{n-1}}.$$

Then (2.8) holds and (2.19) reduces to

$$\beta\lambda^3 > (3n-5)\bigl((n-1)!\bigr)^3,$$

and thus, by Corollary 1, every nonoscillatory solution $x(t)$ of (2.20) tends to zero as $t \to \infty$.

Our results are based on Lemma 1, *i.e.*, we essentially utilize the estimate (2.2). It is easy to see that for $x(t) = t^{1/2}$ and $\tau(t) = 2t$, estimate (2.2) does not hold, that is, for

$$x(t) > 0, \qquad x'(t) > 0, \qquad x''(t) < 0, \tag{2.21}$$

relationship (2.2) fails, and so our result here cannot be applied for n even. Hence, it remains an open problem how to obtain the corresponding results also for n even.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Received: 18 January 2014 Accepted: 5 September 2014 Published online: 26 September 2014

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doi:10.1186/s13661-014-0214-3
Cite this article as: Baculíková and Džurina: On the oscillation of odd order advanced differential equations. *Boundary Value Problems* 2014 **2014**:214.