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Scattering of cylindrical Gaussian pulse near an absorbing half-plane in a moving fluid

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Abstract

We investigate the scattering of Gaussian pulse by an absorbing half-plane satisfying Myers' impedance conditions. The model problem is considered for a subsonic flow in a moving fluid. The Wiener-Hopf technique followed by the spatial and temporal Fourier transforms and method of Steepest descent enables us to develop the far field solution analytically. It is observed that the Myers' impedance condition found higher-order accuracy of Mach number as compared with the results obtained while using Ingard's condition. The solution to the underlying problem leads itself to the variety of problems thereby including the effects of Gaussian pulses.

Keywords: scattering; Gaussian pulse; Myers' impedance condition

1 Introduction

The impedance boundary condition (IBC) was first introduced by Leontovich in attempt to solve the problems of radio wave propagation over the earth. The IBCs are the approximate boundary conditions that relate the field outside the scatterer only, and thus analysis of the related problem is much more simplified [1]. These IBCs have been utilized by many researchers in the field of electromagnetics and acoustics; refer, for instance, to Wang [1], Nawaz et al. [2, 3], Rawlins [4], Ahmad [5], Buyukaksoy et al. [6], Ayub et al. [7], etc. Rawlins [4] used Ingard's condition [8] to model the impedance conditions that arise in the noise reduction problems by barriers. Ahmad [5] reconsidered Rawlins problem [4] and showed that Myers' condition [9] gives better results than Ingard's conditions when the diffraction problems of acoustic waves related to noise reduction by barriers are considered in a moving fluid regime. Myers' condition [9] contains a correction term and thus allows a straightforward manipulation of the condition into a form that is more convenient to apply than Ingard's condition. In this paper, we focus ourselves on the diffraction of cylindrical Gaussian pulse by an absorbing half-plane in a moving fluid satisfying Myers' impedance condition.

Gaussian functions and integrals frequently occur in many problems of mathematics, physics, statistics, and also in other branches of science and technology. To name a few, Gaussian integrals occur in normal distributions (also known as Gaussian distributions), which occupy a central position in statistical inference, sampling distributions and are excellent approximations to several other distributions. In quantum field theory, the Gaus-

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sian integrals involve ordinary real or complex variables or the Grassmann variables. Because Gaussian beams have favorable propagation characteristics and represent physically observable entities, these have played a vital role in many modeling schemes; see, for instance, [10]. In particular, Gaussian and comb functions are the best known examples of self-Fourier functions [11]. Keeping in view the importance of Gaussian functions, the diffraction of cylindrical Gaussian pulse near an absorbing half-plane in a moving fluid regime is examined mathematically.

While investigating the diffraction problems of acoustic/electromagnetic/elastic waves, harmonic time variation is assumed and suppressed throughout the analysis. Although time harmonic waves are of great importance, yet there are significant fields whose time variation is nonharmonic. The time-dependant wave phenomenon is also an important aspect in the wave motion theory and gives a more transparent picture of wave motion phenomenon. A good account of transient problems can be found in the books of Friedlander [12] and Jones [13]. The transient wave phenomenon is also important due to its ability to produce short electromagnetic pulses, which are used as a diagnostic tool for identification/location of cracks or other defects, implosion and seismological prospects like bore hole sounding and nondestructive testing [14]. Keeping in view the importance of transient wave motion, many scientists have contributed transient wave problems in diffraction theory, to name a few, for example, Haris [15, 16], Rienstra [17], Kriegsmann et al. [18], Ahmad [19], Ishii and Tanaka [20], Alford et al. [21], and Ayub et al. [22, 23]. Moreover, Marin et al. [24] and Marin [25] have also focused themselves on related studies by considering nonsimple material problems.

The solution to the underlying problem is presented while using spatial and temporal Fourier transforms, the Wiener-Hopf technique [26], and the method of steepest descent [13]. Firstly, temporal Fourier transform is applied to obtain the transfer function in frequency domain, and then following the approach of Sun et al. [27], finally, the inverse transform is used to get the results in time domain. The results for the rigid barrier and still air can be computed as a special case from the given diffracted field.

2 Statement of the problem

We consider the scattering of acoustic waves radiated by a cylindrical Gaussian pulse by an absorbing half-plane located at $x \geq 0$, $y = 0$. The cylindrical Gaussian pulse is located at (x_0, y_0), whereas the half-plane is of negligible thickness that satisfies Myers' impedance condition [9]

$$u_n = \left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right]\frac{g}{|\nabla_\alpha|} \tag{1}$$

with

$$\frac{\partial g}{\partial t} = -\frac{p}{z_a}|\nabla_\alpha|, \tag{2}$$

where u_n is the normal derivative of the perturbation velocity, p is the surface pressure, z_a is the acoustic impedance, and n is the normal vector pointing from the surface into the fluid. The whole system is assumed to be in moving fluid with subsonic velocity U parallel to x-axis. The equations of motion are linearized, and the effects of viscosity, thermal conductivity, and gravity are neglected. The perturbation velocity $\mathbf{u}$ of the irrotational sound wave can be written in terms of the velocity potential ϕ as $\mathbf{u} = \nabla\phi$, and the resulting pressure p of the sound field is given by

$$p = -\rho_0\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\phi, \tag{3}$$

where ρ_0 is the density of the undisturbed stream. The Gaussian pulse of strength $\frac{s}{\sqrt{\pi}}$ is considered parallel to the edge at the point (x_0, y_0). The governing convective wave equation is given by

$$\left[\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - \left(\frac{1}{c}\frac{\partial}{\partial t} + M\frac{\partial}{\partial x}\right)^2\right]\phi(x,y,t) = \frac{s}{\sqrt{\pi}}\delta(x-x_0)\delta(y-y_0)e^{-s^2t^2}. \tag{4}$$

3 Problem in frequency domain

Let us define the temporal Fourier transform pair as

$$\begin{aligned}\psi(x,y,\omega) &= \int_{-\infty}^{\infty}\phi(x,y,t)e^{i\omega t}\,dt,\\ \phi(x,y,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi(x,y,\omega)e^{-i\omega t}\,d\omega.\end{aligned} \tag{5}$$

Now transforming equation (4) by using equation (5), we obtain

$$\left[\left(1-M^2\right)\frac{\partial^2}{\partial x^2} + 2ikM\frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} + k^2\right]\psi(x,y,\omega) = \delta(x-x_0)\delta(y-y_0)e^{-\omega^2t^2}. \tag{6}$$

The Fourier temporal transform of the boundary condition (1) yields

$$\left[\frac{\partial}{\partial y} \mp 2\beta M\frac{\partial}{\partial x} \pm ik\beta \mp \frac{i\beta M^2}{k}\frac{\partial^2}{\partial x^2}\right]\psi\left(x,0^{\pm},\omega\right) = 0, \quad x > 0. \tag{7}$$

The continuity conditions for pressure and normal component of velocity are given by

$$\begin{cases}\psi(x,0^+,\omega) = \psi(x,0^-,\omega), \\ \frac{\partial\psi}{\partial y}(x,0^+,\omega) = \frac{\partial\psi}{\partial y}(x,0^-,\omega),\end{cases} \quad x < 0, \tag{8}$$

where $k = \frac{\omega}{c}$ is the wave number, c is the speed of sound, $\beta = \frac{\rho_0 c}{Z_a}$ is the specific complex admittance of the material of which half-plane is made up of, $M = \frac{U}{c}$ is the Mach number, and Z_a is the acoustic impedance. We assume that the flow is subsonic, that is, $|M| < 1$ and $\operatorname{Re}\beta > 0$, which is a necessary condition for an absorbing surface [5].

4 Nondimensional form

Through equations (6)-(8), we observe that the boundary value problem in the transformed plane ω is quite analogous to the boundary value problem in [5], with an additional factor $e^{-\frac{\omega^2}{4s^2}}$ and where k $(= \frac{\omega}{c})$ is a function of ω rather than a constant. This fact results in much greater complexity while working the diffracted field. Thus, in order to avoid repetition, we refer to [5] for details of calculations and restrict ourselves to important calculation steps only. To ease the solution procedure, in first attempt, we shall undimensionalize the problem defined through equations (6)-(8) with the following real substitutions [5]:

$$x = \sqrt{1-M^2}X, \qquad x_0 = \sqrt{1-M^2}X_0, \qquad y = Y, \qquad y_0 = Y_0,$$
$$\beta = \sqrt{1-M^2}B, \qquad k = \sqrt{1-M^2}K,$$

and

$$\psi(x,y,\omega) = \Psi(X,Y,\omega)e^{-ifKMX},$$

which results in

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + K^2\right)\Psi(X,Y,\omega) = \frac{\delta(X-X_0)\delta(Y-Y_0)}{\sqrt{1-M^2}}e^{-\frac{\omega^2}{4s^2}+iKMX_0}, \tag{9}$$

subject to the boundary conditions

$$\left[\frac{\partial}{\partial Y} \mp 2BM\frac{\partial}{\partial X} \pm iKB\left(1+M^2\right) \pm \frac{iBM^2}{(1-M^2)K}\frac{\partial^2}{\partial X^2}\right]\Psi\left(X,0^{\pm},\omega\right) = 0, \quad X > 0, \tag{10}$$

and continuity conditions

$$\begin{cases} \frac{\partial\Psi}{\partial Y}(X,0^+,\omega) = \frac{\partial\Psi}{\partial Y}(X,0^-,\omega), \\ \Psi(X,0^+,\omega) = \Psi(X,0^-,\omega), \end{cases} \quad X < 0. \tag{11}$$

5 Analytic solution

It is pertinent to mention that the total field carries two effects, one in terms of incident and the other in terms of diffracted field. Therefore, the total field $\Psi(X,Y,\omega)$ is split as

$$\Psi(X,Y,\omega) = \Psi_0(X,Y,\omega) + \Psi_d(X,Y,\omega), \tag{12}$$

where $\Psi_0(X,Y,\omega)$ satisfies the inhomogeneous equation

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + K^2\right)\Psi_0(X,Y,\omega) = \frac{\delta(X-X_0)\delta(Y-Y_0)}{\sqrt{1-M^2}}e^{-\frac{\omega^2}{4s^2}+iKMX_0}, \tag{13}$$

and $\Psi_d(X,Y,\omega)$ satisfies the homogeneous equation

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + K^2\right)\Psi_d(X,Y,\omega) = 0. \tag{14}$$

Now with the help of Green's function method [26] solution of equation (13) results in

$$\Psi_0(X,Y,\omega) = \frac{a}{4i}H_0^1(KR) = \frac{a}{4\pi i}\int_{-\infty}^{\infty}\frac{1}{\kappa}e^{i[\nu(X-X_0)+\kappa|Y-Y_0|]}\,d\nu, \tag{15}$$

where $a = \frac{e^{iKMX_0}}{\sqrt{1-M^2}}$, $R = \sqrt{(X-X_0)^2 + (Y-Y_0)^2}$, $\kappa = \sqrt{K^2 - \nu^2}$ is the wave number, and ν is the Fourier transform variable introduced by the following relations:

$$\begin{cases} \bar{\Psi}(\nu, Y, \omega) = \int_{-\infty}^{\infty} \Psi(X, Y, \omega) e^{i\nu X}\, dX, \\ \Psi(X, Y, \omega) = \int_{-\infty}^{\infty} \bar{\Psi}(\nu, Y, \omega) e^{-i\nu X}\, d\nu. \end{cases}$$

Using this relation, the solution to equations (13) and (14) can easily be computed. Using the standard WH procedure [26], we shall yield the solution of equation (9) subject to boundary conditions (10) and continuity conditions (11) after using equation (12) as follows:

$$\begin{aligned} &\Psi(X, Y, \omega) \\ &\quad = \frac{\exp[-iKM(X-X_0) - \frac{\omega^2}{4s^2}]}{4\pi i\sqrt{(1-M^2)}} \int_{-\infty}^{\infty} \frac{1}{\kappa} e^{i[\nu(X-X_0)+\kappa|Y-Y_0|]}\, d\nu \\ &\qquad + \frac{e^{-iKM(X-X_0)-\frac{\omega^2}{4s^2}}}{8\pi^2\sqrt{1-M^2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\nu, \xi, \omega) e^{-i\xi X_0 + i\sqrt{K^2-\xi^2}|Y_0|} e^{i\kappa|Y|+i\nu X}\, d\xi\, d\nu, \end{aligned} \tag{16}$$

where

$$G(\nu, \xi, \omega) = \frac{B[K(1+M^2) + 2\xi M + \frac{\xi^2 M^2}{(1-M^2)K}] - \sqrt{K-\nu}\sqrt{K+\xi} \quad \operatorname{sgn}|Y| \operatorname{sgn}|Y_0|}{L_+(\nu) L_-(\xi)(\xi-\nu)\sqrt{K^2-\nu^2}\sqrt{K^2-\xi^2}}. \tag{17}$$

Here $L_\pm(\nu)$ are the factors of the kernel function $L(\nu)$ arising due to application of WH procedure (explicit expressions of $L_\pm(\nu)$ can found in [5]), $\kappa(\nu) = \sqrt{K^2-\nu^2} = \sqrt{K-\nu}\sqrt{K+\nu}$ are such that $\kappa_-(\nu) = \sqrt{K-\nu}$ and $\kappa_+(\nu) = \sqrt{K+\nu}$; $L_+(\nu)$ and $\kappa_+(\nu)$ are regular in the upper half-plane $\operatorname{Im}\nu > \operatorname{Im}(-K)$, $L_-(\nu)$ and $\kappa_-(\nu)$ are regular in the lower half-plane $\operatorname{Im}\nu < \operatorname{Im}(K)$, and $L(\nu)$ and $\kappa(\nu)$ are regular in the common strip of analyticity $\operatorname{Im}(-K) < \operatorname{Im}\nu < \operatorname{Im}(K)$. Letting

$$\Psi(X, Y, \omega) = \widetilde{I}_1 + \widetilde{I}_2, \tag{18}$$

where

$$\widetilde{I}_1(X, Y, \omega) = \frac{\exp[-iKM(X-X_0) - \frac{\omega^2}{4s^2}]}{4\pi i\sqrt{(1-M^2)}} \int_{-\infty}^{\infty} \frac{1}{\kappa} e^{i[\nu(X-X_0)+\kappa|Y-Y_0|]}\, d\nu, \tag{19}$$

$$\begin{aligned} &\widetilde{I}_2(X, Y, \omega) \\ &\quad = +\frac{e^{-iKM(X-X_0)-\frac{\omega^2}{4s^2}}}{8\pi^2\sqrt{1-M^2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\nu, \xi, \omega) e^{-i\xi X_0 + i\sqrt{K^2-\xi^2}|Y_0|} e^{i\kappa|Y|+i\nu X}\, d\xi\, d\nu, \end{aligned} \tag{20}$$

In order to calculate the total field $\phi(x, y, t)$, we need to find out the inverse temporal Fourier transform of (16). For that, let us rearrange equation (16) in terms of expressions (19)-(20) for the sake of convenience. Introducing the substitutions $X - X_0 = R'\cos\Theta'$, $|Y - Y_0| = R'\sin\Theta'$, $\nu = K\cos(\Theta' + i\varsigma)$ for equation (19) and keeping in mind the fact that

K is a function of ω, we arrive at

$$I_1(X,Y,t) = -\frac{1}{8\pi^2\sqrt{1-M^2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i\omega}{C}(R'\cosh\varsigma - MR'\cos\Theta') - \frac{\omega^2}{4s^2} - i\omega t}\, d\omega\, d\varsigma, \tag{21}$$

where $C = c\sqrt{1-M^2}$. The integral appearing in equation (21) can be solved completely by following the procedure outlined in [27], and we arrive at

$$I_1(X,Y,t) = -\frac{\sqrt{2\pi}\,c}{4\pi^2 C\sqrt{(t+\frac{MR'\cos\Theta'}{C})^2 - \frac{(R')^2}{C^2}}}. \tag{22}$$

Now, before finding the inverse temporal Fourier transform of $\widetilde{I}_2$, we shall calculate the double integral appearing in equation (20). To do so, we introduce the polar coordinates

$$X = R\cos\Theta, \qquad |Y| = R\sin\Theta,$$
$$X_0 = R_0\cos\Theta_0, \qquad |Y_0| = R_0\sin\Theta_0,$$

and thereby using the transformation $\xi = -K\cos(\Theta_0 + ip)$, which changes the contour of integration over ξ into a hyperbola through the point $\xi = -K\cos\Theta_0$. Similarly, by the change of variable $\nu = K\cos(\Theta + iq)$ the contour of integration can be converted from ν into a hyperbola through the point $\nu = K\cos\Theta$. Thus, omitting the details of calculations, we obtain

$$\widetilde{I}_2 = \frac{-ic}{4\pi}\left[\frac{B\{(1+M^2) - 2M\cos\Theta_0 + \frac{M^2\cos^2\Theta_0}{(1-M^2)}\} - 2\sin\frac{\Theta}{2}\sin\frac{\Theta_0}{2}}{L_+(K\cos\Theta)L_-(-K\cos\Theta_0)(\cos\Theta+\cos\Theta_0)\omega\sqrt{RR_0}}\right]$$
$$\times e^{iK(R+R_0-M(X-X_0)) - \frac{\omega^2}{4s^2}}, \tag{23}$$

which, after taking the inverse Fourier transform, by using the method detailed in [28, 29] yields

$$I_2(X,Y,t) = \frac{-ic\widetilde{A}}{8\pi^2}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{\frac{i\omega(R+R_0-MR'\cos\Theta')}{C} - \frac{\omega^2}{4s^2} - i\omega t}\, d\omega, \tag{24}$$

where

$$\widetilde{A} = \left[\frac{B\{(1+M^2) - 2M\cos\Theta_0 + \frac{M^2\cos^2\Theta_0}{(1-M^2)}\} - 2\sin\frac{\Theta}{2}\sin\frac{\Theta_0}{2}}{L_+(K\cos\Theta)L_-(-K\cos\Theta_0)(\cos\Theta+\cos\Theta_0)\sqrt{RR_0}}\right]. \tag{25}$$

Thus, equation (24) can be simplified to

$$I_2(X,Y,t) = \frac{-ic\widetilde{A}}{8\pi^2}\int_{-\infty}^{\infty}\frac{1}{\omega}e^{i\omega\zeta - \frac{\omega^2}{4s^2}}\, d\omega, \tag{26}$$

where $\zeta = \frac{(R+R_0-MR'\cos\Theta')}{C} - t$. Using the results of [28, 29], we have

$$\int_{-\infty}^{\infty} e^{-ax^2+ibx}\, dx = \sqrt{\frac{\pi}{a}}e^{-\frac{b^2}{4a}} \tag{27a}$$

and

$$\int_{\zeta}^{\infty} e^{i\omega t}\, dt = 2\pi\,\delta(\omega) + \frac{i}{\omega} e^{i\zeta\omega}. \tag{27b}$$

Similarly, equation (26) can be simplified to the form

$$2\int_{\zeta}^{\infty} s\sqrt{\pi} e^{-s^2\zeta^2}\, dt = 2\pi + i\int_{-\infty}^{\infty} \frac{e^{i\omega\zeta-\frac{\omega^2}{4s^2}}}{\omega}\, d\omega. \tag{28}$$

Now the integral appearing on the left-hand side of equation (28) can be expressed in terms of complementary error function as follows:

$$\int_{-\infty}^{\infty} \frac{e^{i\omega\zeta-\frac{\omega^2}{4s^2}}}{\omega}\, d\omega = -is\pi \operatorname{erf} c(\zeta) + 2\pi i, \tag{29}$$

where

$$\operatorname{erf} c(\zeta) = \frac{2}{\sqrt{\pi}} \int_{\zeta}^{\infty} e^{-\vartheta^2}\, d\vartheta.$$

Thereby substituting equation (29) into equation (26) reveals that

$$I_2(X,Y,t) = \frac{c\widetilde{A}}{8\pi^2}\bigl[2\pi - \pi \operatorname{erf}(s\zeta)\bigr]. \tag{30}$$

Hence, the total diffracted field of a Gaussian pulse by an absorbing half-plane satisfying Myers' impedance condition is given by

$$\phi(x,y,t) = -\frac{\sqrt{2\pi}\,c}{4\pi^2 C\sqrt{(t+\frac{MR'\cos\Theta'}{C})^2 - \frac{(R')^2}{C^2}}} + \frac{c\widetilde{A}}{8\pi^2}\bigl[2\pi - \pi \operatorname{erf}(s\zeta)\bigr]. \tag{31}$$

6 Discussion

We have calculated the total diffracted field of the transient nature in terms of equation (31), where the first term represents the field at the observation point coming directly from the Gaussian line source, and the second term contains the effects of Gaussian pulse through the term $\operatorname{erf}(s\zeta)$. It is noted that the effect of modified absorbent half-plane is represented through the term

$$\biggl[\frac{B\{(1+M^2) - 2M\cos\Theta_0 + \frac{M^2\cos^2\Theta_0}{(1-M^2)}\} - 2\sin\frac{\Theta}{2}\sin\frac{\Theta_0}{2}}{L_+(K\cos\Theta)L_-(-K\cos\Theta_0)(\cos\Theta+\cos\Theta_0)\sqrt{RR_0}}\biggr].$$

An important feature of the presented analysis is that the diffracted field contains the term of order M^2, which gives the diffracted field in an improved form when compared with the diffracted field obtained by Rawlins [4] that retained the terms of order M. This is because of the consideration of Ingard's boundary conditions in the form $[\frac{\partial}{\partial y} \mp 2\beta M\frac{\partial}{\partial x} \pm ik\beta]\psi(x,0^{\pm},\omega) = 0$, whereas Myers' impedance boundary conditions are of the form $[\frac{\partial}{\partial y} \mp 2\beta M\frac{\partial}{\partial x} \pm ik\beta \mp \frac{i\beta M^2}{k}\frac{\partial^2}{\partial x^2}]\psi(x,0^{\pm},\omega) = 0$. Explicitly, the term $\frac{i\beta M^2}{k}\frac{\partial^2}{\partial x^2}$ contains the effects of the Mach number parameter up to order M^2, which is known as the correction/perturbation term in the literature. With this we conclude that the diffracted field obtained herein contains higher-order accuracy of the Mach number as compared with the results of Rawlins [4]. Also, it is observed that the diffracted field starts reaching the point (x, y) after the time lapse $t' > \frac{1}{C}(R + R_0)$ where $t' = t + \frac{MR' \cos\Theta'}{C}$, and the strength of the field decays as $\frac{1}{\sqrt{RR_0}}$. Moreover, it is observed that results for the rigid barrier can be obtained while taking the parameter $\beta = 0$, whereas the results for still air case can be obtained by neglecting the Mach number M ($M = 0$).

Competing interests
Authors declare that they have no competing interests.

Authors' contributions
All authors participated in drafting, revising, and commenting the manuscript. Also, each of the authors read and approved the final draft of manuscript.

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