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Existence of random attractors for the floating beam equation with strong damping and white noise

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Abstract
In this paper, we investigate the existence of a compact random attractor for the random dynamical system generated by a model for nonlinear oscillations in a floating beam equation with strong damping and white noise.

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Keywords: floating beam equation; random dynamical system; random attractors

1 Introduction

In this paper, we consider the following stochastic floating beam equations:

$$\begin{cases} u_{tt} + \triangle^2 u + \triangle^2 u_t + bu^+ + f(u) = q(x)\dot{W}, & \text{in } \Omega \times [\tau, +\infty), \tau \in \mathbb{R}, \\ \triangle u(x,t) = \nabla \triangle u(x,t) = 0, & x \in \partial\Omega, t \geq \tau, \\ u(x,\tau) = u_0(x), & u_t(x,\tau) = u_1(x), \end{cases} \tag{1.1}$$

where $b > 0$ is a measure of the cross section of the floating beam, Ω is an open bounded subset of $\mathbb{R}^2$ with sufficiently smooth boundary $\partial\Omega$. $u = u(x,t)$ represents the depth of the bottom of the floating beam as it floats, $u^+ = u$ for $u \geq 0$ and $u^+ = 0$ for $u < 0$. $q(x) \in H^3(\Omega)$ is not identically equal to zero, f is a nonlinear function satisfying certain conditions. $\dot{W}$ is the derivative of a one-dimensional two-sided real-valued Wiener process $W(t)$ and $q(x)\dot{W}$ formally describes white noise.

We assume that the nonlinear function $f \in C^2(\mathbb{R},\mathbb{R})$ with $f(0) = 0$, which satisfies the following assumptions.

(a) Growth conditions:

$$|f(s)| \leq C_0(1 + |s|^p), \quad p \geq 1, \forall s \in \mathbb{R}, \tag{1.2}$$

where C_0 is a positive constant. For example, obviously, $f(s) = |s|^{p-1}s$ satisfies (1.2).

(b) Dissipation conditions:

$$F(s) := \int_0^s f(r)\,dr \geq C_1(|s|^{p+1} - 1), \quad p \geq 1, \forall s \in \mathbb{R} \tag{1.3}$$

and

$$sf(s) \geq C_2\big(F(s) - 1\big), \quad \forall s \in \mathbb{R}, \tag{1.4}$$

where C_1, C_2 are positive constants.

When $f(u) \equiv 0$ and $q(x) \equiv 0$, equation (1.1) is regarded as a model of naval structures, which is originally in [1] introduced by Lazer and McKenna. In the actual problems, it can be presented as ships, submarines, hovercraft, gliders *etc.* To the best of our knowledge, the author investigated the existence of a global attractor for the deterministic floating beam in [2], that is, the 'noise' is absent in (1.1). Until now, we find that no one else has studied the long-time behavior of the solutions about these problems, it is just our interest in this paper. As far as the other related problems are concerned, we refer the reader to [3–10] and the references therein.

It is well known that Crauel and Flandoli originally introduced the random attractor for the infinite-dimensional RDS [11, 12]. A random attractor of RDS is a measurable and compact invariant random set attracting all orbits. It is the appropriate generalization of the now classical attractor from the deterministic dynamical systems to RDS. The reason is that if such a random attractor exists, it is the smallest attracting compact set and the largest invariant set [13]. These abstract results have been successfully applied to many stochastic dissipative partial differential equations. For instance, Fan [14] proved the existence of a random attractor for a damped Sine-Gordon equation with white noise. The existence of random attractors for the wave equations has been investigated by several authors [15–17]. Yang *et al.* [18] studied random attractors for stochastic semi-linear degenerate parabolic equations. Ma and Ma [19] investigated attractors for stochastic strongly damped plate equations with additive noise. In this article, we study the existence of random attractors for the floating beam equation with white noise by means of the methods established in [11–13].

The outline of this paper is as follows. Background material on RDS and random attractors is iterated in Section 2. We present the existence and uniqueness of the solution corresponding to system (1.1) which determines RDS in Section 3. Finally, the existence of random attractors for RDS is shown in the last section.

2 Random dynamical system

In this section, we recall some basic concepts related to RDS and a random attractor for RDS in [11–13], which are important for getting our main results.

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ-algebra $\mathcal{B}(X)$, and let $(\Omega, \mathcal{F}, P)$ be a probability space. $\theta_t : \Omega \to \Omega, t \in \mathbb{R}$ is a family of measure preserving transformations such that $(t, \omega) \mapsto \theta_t\omega$ is measurable, $\theta_0 = \text{id}$ and $\theta_{t+s} = \theta_t\theta_s$ for all $t, s \in \mathbb{R}$. The flow θ_t together with the probability space $(\Omega, \mathcal{F}, P, (\theta_t)_{t\in\mathbb{R}})$ is called a metric dynamical system.

Definition 2.1 Let $(\Omega, \mathcal{F}, P, (\theta_t)_{t\in\mathbb{R}})$ be a metric dynamical system. Suppose that the mapping $\phi : \mathbb{R}^+ \times \Omega \times X \to X$ is $(\mathcal{B}(\mathbb{R}^+) \times \mathcal{F} \times \mathcal{B}(X), \mathcal{B}(X))$-measurable and satisfies the following properties:

(i) $\phi(0, \omega)x = x$, $x \in X$ and $\omega \in \Omega$;

(ii) $\phi(t + s, \omega) = \phi(t, \theta_s\omega) \circ \phi(s, \omega)$ for all $t, s \in \mathbb{R}^+$, $x \in X$ and $\omega \in \Omega$.

Then ϕ is called a random dynamical system (RDS). Moreover, ϕ is called a continuous RDS if ϕ is continuous with respect to x for $t \geq 0$ and $\omega \in \Omega$.

Definition 2.2 A set-valued map $D:\Omega\to 2^X$ is said to be a closed (compact) random set if $D(\omega)$ is closed (compact) for P-a.s. $\omega\in\Omega$, and $\omega\mapsto \mathrm{d}(x,D(\omega))$ is P-a.s. measurable for all $x\in X$.

Definition 2.3 If K and B are random sets such that for P-a.s. ω there exists a time $t_B(\omega)$ such that for all $t\geq t_B(\omega)$,

$$\phi(t,\theta_{-t}\omega)B(\theta_{-t}\omega)\subset K(\omega),$$

then K is said to absorb B, and $t_B(\omega)$ is called the absorption time.

Definition 2.4 A random set $\mathcal{A}=\{A(\omega)\}_{\omega\in\Omega}\subset X$ is called a random attractor associated to the RDS ϕ if P-a.s.:

(i) $\mathcal{A}$ is a random compact set, *i.e.*, $A(\omega)$ is compact for P-a.s. $\omega\in\Omega$, and the map $\omega\mapsto \mathrm{d}(x,A(\omega))$ is measurable for every $x\in X$;

(ii) $\mathcal{A}$ is ϕ-invariant, *i.e.*, $\phi(t,\omega)A(\omega)=A(\theta_t\omega)$ for all $t\geq 0$ and P-a.s. $\omega\in\Omega$;

(iii) $\mathcal{A}$ attracts every set B in X, *i.e.*, for all bounded (and non-random) $B\subset X$,

$$\lim_{t\to\infty}\mathrm{d}\bigl(\phi(t,\theta_{-t}\omega)B(\theta_{-t}\omega),A(\omega)\bigr)=0,$$

where $\mathrm{d}(\cdot,\cdot)$ denotes the Hausdorff semi-distance:

$$\mathrm{d}(A,B)=\sup_{x\in A}\inf_{y\in B}\mathrm{d}(x,y),\qquad A,B\in X.$$

Note that $\phi(t,\theta_{-t}\omega)x$ can be interpreted as the position of the trajectory which was in x at time $-t$. Thus, the attraction property holds from $t=-\infty$.

Theorem 2.5 (Existence of a random attractor [11]) *Let ϕ be a continuous random dynamical system on X over $(\Omega,\mathcal{F},P,(\theta_t)_{t\in\mathbb{R}})$. Suppose that there exists a random compact set $K(\omega)$ absorbing every bounded non-random set $B\subset X$. Then the set*

$$\mathcal{A}=\bigl\{A(\omega)\bigr\}_{\omega\in\Omega}=\overline{\bigcup_{B\subset X}\Lambda_B(\omega)}$$

is a global random attractor for ϕ, where the union is taken over all bounded $B\subset X$, and $\Lambda_B(\omega)$ is the ω-limits set of B given by

$$\Lambda_B(\omega)=\bigcap_{s\geq 0}\overline{\bigcup_{t\geq s}\phi(t,\theta_{-t}\omega)B(\theta_{-t}\omega)}.$$

3 Existence and uniqueness of solutions

With the usual notation, we denote

$$H=L^2(\Omega),\qquad V=H^2(\Omega),$$
$$D(A)=H^2(\Omega)\cap H_0^1(\Omega),$$
$$D\bigl(A^2\bigr)=\bigl\{u\in H^4(\Omega):A^2u\in L^2(\Omega)\bigr\},$$

where $A = -\triangle$, $A^2 = \triangle^2$. We denote H, V with the following inner products and norms, respectively:

$$(u,v) = \int_\Omega uv\,dx, \qquad \|u\|^2 = (u,u), \quad \forall u,v \in H,$$
$$\big((u,v)\big) = \int_\Omega \triangle u \triangle v\,dx, \qquad \|u\|_2^2 = \big((u,u)\big), \quad \forall u,v \in V.$$

And we introduce the space $E = D(A) \times H$ which is used throughout the paper and endow the space E with the following usual scalar product and norm:

$$(y_1,y_2)_E = \big((u_1,u_2)\big) + (v_1,v_2), \qquad \|y\|_E^2 = (y,y)_E$$

for all $y_i = (u_i,v_i)^T, y = (u,v)^T \in E$, here T denotes the transposition. Moreover, the norm of $L^p(\Omega)$ is written as $\|\cdot\|_p$.

Let $\lambda > 0$ be the eigenvalue of $A^2 v = \lambda v$, $\triangle v(x,t) = \nabla\triangle v(x,t) = 0$, $x \in \partial\Omega$, by the Poincaré inequality, we have

$$\|u\|_2^2 \geq \lambda\|u\|^2, \quad \forall u \in D(A).$$

It is convenient to reduce (1.1) to an evolution equation of the first order in time

$$\begin{cases} u_t = v, \\ v_t = -A^2 u - A^2 v - bu^+ - f(u) + q(x)\dot{W}, \\ u(x,\tau) = u_0(x), \qquad u_t(x,\tau) = u_1(x), \quad x \in \Omega, \end{cases} \tag{3.1}$$

whose equivalent Itó equation is

$$\begin{cases} du = v\,dt, \\ dv = -A^2 u\,dt - A^2 v\,dt - bu^+\,dt - f(u)\,dt + q(x)\,dW, \\ u(x,\tau) = u_0(x), \qquad u_t(x,\tau) = u_1(x), \quad x \in \Omega, \end{cases} \tag{3.2}$$

where $W(t)$ is a one-dimensional two-sided real-valued Wiener process on $(\Omega, \mathcal{F}, P, (\theta_t)_{t\in\mathbb{R}})$. We can assume without loss of generality that

$$\Omega = \big\{\omega(t) = W(t) \in C(\mathbb{R},\mathbb{R}) : \omega(0) = 0\big\},$$

that P is a Wiener measure. We can define a family of measure preserving and ergodic transformations (flow) $\{\theta_t\}_{t\in\mathbb{R}}$ by

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), \quad t \in \mathbb{R}, \omega \in \Omega.$$

Let $z = v - q(x)W$, then $v = z + q(x)W$. We consider the random partial differential equation equivalent to (3.2)

$$\begin{cases} \frac{du}{dt} = z + q(x)W, \\ \frac{dz}{dt} = -A^2 u - A^2 z - bu^+ - f(u) - A^2 q(x)W, \\ u(x,\tau) = u_0(x), \qquad z(\tau,\omega) = z(x,\tau,\omega) = u_1(x) - q(x)W(\tau), \quad x \in \Omega. \end{cases} \tag{3.3}$$

It apparently contrasts to the stochastic differential equation (3.2), no stochastic differential appears here. Let

$$\varphi = \begin{pmatrix} u \\ z \end{pmatrix}, \qquad L = \begin{pmatrix} 0 & I \\ -A^2 & -A^2 \end{pmatrix}$$

and

$$F(\varphi,\omega) = \begin{pmatrix} q(x)W \\ -bu^+ - f(u) - A^2 q(x)W \end{pmatrix},$$

then (3.3) can be written as

$$\dot{\varphi} = L\varphi + F(\varphi,\omega), \qquad \varphi(\tau,\omega) = \bigl(u_0, z(\tau,\omega)\bigr)^T. \tag{3.4}$$

We know from [20] that L is the infinitesimal generators of C_0-semigroup e^{Lt} on E. It is not difficult to check that the function $F(\cdot,\omega): E \mapsto E$ is locally Lipschitz continuous with respect to φ and bounded for every $\omega \in \Omega$. By the classical semigroup theory of existence and uniqueness of solutions of evolution differential equations [20], the random partial differential equation (3.4) has a unique solution in the mild sense

$$\varphi(t,\omega) = e^{L(t-\tau)}\varphi(\tau,\omega) + \int_\tau^t e^{L(t-s)} F\bigl(\varphi(s),\omega\bigr)\,ds$$

for any $\varphi(\tau,\omega) \in E$. We can prove that for P-a.s. every $\omega \in \Omega$ the following statements hold for all $T > 0$:

(i) If $\varphi(\tau,\omega) \in E$, then $\varphi(t,\omega) \in C([\tau,\tau+T);D(A)) \times C([\tau,\tau+T);H)$.
(ii) $\varphi(t,\varphi(\tau,\omega))$ is continuous in t and $\varphi(\tau,\omega)$.
(iii) The solution mapping of (3.4) satisfies the properties of RDS.

Equation (3.4) has a unique solution for every $\omega \in \Omega$. Hence the solution mapping

$$\bar{S}(t,\omega): \varphi(\tau,\omega) \mapsto \varphi(t,\omega) \tag{3.5}$$

generates a random dynamical system. So the transformation

$$S(t,\omega): \varphi(\tau,\omega) + \bigl(0, q(x)W(\tau)\bigr)^T \mapsto \varphi(t,\omega) + \bigl(0, q(x)W(t)\bigr)^T \tag{3.6}$$

also determines a random dynamical system corresponding to equation (3.1).

4 Existence of a random attractor

In this section, we prove the existence of a random attractor for RDS (3.6) in the space E.

Let $\bar{z} = z + \varepsilon u$, $\psi = (u,\bar{z})^T$, where

$$\varepsilon = \frac{\lambda^2}{4\lambda^2 + 3\lambda + 4}. \tag{4.1}$$

Hence equation (3.3) can be written as

$$\dot{\psi} + Q\psi = \bar{F}(\psi,\omega), \qquad \psi(\tau,\omega) = \bigl(u_0, z(\tau,\omega) + \varepsilon u_0\bigr)^T, \quad t \geq \tau, \tag{4.2}$$

where

$$Q = \begin{pmatrix} \varepsilon I & -I \\ (1-\varepsilon)A^2 + \varepsilon^2 I & A^2 - \varepsilon I \end{pmatrix},$$

$$\bar{F}(\psi, \omega) = \begin{pmatrix} q(x)W \\ -bu^+ - f(u) - A^2 q(x)W + \varepsilon q(x)W \end{pmatrix}.$$

The mapping

$$\bar{S}_\varepsilon(t,\omega) : \bigl(u_0, z(\tau,\omega) + \varepsilon u_0\bigr)^T \mapsto \bigl(u(t), z(t) + \varepsilon u(t)\bigr)^T, \qquad E \to E, \quad t \geq \tau$$

is defined by (4.2).

To show the conjugation of the solution of the stochastic partial differential equation (1.1) and the random partial differential equation (4.2), we introduce the homeomorphism

$$R_\varepsilon : (u,z)^T \mapsto (u, z + \varepsilon u)^T$$

with the inverse homeomorphism $R_{-\varepsilon}$. Then the transformation

$$\bar{S}_\varepsilon(t,\omega) = R_\varepsilon S(t,\omega) R_{-\varepsilon} \tag{4.3}$$

also determines RDS corresponding to equation (1.1). Therefore, for RDS (3.6) we only need consider the equivalent random dynamical system $S_\varepsilon(t,\omega) = R_\varepsilon S(t,\omega)R_{-\varepsilon}$, where $S_\varepsilon(t,\omega)$ is decided by

$$\eta_t + Q\eta = G(\eta,\omega), \qquad \eta(\tau,\omega) = (u_0, u_1 + \varepsilon u_0)^T, \quad t \geq \tau, \tag{4.4}$$

where $\eta(t) = (u(t), u_t(t) + \varepsilon u(t))^T$ and

$$G(\eta,\omega) = \begin{pmatrix} 0 \\ -bu^+ - f(u) + q(x)\dot{W} \end{pmatrix}.$$

Next, we prove a positivity property of the operator Q in E that plays a vital role throughout the paper.

Lemma 4.1 *For any* $\varphi = (u,z)^T \in E$, *there holds*

$$(Q\varphi, \varphi)_E \geq \frac{\varepsilon}{2}\|\varphi\|_E^2 + \frac{\varepsilon}{4}\|u\|_2^2 + \frac{\lambda}{2}\|z\|^2.$$

Proof Since $Q\varphi = (\varepsilon u - z, (1-\varepsilon)A^2 u + \varepsilon^2 u + A^2 z - \varepsilon z)^T$, using the Poincaré inequality and the Young inequality, we get

$$\begin{aligned}(Q\varphi,\varphi)_E &= \varepsilon\|u\|_2^2 - \varepsilon(Au, Az) + \varepsilon^2(u,z) + \|Az\|^2 - \varepsilon\|z\|^2 \\ &\geq \varepsilon\|u\|_2^2 - \frac{\varepsilon}{8}\|u\|_2^2 - 2\varepsilon\|Az\|^2 - \frac{\varepsilon}{8}\|u\|_2^2 - \frac{2\varepsilon^3}{\lambda}\|z\|^2 + \|Az\|^2 - \varepsilon\|z\|^2\end{aligned}$$

$$\geq \frac{\varepsilon}{2}\|\varphi\|_E^2 + \frac{\varepsilon}{4}\|u\|_2^2 + (1-2\varepsilon)\lambda\|z\|^2 - \left(\frac{2\varepsilon}{\lambda} + \frac{3\varepsilon}{2}\right)\|z\|^2,$$

$$= \frac{\varepsilon}{2}\|\varphi\|_E^2 + \frac{\varepsilon}{4}\|u\|_2^2 + \frac{\lambda}{2}\|z\|^2,$$

noting that we used the fact $\varepsilon = \frac{\lambda^2}{4\lambda^2+3\lambda+4}$ in the last inequality. □

Lemma 4.2 *Let* (1.2)-(1.4) *hold. There exist a random variable* $r_1(\omega) > 0$ *and a bounded ball* B_0 *of* E *centered at* 0 *with random radius* $r_0(\omega) > 0$ *such that for any bounded non-random set* B *of* E, *there exists a deterministic* $T(B) \leq -1$ *such that the solution* $\psi(t,\omega;\psi(\tau,\omega)) = (u(t,\omega),\bar{z}(t,\omega))^T$ *of* (4.2) *with initial value* $(u_0, u_1 + \varepsilon u_0)^T \in B$ *satisfies, for P-a.s.* $\omega \in \Omega$,

$$\|\psi(-1,\omega;\psi(\tau,\omega))\|_E \leq r_0(\omega), \quad \tau \leq T(B),$$

and for all $\tau \leq t \leq 0$,

$$\begin{aligned}&\|\psi(t,\omega;\psi(\tau,\omega))\|_E^2 \\ &\quad \leq 2e^{-\varepsilon_1(t-\tau)}\left(\|u_0\|_2^2 + \|u_1+\varepsilon u_0\|^2 + \|q\|^2|W(\tau)|^2 + \int_\Omega F(u_0)\,dx\right) + r_1^2(\omega),\end{aligned} \tag{4.5}$$

where $\bar{z}(t,\omega) = u_t(t) + \varepsilon u(t) - q(x)W(t)$.

Besides, it is easy to deduce a similar absorption result for

$$\eta(-1) = (\eta_1,\eta_2) = \bigl(u(-1), u_t(-1) + \varepsilon u(-1)\bigr)^T$$

instead of $\psi(-1)$.

Proof Taking the inner product in E of (4.2) with $\psi = (u,\bar{z})^T$, in which $\bar{z} = u_t + \varepsilon u - q(x)W$, we find that

$$\frac{1}{2}\frac{d}{dt}\|\psi\|_E^2 + (Q\psi,\psi)_E = \bigl(\bar{F}(\psi,\omega),\psi\bigr)_E, \quad \forall t \geq \tau, \tag{4.6}$$

where

$$\bigl(\bar{F}(\psi,\omega),\psi\bigr)_E = \bigl(\bigl(u,q(x)W\bigr)\bigr) - b\bigl(u^+,\bar{z}\bigr) - \bigl(f(u),\bar{z}\bigr) - \bigl(A^2q(x)W,\bar{z}\bigr) + \varepsilon\bigl(q(x)W,\bar{z}\bigr). \tag{4.7}$$

We deal with the terms in (4.7) one by one as follows:

$$\bigl(\bigl(u,q(x)W\bigr)\bigr) \leq \frac{\varepsilon}{4}\|u\|_2^2 + \frac{\|q\|_2^2}{\varepsilon}\bigl|W(t)\bigr|^2; \tag{4.8}$$

$$\begin{aligned}-b\bigl(u^+,\bar{z}\bigr) &= -b\bigl(u^+, u_t + \varepsilon u - q(x)W\bigr) \\ &= -\frac{1}{2}\frac{d}{dt}b\bigl\|u^+\bigr\|^2 - \varepsilon b\bigl\|u^+\bigr\|^2 + b\bigl(u^+,q(x)W\bigr) \\ &\leq -\frac{1}{2}\frac{d}{dt}b\bigl\|u^+\bigr\|^2 - \frac{\varepsilon b}{2}\bigl\|u^+\bigr\|^2 + \frac{b\|q\|^2}{2\varepsilon}\bigl|W(t)\bigr|^2;\end{aligned} \tag{4.9}$$

$$\left|-\left(A^2 q(x) W, \bar{z}\right)\right| \le \frac{\|q\|_4^2}{\lambda}\left|W(t)\right|^2 + \frac{\lambda}{4}\|\bar{z}\|^2; \tag{4.10}$$

$$\varepsilon\left(q(x) W, \bar{z}\right) \le \frac{\varepsilon^2\|q\|^2}{\lambda}\left|W(t)\right|^2 + \frac{\lambda}{4}\|\bar{z}\|^2. \tag{4.11}$$

Using (1.2)-(1.3) and the Hölder inequality, we conclude that

$$\begin{aligned}
&\left(f(u), q(x) W(t)\right) \\
&\quad \le C_0 \int_\Omega \left(1 + |u|^p\right) q(x) W(t)\,dx \\
&\quad \le C_0\|q\|\left|W(t)\right| + C_0\left(\int_\Omega |u|^{p+1}\,dx\right)^{\frac{p}{p+1}}\|q\|_{p+1}\left|W(t)\right| \\
&\quad \le C_0\|q\|\left|W(t)\right| + C_0 C_1^{-\frac{p}{p+1}}\left(\int_\Omega \left(F(u) + C_1\right)dx\right)^{\frac{p}{p+1}}\|q\|_{p+1}\left|W(t)\right| \\
&\quad \le C_0\|q\|\left|W(t)\right| + \frac{\varepsilon C_0 C_1^{-1}}{2}\int_\Omega F(u)\,dx + \frac{C_0}{2\varepsilon}\|q\|_{p+1}^{p+1}\left|W(t)\right|^{p+1} + \frac{\varepsilon C_0|\Omega|}{2}.
\end{aligned} \tag{4.12}$$

Inequality (1.4) together with (4.12) yields

$$\begin{aligned}
&-\left(f(u), \bar{z}\right) \\
&\quad = -\left(f(u), u_t + \varepsilon u - qW(t)\right) \\
&\quad \le -\frac{d}{dt}\int_\Omega F(u)\,dx - \varepsilon C_2\int_\Omega F(u)\,dx + \varepsilon C_2|\Omega| + \left(f(u), qW(t)\right) \\
&\quad \le -\frac{d}{dt}\int_\Omega F(u)\,dx - \frac{\varepsilon(2C_2 - C_0 C_1^{-1})}{2}\int_\Omega F(u)\,dx + C_0\|q\|\left|W(t)\right| \\
&\qquad + \frac{C_0}{2\varepsilon}\|q\|_{p+1}^{p+1}\left|W(t)\right|^{p+1} + \frac{\varepsilon(C_0 + 2C_2)}{2}|\Omega|.
\end{aligned} \tag{4.13}$$

Therefore, collecting with (4.6)-(4.13) and Lemma 4.1, we get that

$$\begin{aligned}
&\frac{d}{dt}\left(\|\psi\|_E^2 + b\left\|u^+\right\|^2 + 2\int_\Omega F(u)\,dx + 2C_1|\Omega|\right) \\
&\qquad + \varepsilon\|\psi\|_E^2 + \varepsilon b\left\|u^+\right\|^2 + \varepsilon\left(2C_2 - C_0 C_1^{-1}\right)\int_\Omega F(u)\,dx + 2\varepsilon C_1|\Omega| \\
&\quad \le M\left(1 + \left|W(t)\right| + \left|W(t)\right|^2 + \left|W(t)\right|^{p+1}\right),
\end{aligned}$$

where $M = \max\{\varepsilon(C_0 + 2C_2 + 2C_1)|\Omega|, 2C_0\|q\|, \frac{2\varepsilon\|q\|_4^2 + 2\lambda\|q\|_2^2 + (2\varepsilon^3 + \lambda b)\|q\|^2}{\varepsilon\lambda}, \frac{C_0}{\varepsilon}\|q\|_{p+1}^{p+1}\}$. Using (1.3), we have the fact $2\int_\Omega F(u)\,dx + 2C_1|\Omega| \ge 0$. Choosing $\varepsilon_1 = \min\{\varepsilon, \frac{\varepsilon(2C_2 - C_0 C_1^{-1})}{2}\}$, and $C_2 > \frac{C_0 C_1^{-1}}{2}$, by the Gronwall lemma, we conclude that

$$\begin{aligned}
&\left\|\psi\left(t, \omega; \psi(\tau, \omega)\right)\right\|_E^2 \\
&\quad \le e^{-\varepsilon_1(t-\tau)}\left(\left\|\psi(\tau, \omega)\right\|_E^2 + b\left\|u_0^+\right\|^2 + 2\int_\Omega F(u_0)\,dx + 2C_1|\Omega|\right) \\
&\qquad + M\int_\tau^t e^{-\varepsilon_1(t-s)}\left(1 + \left|W(s)\right| + \left|W(s)\right|^2 + \left|W(s)\right|^{p+1}\right)ds
\end{aligned}$$

$$\le 2e^{-\varepsilon_1(t-\tau)}\left(\|u_0\|_2^2 + \|u_1+\varepsilon u_0\|^2 + \|q\|^2\big|W(\tau)\big|^2 + b\big\|u_0^+\big\|^2 + \int_\Omega F(u_0)\,dx + C_1|\Omega|\right)$$
$$+ M\int_\tau^t e^{-\varepsilon_1(t-s)}\big(1+\big|W(s)\big|+\big|W(s)\big|^2+\big|W(s)\big|^{p+1}\big)\,ds. \tag{4.14}$$

Take

$$r_0^2(\omega) = 2\Big(1+\sup_{\tau\le -1} e^{\varepsilon_1\tau}\|q\|^2\big|W(\tau)\big|^2\Big) + \frac{M}{\varepsilon_1}$$
$$+ M\int_{-\infty}^{-1} e^{-\varepsilon_1(-1-s)}\big(\big|W(s)\big|+\big|W(s)\big|^2+\big|W(s)\big|^{p+1}\big)\,ds$$

and

$$r_1^2(\omega) = \frac{M}{\varepsilon_1} + M\int_{-\infty}^{0} e^{\varepsilon_1 s}\big(\big|W(s)\big|+\big|W(s)\big|^2+\big|W(s)\big|^{p+1}\big)\,ds.$$

Since $\lim_{t\to\infty}\frac{W(t)}{t} = 0$, $r_0^2(\omega)$ and $r_1^2(\omega)$ are finite P-a.s., given a bounded set B of E, choose $T(B) \le -1$ such that

$$e^{-\varepsilon_1(-1-\tau)}\left(\|u_0\|_2^2 + \|u_1+\varepsilon u_0\|^2 + b\big\|u_0^+\big\|^2 + \int_\Omega F(u_0)\,dx + C_1|\Omega|\right) \le 1 \tag{4.15}$$

for all $(u_0, u_1+\varepsilon u_0)^T \in B$, and

$$e^{\varepsilon_1\tau}\left(\|u_0\|_2^2 + \|u_1+\varepsilon u_0\|^2 + b\big\|u_0^+\big\|^2 + \int_\Omega F(u_0)\,dx + C_1|\Omega|\right) \le 1 \tag{4.16}$$

for all $(u_0, u_1+\varepsilon u_0)^T \in B$, and for all $\tau \le T(B)$.

This completes the proof of Lemma 4.2. □

Let $u(t)$ be a solution of problem (1.1) with initial value $(u_0, u_1+\varepsilon u_0)^T \in B$. We make the decomposition $u(t) = y_1(t) + y_2(t)$, where $y_1(t)$ and $y_2(t)$ satisfy

$$\begin{cases} y_{1tt} + \triangle^2 y_1 + \triangle^2 y_{1t} = 0, & \text{in } \Omega\times[\tau,+\infty), \tau\in\mathbb{R}, \\ \triangle y_1(x,t) = \nabla\triangle y_1(x,t) = 0, & x\in\partial\Omega, t\ge\tau, \\ y_1(x,\tau) = u_0(x), \quad y_{1t}(x,\tau) = u_1(x), & x\in\Omega \end{cases} \tag{4.17}$$

and

$$\begin{cases} y_{2tt} + \triangle^2 y_2 + \triangle^2 y_{2t} + bu^+ + f(u) = q(x)\dot{W}, & \text{in } \Omega\times[\tau,+\infty), \tau\in\mathbb{R}, \\ \triangle y_2(x,t) = \nabla\triangle y_2(x,t) = 0, & x\in\partial\Omega, t\ge\tau, \\ y_2(x,\tau) = 0, \quad y_{2t}(x,\tau) = 0, & x\in\Omega. \end{cases} \tag{4.18}$$

Lemma 4.3 *Let B be a bounded non-random subset of E. For any $(u_0, u_1+\varepsilon u_0)^T \in B$,*

$$\big\|Y_1(0)\big\|_E^2 = \big\|y_1(0)\big\|_2^2 + \big\|y_{1t}(0)+\varepsilon y_1(0)\big\|^2 \le \big(\|u_0\|_2^2 + \|u_1+\varepsilon u_0\|^2\big)\frac{e^{\varepsilon\tau}}{1-\varepsilon}, \tag{4.19}$$

where $Y_1 = (y_1, y_{1t}+\varepsilon y_1)^T$ satisfies (4.17).

Proof Taking the scalar product in $L^2(\Omega)$ of (4.17) with $v = y_{1t} + \varepsilon y_1$, we conclude that

$$\frac{1}{2}\frac{d}{dt}\big((1-\varepsilon)\|y_1\|_2^2 + \|v\|^2\big) + \varepsilon(1-\varepsilon)\|y_1\|_2^2 + \|v\|_2^2 - \varepsilon\|v\|^2 + \varepsilon^2(y_1, v) = 0. \tag{4.20}$$

Due to (4.1), using the Hölder inequality and the Young inequality, we get that

$$\begin{aligned} &\varepsilon(1-\varepsilon)\|y_1\|_2^2 + \|v\|_2^2 - \varepsilon\|v\|^2 + \varepsilon^2(y_1, v) \\ &\quad\geq \varepsilon(1-\varepsilon)\|y_1\|_2^2 + \|v\|_2^2 - \varepsilon\|v\|^2 - \frac{\varepsilon(1-\varepsilon)}{2}\|y_1\|_2^2 - \frac{\varepsilon^3}{2(1-\varepsilon)\lambda}\|v\|^2 \\ &\quad\geq \frac{\varepsilon(1-\varepsilon)}{2}\|y_1\|_2^2 + \left(\lambda - \frac{\varepsilon}{2\lambda} - \varepsilon\right)\|v\|^2 \\ &\quad\geq \frac{\varepsilon(1-\varepsilon)}{2}\|y_1\|_2^2 + \frac{\varepsilon}{2}\|v\|^2. \end{aligned} \tag{4.21}$$

Associating (4.20) with (4.21), we have that

$$\frac{d}{dt}\big((1-\varepsilon)\|y_1\|_2^2 + \|v\|^2\big) + \varepsilon\big((1-\varepsilon)\|y_1\|_2^2 + \|v\|^2\big) \leq 0.$$

The Gronwall lemma leads to (4.19). □

Lemma 4.4 *Assume that* (1.2) *holds, there exists a random radius* $r_2(\omega)$ *such that for P-a.s.* $\omega \in \Omega$,

$$\big\|A^{\frac{1}{2}}Y_2\big(0, \omega; Y_2(\tau, \omega)\big)\big\|_E^2 \leq r_2^2(\omega), \tag{4.22}$$

where $Y_2 = (y_2, y_{2t} + \varepsilon y_2 - q(x)W)^T$ *satisfies* (4.18).

Proof Provided that $Y_2 = (y_2, y_{2t} + \varepsilon y_2 - q(x)W)^T$, then equation (4.18) can be reduced to

$$Y_{2t} + QY_2 = H(Y_2, \omega), \qquad Y_2(\tau) = \big(0, -q(x)W(\tau)\big)^T, \quad t \geq \tau, \tag{4.23}$$

where

$$H(Y_2, \omega) = \begin{pmatrix} q(x)W(t) \\ -bu^+ - f(u) - A^2q(x)W(t) + \varepsilon q(x)W(t) \end{pmatrix}.$$

Taking the inner product in E of (4.23) with AY_2, we have that

$$\frac{1}{2}\frac{d}{dt}\big\|A^{\frac{1}{2}}Y_2\big\|_E^2 + (QY_2, AY_2)_E = \big(H(Y_2, \omega), AY_2\big)_E, \tag{4.24}$$

where

$$\begin{aligned} &\big(H(Y_2, \omega), AY_2\big)_E \\ &\quad= \big(\big(Ay_2, q(x)W\big)\big) \\ &\qquad - \big(bu^+ + f(u) + A^2q(x)W(t) - \varepsilon q(x)W(t), A\big(y_{2t} + \varepsilon y_2 - q(x)W\big)\big). \end{aligned} \tag{4.25}$$

According to Lemma 4.1, we have that

$$(QY_2, AY_2)_E \geq \frac{\varepsilon}{2}\|A^{\frac{1}{2}}Y_2\|_E^2 + \frac{\varepsilon}{4}\|A^{\frac{1}{2}}y_2\|_2^2 + \frac{\lambda}{2}\|A^{\frac{1}{2}}(y_{2t} + \varepsilon y_2 - q(x)W)\|^2. \tag{4.26}$$

Thanks to the Young inequality, we obtain that

$$\big((Ay_2, q(x)W)\big) \leq \frac{\varepsilon}{4}\|A^{\frac{1}{2}}y_2\|_2^2 + \frac{1}{\varepsilon}\|A^{\frac{1}{2}}q\|_2^2|W(t)|^2; \tag{4.27}$$

$$\begin{aligned}&\big|-\big(bu^+, A(y_{2t} + \varepsilon y_2 - q(x)W)\big)\big| \\ &\quad \leq \frac{2b^2}{\lambda}\|A^{\frac{1}{2}}u^+\|^2 + \frac{\lambda}{8}\|A^{\frac{1}{2}}(y_{2t} + \varepsilon y_2 - q(x)W)\|^2;\end{aligned} \tag{4.28}$$

$$\begin{aligned}&\big|-\big(A^2q(x)W(t), A(y_{2t} + \varepsilon y_2 - q(x)W)\big)\big| \\ &\quad \leq \frac{2}{\lambda}\|A^{\frac{5}{2}}q\|^2|W(t)|^2 + \frac{\lambda}{8}\|A^{\frac{1}{2}}(y_{2t} + \varepsilon y_2 - q(x)W)\|^2;\end{aligned} \tag{4.29}$$

$$\begin{aligned}&\big|\big(\varepsilon q(x)W(t), A(y_{2t} + \varepsilon y_2 - q(x)W)\big)\big| \\ &\quad \leq \frac{2\varepsilon^2}{\lambda}\|A^{\frac{1}{2}}q\|^2|W(t)|^2 + \frac{\lambda}{8}\|A^{\frac{1}{2}}(y_{2t} + \varepsilon y_2 - q(x)W)\|^2.\end{aligned} \tag{4.30}$$

By (1.2), (4.5) and the Sobolev embedding theorem, we show that $f(s)$ is uniformly bounded in L^∞, that is, there exists a constant $M > 0$ such that

$$|f'(s)|_{L^\infty} \leq M. \tag{4.31}$$

Combining with (4.31), the Young inequality and the Sobolev embedding theorem, it follows that

$$\begin{aligned}&\big|-\big(f(u), A(y_{2t} + \varepsilon y_2 - q(x)W)\big)\big| \\ &\quad \leq \|A^{\frac{1}{2}}f(u)\|\,\|A^{\frac{1}{2}}(y_{2t} + \varepsilon y_2 - q(x)W)\| \\ &\quad \leq \frac{2\mu M^2}{\lambda}\|u\|_2^2 + \frac{\lambda}{8}\|A^{\frac{1}{2}}(y_{2t} + \varepsilon y_2 - q(x)W)\|^2,\end{aligned} \tag{4.32}$$

where μ is a positive constant. Thus, collecting all (4.25)-(4.32) and (4.5), from (4.24) we have, for $\tau \leq T(B)$,

$$\begin{aligned}&\frac{d}{dt}\|A^{\frac{1}{2}}Y_2\|_E^2 + \|A^{\frac{1}{2}}Y_2\|_E^2 \\ &\quad \leq \frac{4\mu(b^2 + M^2)}{\lambda}\|u\|_2^2 + \left(\frac{2}{\varepsilon}\|A^{\frac{1}{2}}q\|_2^2 + \frac{4}{\lambda}\|A^{\frac{5}{2}}q\|^2 + \frac{4\varepsilon^2}{\lambda}\|A^{\frac{1}{2}}q\|^2\right)|W(t)|^2 \\ &\quad \leq \frac{4\mu(b^2 + M^2)}{\lambda}\left(2e^{-\varepsilon_1(t-\tau)}\left(\|u_0\|_2^2 + \|u_1 + \varepsilon u_0\|^2 + \|q\|^2|W(\tau)|^2\right.\right. \\ &\qquad \left.\left. + \int_\Omega F(u_0)\,dx\right) + r_1^2(\omega)\right) \\ &\qquad + \left(\frac{2}{\varepsilon}\|A^{\frac{1}{2}}q\|_2^2 + \frac{4}{\lambda}\|A^{\frac{5}{2}}q\|^2 + \frac{4\varepsilon^2}{\lambda}\|A^{\frac{1}{2}}q\|^2\right)|W(t)|^2, \quad \tau \leq t \leq 0.\end{aligned}$$

Applying the Gronwall lemma, we obtain that

$$\begin{aligned}
&\|A^{\frac{1}{2}}Y_2(0,\omega;Y_2(\tau,\omega))\|_E^2 \\
&\quad\le e^{\varepsilon\tau}\|A^{\frac{1}{2}}q\|^2|W(\tau)|^2+\frac{4\mu(b^2+M^2)}{\lambda}\left(\frac{2e^{\varepsilon_1\tau}}{\varepsilon-\varepsilon_1}\left(\|u_0\|_2^2+\|u_1+\varepsilon u_0\|^2\right.\right. \\
&\quad\quad\left.\left.+\|q\|^2|W(\tau)|^2+\int_\Omega F(u_0)\,dx\right)+\frac{r_1^2(\omega)}{\varepsilon}\right) \\
&\quad\quad+\left(\frac{2}{\varepsilon}\|A^{\frac{1}{2}}q\|_2^2+\frac{4}{\lambda}\|A^{\frac{5}{2}}q\|^2+\frac{4\varepsilon^2}{\lambda}\|A^{\frac{1}{2}}q\|^2\right)\int_\tau^0 e^{\varepsilon s}|W(s)|^2\,ds. \qquad (4.33)
\end{aligned}$$

Set

$$\begin{aligned}
r_2^2(\omega)={}&\|A^{\frac{1}{2}}q\|^2\sup_{\tau\le 0}e^{\varepsilon\tau}|W(\tau)|^2 \\
&+\frac{4\mu(b^2+M^2)}{\lambda(\varepsilon-\varepsilon_1)}\left(2+2\|q\|^2\sup_{\tau\le 0}e^{\varepsilon_1\tau}|W(\tau)|^2+r_1^2(\omega)\right) \\
&+\left(\frac{2}{\varepsilon}\|A^{\frac{1}{2}}q\|_2^2+\frac{4}{\lambda}\|A^{\frac{5}{2}}q\|^2+\frac{4\varepsilon^2}{\lambda}\|A^{\frac{1}{2}}q\|^2\right)\int_{-\infty}^0 e^{\varepsilon s}|W(s)|^2\,ds.
\end{aligned}$$

Since $\lim_{t\to\infty}\frac{W(t)}{t}=0$, $r_2^2(\omega)$ is finite P-a.s., together with (4.16) and (4.33), we have that

$$\|A^{\frac{1}{2}}Y_2(0,\omega;Y_2(\tau,\omega))\|_E^2\le r_2^2(\omega)$$

for all $(u_0,u_1+\varepsilon u_0)^T\in B$ and all $\tau\le T(B)$. □

Theorem 4.5 *Let* (1.2)-(1.4) *hold,* $q(x)\in H^3(\Omega)$. *Then the random dynamical system* $S_\varepsilon(t,\omega)$ *possesses a nonempty compact random attractor* $\mathcal{A}$.

Proof Let $B_1(\omega)$ be the ball of $H^3(\Omega)\times(H^2(\Omega)\cap H_0^1(\Omega))$ of radius $r_2(\omega)$. From the compact embedding $H^3(\Omega)$, it follows that $B_1(\omega)$ is compact in E. For every bounded non-random set B of E and any $\psi(0)\in\bar{S}_\varepsilon(t,\theta_{-t}\omega)B$, from Lemma 4.4, we know that $Y_2(0)=\psi(0)-Y_1(0)\in B_1(\omega)$, where $Y_2(t,\omega)$ is given by (4.18). Therefore, for $\tau\le 0$,

$$\inf_{l(0)\in B_1(\omega)}\|\psi(0)-l(0)\|_E^2\le\|Y_1(0)\|_E^2\le\left(\|u_0\|_2^2+\|u_1+\varepsilon u_0\|^2\right)\frac{e^{\varepsilon\tau}}{1-\varepsilon}.$$

Furthermore, for all $t\ge 0$,

$$\mathrm{d}\left(\bar{S}_\varepsilon(t,\theta_{-t}\omega)B,B_1(\omega)\right)\le\left(\|u_0\|_2^2+\|u_1+\varepsilon u_0\|^2\right)\frac{e^{-\varepsilon t}}{1-\varepsilon}.$$

Finally, from relation (4.3) between $S_\varepsilon(t,\omega)$ and $\bar{S}_\varepsilon(t,\omega)$, one can easily obtain that for any non-random bounded $B\subset E$ P-a.s.,

$$\mathrm{d}\left(S_\varepsilon(t,\theta_{-t}\omega)B,B_1(\omega)\right)\to 0\quad\text{as } t\to+\infty.$$

Hence, the RDS $S_\varepsilon(t,\omega)$ associated with (3.6) possesses a uniformly attracting compact set $B_1(\omega)\subset E$. Then applying Theorem 2.5 we complete the proof. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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References

1. Lazer, AC, McKenna, PJ: Large-amplitude periodic oscillations in suspension bridges: some new connection with nonlinear analysis. SIAM Rev. **32**(4), 537-578 (1990)
2. Ma, QZ: Asymptotic behavior of the solution for the floating beam. J. Northwest Norm. Univ. Nat. Sci. **41**(6), 4-7 (2005) (in Chinese)
3. Zhong, CK, Ma, QZ, Sun, CY: Existence of strong solutions and global attractors for the suspension bridge equations. Nonlinear Anal. **67**, 442-454 (2007)
4. Park, JY, Kang, JR: Global attractors for the suspension bridge equations with nonlinear damping. Q. Appl. Math. **69**, 465-475 (2011)
5. Park, JY, Kang, JR: Pullback $\mathcal{D}$-attractors for non-autonomous suspension bridge equations. Nonlinear Anal. **71**, 4618-4623 (2009)
6. Park, JY, Kang, JR: Uniform attractor for non-autonomous suspension bridge equations with localized damping. Math. Methods Appl. Sci. **34**, 487-496 (2011)
7. Ma, QZ, Zhong, CK: Existence of global attractors for the coupled suspension bridge equations. J. Math. Anal. Appl. **308**, 365-379 (2005)
8. Ma, QZ, Zhong, CK: Existence of strong solutions and global attractors for the coupled suspension bridge equations. J. Differ. Equ. **246**, 3755-3775 (2009)
9. Ma, QZ, Wang, SP, Chen, XB: Uniform attractors for the coupled suspension bridge equations. Appl. Math. Comput. **217**, 6604-6615 (2011)
10. Ma, QZ, Wang, BL: Existence of pullback attractors for the coupled suspension bridge equations. Electron. J. Differ. Equ. **2011**, 16 (2011)
11. Crauel, H, Flandoli, F: Attractors for random dynamical systems. Probab. Theory Relat. Fields **100**, 365-393 (1994)
12. Crauel, H, Debussche, A, Flandoli, F: Random attractors. J. Dyn. Differ. Equ. **9**, 307-314 (1997)
13. Arnold, L: Random Dynamical Systems. Springer, New York (1998)
14. Fan, XM: Random attractor for a damped sine-Gordon equation with white noise. Pac. J. Math. **216**, 63-76 (2004)
15. Fan, XM, Wang, YG: Fractal dimensional of attractors for a stochastic wave equation with nonlinear damping and white noise. Stoch. Anal. Appl. **25**, 381-396 (2007)
16. Fan, XM: Random attractor for a damped stochastic wave equation with multiplicative noise. Int. J. Math. **19**, 421-437 (2008)
17. Yang, MH, Duan, JQ, Kloeden, PE: Asymptotic behavior of solutions for random wave equation with nonlinear damping and white noise. Nonlinear Anal., Real World Appl. **12**, 464-478 (2011)
18. Yang, MH, Kloeden, PE: Random attractors for stochastic semi-linear degenerate parabolic equation. Nonlinear Anal., Real World Appl. **12**, 2811-2821 (2011)
19. Ma, WJ, Ma, QZ: Attractors for stochastic strongly damped plate equations with additive noise. Electron. J. Differ. Equ. **2013**, 111 (2013)
20. Pazy, A: Semigroup of Linear Operators and Applications to Partial Differential Equations. Appl. Math. Sci. Springer, New York (1983)