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Entropy solution of fractional dynamic cloud computing system associated with finite boundary condition

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Abstract

Cloud computing is relevant for the applications transported as services over the hardware and for the Internet and systems software in the datacenters that deliver those services. The major problem for this state is computing the capacity and the amplitude of the dynamic system of these services. In this effort, we process an algorithm based on fractional differential stochastic equation (fractional Fokker-Planck equation (FFPE)) to find the fractional entropy solutions. Our tool is based on Mellin-Laplace transforms. Also, we suggest a fractional functional entropy formula by using the Tsallis entropy. Approximate outcomes are illustrated and discussed. The convergence of the method is investigated.

Keywords: fractional calculus; fractional dynamical system; fractional entropy; cloud computing

1 Introduction

Fractional calculus has many applications, not only in mathematics, but in other sciences, engineering, economics, and social studies. It covenants with differential and integral operators involving arbitrary powers; real and complex. It is associated with many well-known names such as Abel, Caputo, Euler, Grunwald, Hadamard, Hardy, Heaviside, Jumarie, Laplace, Leibniz, Letnikov, Liouville, Riemann, Riesz, and Weyl. The central purpose, or, at least, one of the chief purposes in considering and studying fractional calculus, is the circumstance that fractional calculus appears to be fairly significant in the investigation of some problems which arise in fractal space-time physics. We have physical schemes at three unalike stages of thought: microscopic, megascopic, and macroscopic. The fractional calculus approximates the classical calculus, and it includes non-commutative derivatives, which appears to be fairly reliable on using non-commutative geometry. This development leads one to generalize the information theory of fractional order. The books of Oldham and Spanier [1], Srivastava and Owa [2], Oustaloup [3], Miller and Ross [4], Samko *et al.* [5], Kiryakova [6], Mainardi [7], Podlubny [8], Hilfer [9], Zaslavsky [10], Kilbas *et al.* [11], Magin [12], Sabatier *et al.* [13], Hilfer [14], Mainardi [15], Monje *et al.* [16], Klafter *et al.* [17], Tarasov [18], Baleanu *et al.* [19], Yang [20], Jumarie [21, 22], *etc.* have enriched all areas of applied sciences. However, certain mathematical problems remain and baffle us. The complications and most of the recognized key mathematical problems in the field have been determined up to a point. There were practically no applied formulations of requirements in different areas. The developments in these areas continue [1–22]. The central substantial advantage of fabricating a procedure of fractional differential equations (ordinary and partial) in scientific modeling is their nonlocal property. It is recognized that the normal derivative is a local, linear operator, while the fractional derivative is nonlocal and non-linear. As a result the subsequent formulation of a system is predisposed by not only its current formal feature, but consistently by all of its preceding ones.

The theory of entropy was introduced in the area of thermodynamics in the 19th century and was utilized by Shannon to improve the information theory. Entropy is a conventional statistic computing concept, showing uniformity and complexity, which achieves promising applications to a widespread variety of reasonable and noisy time series data. The development was motivated by data length restraints that are commonly challenging. Investigators stressed its employment and amplification, and its utility to differentiate associated stochastic processes and models. They deliberated its impact and are stimulated to apply it in a statistically usable manner, such as marginal probability distributions and other methods. The major outcome is that the density of information so convoluted is formulated by the derivative of the function or its fractional derivative, depending upon whether it is differentiable or not. As regards information theory, one may compare the perspectives between the probability density and the derivative of a function. Fractional entropy appeared due to Tsallis (1988) (see [23]). Many investigators published different studies to improve this concept (see [24–32]).

Recently, cloud computing (CC) has developed as one of the newest and most general network computing models in various areas, such as academic circles, governments, information industry, *etc.* It is now essential for the new compeers information technology modification, and it expresses the progress of great scales, increasing focus on the relevance in IT studies. In an environment of CC, data is stockpiled on the cloud and manipulators can attain the influential computing capability from the cloud, devoid of getting those costly substructures. A manipulator would purchase the service of CC and attain demand as extended as suggested to the cloud service supplier and paying the lowest price. The experimental results show that the entropy is the best method to select the service of CC (see [32, 33]). This study leads to the probability capacity and the amplitude of the dynamic system of these services.

In this work, we develop an algorithm based on the fractional differential stochastic equation (FFPE) to find the fractional entropy solutions. These solutions are employed to compute the capacity and the amplitude of fractional dynamic systems. Our tool is based on the Mellin-Laplace transforms. Moreover, we propose the fractional functional entropy formulated by the Tsallis concept of entropy. Numerical results are presented for illustration. The convergence of the method is investigated.

2 Processing

Our approach deals with the following concepts.

2.1 The fractional Fokker-Planck equation (FFPE)

In this effort, we consider the following fractional differential equation with diffusion coefficients:

$$D_t^{\nu}\wp(\Lambda,t) = -\frac{\partial}{\partial\Lambda}\big[\mu(\Lambda)\wp(\Lambda,t)\big] + \frac{1}{2}\frac{\partial^2}{\partial\Lambda^2}\big[\sigma^2(\Lambda)\wp(\Lambda,t)\big], \tag{1}$$

subject to the initial condition

$$\wp(\Lambda,0) := \overline{\wp}(\Lambda)$$

and the boundary condition of $\wp(\Lambda,t)$ is finite as $\Lambda \to 0$,

$$\begin{aligned}&\epsilon_1\wp(0,\tilde{t}) + \epsilon_2\wp(0,t) = \varrho\\ &\quad\bigl(\epsilon_1+\epsilon_2 \neq 0, 0 < \tilde{t} < t \in J = [0,T], T < \infty, \varrho < \infty\bigr)\end{aligned}$$

and

$$\wp(\Lambda,t) \to 0, \qquad \frac{\partial\wp(\Lambda,t)}{\partial\Lambda} \to 0, \quad \Lambda \to \infty,$$

where $\overline{\wp}(\Lambda)$ is a function of the amplitude response Λ, $\wp(\Lambda,t)$ is the density function of Λ with respect time $t \in J$, D_t^{ν} is referred to the Riemann-Liouville calculus introduced by the formula

$$D_s^{\nu}\wp(s) = \frac{d}{ds}\int_0^s \frac{(s-\varsigma)^{-\nu}}{\Gamma(1-\nu)}\wp(\varsigma)\,d\varsigma,$$

which coincides with the fractional integral operator

$$I_a^{\nu}\wp(s) = \int_a^s \frac{(s-\varsigma)^{\nu-1}}{\Gamma(\nu)}\wp(\varsigma)\,d\varsigma,$$

such that $\nu \in (0,1)$. Moreover, Λ is the capacity of the outcome of the system, $\wp$ is the probability density with respect to the capacity of the system, and the diffusion coefficients are polynomials in Λ such that

$$\mu(\Lambda) = \sum_{k=1}^{n_1}\alpha_k\Lambda^k, \tag{2}$$

$$\sigma^2(\Lambda) = \beta_0 + \sum_{k=1}^{n_2}\beta_k\Lambda^k, \tag{3}$$

where α_k and β_k are polynomial coefficients. Our aim is to solve equation (1), by using the fractional entropy (of Tsallis type) subject to boundary and initial conditions.

2.2 The fractional entropy

The Tsallis fractional entropy [23] is formulated by

$$\top_\lambda(\wp) = \frac{\int_\Lambda[\wp(\Lambda)]^\lambda\,d\Lambda - 1}{1-\lambda}, \quad \lambda \neq 1,$$

or in discrete form

$$\top_\lambda(\wp) = \frac{1}{\lambda-1}\left(1-\sum_{k=1}^{m}\wp_k^\lambda\right), \quad \lambda \neq 1.$$

On this level, we require a computation of an applicable aggregate of the information created by noticing the entrance of an occurrence having probability $\wp \in [0,1]$. In this discussion, we suggest the functional entropy formula

$$\top_\lambda(\wp)(\Lambda,t) = \frac{\int_\Lambda [\wp(\Lambda,t)]^\lambda \, d\Lambda - 1}{1-\lambda}, \quad \lambda \neq 1.$$

Fractional entropy introduces in a natural way supplementary information as regards the implication of individual processes and to regulator modification. If λ has a large positive rate this measure is additional slight to records that arise often; nevertheless for large negative λ it is slighter to the processes which arise seldom. Obviously dimension methods are attractive considering a construction in the entropy calculations. The hypothetical work was surpassing to successfully differentiate dynamical systems expecting finite, noisy data, or to confirm a deterministic background. Accordingly, in these approaches and actions, the collection of data, which is naturally significant to appreciate convergence, is impossibly large. By employing the fractional entropy on $\wp(\Lambda,t)$, we have the fractional entropy system

$$D_t^\nu \top_\lambda(\wp)(\Lambda,t) = -\frac{\partial}{\partial \Lambda}\big[\mu(\Lambda)\top_\lambda(\wp)(\Lambda,t)\big] + \frac{1}{2}\frac{\partial^2}{\partial \Lambda^2}\big[\sigma^2(\Lambda)\top_\lambda(\wp)(\Lambda,t)\big]. \tag{4}$$

2.3 The fractional transform

In this effort, we shall utilize the Mellin transform. This transform has a big capability in many areas such as digital data structures, probabilistic algorithms, asymptotic estimation of integral forms, asymptotic analysis of algorithms and communication theory. By applying the concept of the Mellin transform of the probability density with respect to the capacity, we have

$$\Xi_{\top_\lambda(\wp)}(\rho-1,t) = \int_0^\infty \top_\lambda(\wp)(\Lambda,t)\Lambda^{\rho-1}\, d\Lambda, \quad \rho = a + \imath b \in \mathbb{C}, \Lambda \in [0,\infty),$$

where $a < 1$ is the real part of ρ, while $b \in \mathbb{R}$ is the imaginary part and $\imath$ is the imaginary value, $\sqrt{-1}$. Obviously, the fraction transform is performed by the moment of the fractional complex power. In a discrete form, the converse reads as follows:

$$\wp(\Lambda,t) \approx \frac{1}{2\gamma}\sum_{j=-n}^{n}\frac{\Xi_\wp(\rho_j-1,t)}{\Lambda^{\rho_j}} = \frac{1}{\gamma}\sum_{j=0}^{n}\frac{\Xi_\wp(\rho_j-1,t)}{\Lambda^{\rho_j}},$$

where $\gamma := \pi/\triangle b$, $\triangle b$ is the discretization level on the imaginary axis and $\rho_j := a + \imath j(\pi/\gamma)$. For $\Lambda \in [e^{-\gamma}, e^{\gamma}]$, a calculation yields

$$\Xi_\wp(\rho_0-1,t) = \frac{2\gamma - \sum_{j=-n,j\neq 0}^{n}[e^{-\gamma\rho_j+\gamma} - e^{\gamma\rho_j-\gamma}]/(1-\rho_j)\Xi_\wp(\rho_j-1,t)}{[e^{-\gamma\rho_0+\gamma} - e^{\gamma\rho_0-\gamma}]/(1-\rho_0)}. \tag{5}$$

In view of equation (5), a comparison of two moments shows different powers with different real parts a_1 and a_2 such that $\triangle a = a_2 - a_1$, and we have the following relation:

$$\Xi_\wp\big(\rho_j^{(1)}-1,t\big) = \frac{1}{2\gamma}\sum_{j=-n}^{n}\Xi_\wp\big(\rho_j^{(2)}-1,t\big)K_j(\triangle a), \tag{6}$$

where

$$\rho_j^{(\kappa)} = a_\kappa + \imath j(\pi/\gamma), \quad \kappa = 1,2$$

and

$$K_j(\triangle a) = \frac{\gamma[e^{\triangle a\gamma + \imath j\pi} - e^{-\triangle a\gamma - \imath j\pi}]}{\triangle a\gamma + \imath j\pi}.$$

Now, we multiply equation (1) by $\Lambda^{\rho-1}$, and integrating the outcome with respect to Λ in the interval $[0,\infty)$, we obtain the fractional system

$$\begin{aligned} D_t^\nu \Xi_\wp(\rho-1,t) &= -\bigl[\mu(\Lambda)\wp(\Lambda,t)\bigr]\big|_0^\infty + (\rho-1)\int_0^\infty \Lambda^{\rho-2}\mu(\Lambda)\wp(\Lambda,t)\,d\Lambda \\ &\quad + \frac{1}{2}\frac{\partial}{\partial\Lambda}\bigl[\sigma^2(\Lambda)\wp(\Lambda,t)\bigr]\Lambda^{\rho-1}\big|_0^\infty - \frac{1}{2}(\rho-1)\Lambda^{\rho-2}\bigl[\sigma^2(\Lambda)\wp(\Lambda,t)\bigr]\big|_0^\infty \\ &\quad + \frac{1}{2}(\rho-1)(\rho-2)\int_0^\infty \Lambda^{\rho-3}\bigl[\sigma^2(\Lambda)\wp(\Lambda,t)\bigr]\,d\Lambda. \end{aligned} \tag{7}$$

By employing the assumptions of the system and utilizing (2) and (3), equation (7) can be considered as follows:

$$\begin{aligned} D_t^\nu \Xi_\wp(\rho-1,t) &= (\rho-1)\sum_{k=1}^{n_1}\alpha_k \Xi_\wp(\rho-2+k,t) \\ &\quad + \frac{1}{2}(\rho-1)(\rho-2)\Biggl[\beta_0\Xi_\wp(\rho-3,t) + \sum_{k=1}^{n_2}\beta_k\Xi_\wp(\rho-3+k,t)\Biggr]. \end{aligned} \tag{8}$$

Since $\wp(\Lambda,t)$ is finite when $\Lambda\to 0$, equation (8) is obtained for some a, where a is the real part of ρ. Corresponding to equation (8), we have the following system:

$$\begin{aligned} D_t^\nu \Xi_{\top_\lambda(\wp)}(\rho-1,t) &= (\rho-1)\sum_{k=1}^{n_1}\alpha_k \Xi_{\top_\lambda(\wp)}(\rho-2+k,t) + \frac{1}{2}(\rho-1)(\rho-2) \\ &\quad \times\Biggl[\beta_0\Xi_{\top_\lambda(\wp)}(\rho-3,t) + \sum_{k=1}^{n_2}\beta_k\Xi_{\top_\lambda(\wp)}(\rho-3+k,t)\Biggr]. \end{aligned} \tag{9}$$

2.4 The entropy system

The cloud computing entropy system can be constructed as a multi-agent system. Therefore, it must deal with the world's natural affinity to ailment. Various applications involve a set of agents that are independently autonomous. In this case, each agent concludes its activities established by its own formula, capacity, and the environment. Typically, agents deliberately openly recognize one another additionally, aim approximately at this sensitivity, and then perform a sensible action. In the FFPE model assessment, coordination as an organism is referred to by the environment; agents undergo modification by the capacity of the system. Procedures in the environment produce assemblies that the agents recognize, thus authorizing ordered performance at the agent level. Naturally, these procedures increase ailment and chaos at the agent level, so that the system converts to being less ordered over time. Entropy introduces a good concept describing such a system.

The minimization method for circumventing discreteness agents has the consequence of choosing at each time step the capacity that best centers on the agents. Thus, the state of the asymptotic system completely depends on the entropy solution of the FFPE model. We aim to study and determine the global behavior (self-organization) of the systems by using the capacity.

For this purpose, we assume that s is the agent and that the request is χ_s, $s = 1,\dots,N$ (the dimension of the cloud system). By using the disposition formal relation (see [34]), equation (9) can be rewritten as follows:

$$D_t^\nu \Xi_{\top_\lambda(\wp)}(\rho_s - 1, t) \approx \Xi_{\top_\lambda(\wp)}(\rho_s - 1, t)\left\{\frac{(\rho_s - 1)}{2\gamma}\sum_{k=1}^{n_1}\alpha_k K_j(1-k) + \frac{(\rho_s - 1)(\rho_s - 2)}{4\gamma}\left[\beta_0 K_j(2) + \sum_{k=1}^{n_2}\beta_k K_j(2-k)\right]\right\}. \quad (10)$$

Note that equation (8) may have a divergent solution, but in virtue of the fractional entropy, all the entropy solutions of equation (10) are convergent and hence the probability of the capacity of the system can be computed. Equation (10) can be solved numerically, by applying any method or by employing fractional complex transforms as suggested in [35, 36].

2.5 Approximate solutions

In matrix form, equation (10) can be written as follows:

$$D^\nu \chi(t) = \Upsilon \chi(t), \quad \Upsilon \neq 0, \quad (11)$$

subject to the initial condition

$$\chi(0) = \tau,$$

where $\chi := (\chi_1,\dots,\chi_N)^T$, τ depends on λ and ρ and

$$\chi(t) = \Xi_{\top_\lambda(\wp)}(\rho - 1, t)$$

and

$$\Upsilon = \frac{(\rho - 1)}{2\gamma}\sum_{k=1}^{n_1}\alpha_k K_j(1-k) + \frac{(\rho - 1)(\rho - 2)}{4\gamma}\left[\beta_0 K_j(2) + \sum_{k=1}^{n_2}\beta_k K_j(2-k)\right].$$

The solution of (11) can be formulated by utilizing the Mittag-Leffler function

$$\chi(t) = \tau E_\nu\bigl(\Upsilon t^\nu\bigr), \quad (12)$$

where

$$E_\nu(t) := \sum_{j=0}^{\infty}\frac{t^j}{\Gamma(1 + j\nu)}.$$

It is well known that E_ν shows the following asymptotic behavior (see [37]; Theorem 1):

$$E_\nu(t) \sim \frac{1}{\nu} e^{t^{1/\nu}}, \quad \nu \neq 0. \tag{13}$$

Hence we have the result

$$\chi(t) \approx \frac{\tau}{\nu} e^{t\Upsilon^{1/\nu}}. \tag{14}$$

3 Results and discussion

The consequences of the adopted method to evaluate the probability of the capacity through the entropy solution is characterized by the discrete symbols. The system converges to the diffusion of the origin. The probability of the capacity of the integer system of (14), for a fixed time, can be computed by the formal expression

$$\wp(\Lambda) = \frac{\theta}{\sigma^2(\Lambda)} \exp\left(\int \frac{2\mu(\Lambda)}{\sigma^2(\Lambda)} \, d\Lambda \right),$$

where θ is the normalized constant,

$$\nu = 1, \qquad \tau = \frac{\theta}{\sigma^2(\Lambda)} \quad \text{and} \quad \Upsilon = \int \frac{2\mu(\Lambda)}{\sigma^2(\Lambda)} \, d\Lambda.$$

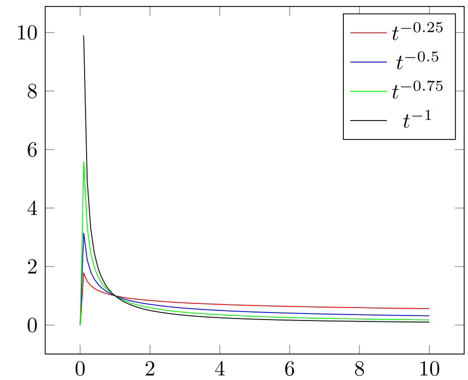
The initial condition adopted above indicates the stationary states at the initial time. The justification of the suggested technique to evaluate the cost response of a system is demonstrated. Moreover, for $\tilde{t} = t$, the boundary condition implies that

$$\wp(0) = \frac{\varrho}{\epsilon_1 + \epsilon_2}, \quad \epsilon_1 + \epsilon_2 \neq 0.$$

Therefore, the initial cost of the cloud system is evaluated by the above equation, which is basically determined by the boundary condition of the system (1). Numerous classes of fractional boundary problems are suggested in [38–40].

Compared with this formal approach, the suggested technique has the benefit that it does not depend on an assortment of series or the fundamental form of the probability density; meanwhile it is constructed based on the moments of the complex fractional power. However, the system measured in the current document is a non-linear system with polynomial damping coefficients and excitation capacity, and the current technique can be employed in problems with non-linear rigidity by presenting a Taylor series expansion of fractional order. Furthermore, the technique of joining the stochastic function, entropy, and Mellin transforms can be straightforwardly prolonged to calculate the passing reaction probability density of the fractional non-linear system by assuming the fractional stochastic formal system. Figure 1 shows the convergence of the entropy solution with respect to time and the entropy fractional order. Moreover, it shows that increasing time and the fractional order λ caused the information as regards the system to increase. This leads to an increase of the capacity. Another advantage of the proposed method is that by the utility of the fractional entropy we avoid any rigorous constraint, because of the convergence of the solution not only in a finite interval, but also for a domain.

Figure 1 Entropy fractional solution.



4 Application

The job scheduling scheme is interesting and one of the essential research fields in cloud computing. It plays a similar role itself in cloud computing. The job scheduling system is accountable to choose the best appropriate resources in a cloud computing users' jobs, by compelling various static and dynamic parameter constraints of the cloud into the deliberation. In this section, we deal with a model that describes the job scheduling system based on the queuing property and cost function considering the users, providers, and the quality of the system (QoS). By utilizing a cloud computing environment, we may assume it as a very influential server. This server holds the user's jobs. For each job one may have a different QoS obligation; typically, the user's jobs have various agencies to be treated. Therefore, we can classify the jobs' urgencies into several classes. Customarily, since the cloud computes resources, customers continuously deliberate which cloud computing resource can encounter their job QoS supplies for computing (such as the paid time of job ruining, the calculating capacity), and how much the cost is that they must feed for the cloud computing resources.

We assume that the cloud computing system has a dynamic according to equation (4). Moreover, we consider the user's jobs in the similar group with urgency to acquiesce to the cloud agreeing to the outcome $\chi_i(t)$, given in (14) for the user $i = 1, \dots, n$. Each group is evaluated by the capacity Λ (for example, if the service has five different jobs, then the capacity obeys $\Lambda = 1, \dots, 5$). Suppose that the total requested groups, the service rate, and intensity of the cloud computing environment are

$$\chi(t) = \sum_{i=1}^{n} \chi_i(t), \qquad f(t) = \sum_{i=1}^{n} f_i(t,\chi), \qquad \phi(t) = \sum_{i=1}^{n} \phi_i(t,\chi), \quad t \in J = [0,1],$$

respectively. Then the total cost function of the cloud service is formulated by

$$\begin{aligned} \Phi\bigl(t, \chi(t), f(t), \phi(t)\bigr) &= \omega_1 \chi(t) + \omega_2 f(t) + \omega_3 \phi(t) \\ &= \sum_{i=1}^{n} \varphi_i \chi_i(t), \end{aligned} \tag{15}$$

where α_i, $i = 1, \dots, n$, are the connection constants in the cloud. The problem for the cloud computing service benefactor is how to utilize a job scheduling system to allocate the appropriate cloud to get the minimum cost value. The minimization of the cost appears

Table 1 The cost function with $\tau = 1$, $\lambda = 2$, $\rho = 2$

Capacity (Λ)	Time	χ_i (14)	Cost: ($\nu = 1$)	$\nu = 0.75$	$\nu = 0.5$	$\nu = 0.25$
1	0.1	30	70.37	57.444	39.712	19.412
2	0.25	60	191.25	156.122	107.928	52.758
3	0.55	90	203.78	166.351	115	56.215
4	0.75	120	420.16	342.687	237.11	115.9
5	1	150	550.89	449.7	310.88	151.969

by using the fractional calculus (see Table 1); here, for user i, we select $\varphi_i \in (0,1]$ for all $i = 1,\ldots,n$ and

$$\Upsilon_i = \frac{(\rho-1)}{2\gamma}\sum_{k=1}^{n_1}\alpha_k K_i(1-k) + \frac{(\rho-1)(\rho-2)}{4\gamma}\left[\beta_0 K_i(2) + \sum_{k=1}^{n_2}\beta_k K_i(2-k)\right],$$

corresponding to the solution χ_i such that

$$K_i(\triangle a) = \frac{\gamma\,[e^{\triangle a\gamma+\imath i\pi} - e^{-\triangle a\gamma-\imath i\pi}]}{\triangle a\gamma + \imath i\pi}.$$

The QoS vector of the unique service i is defined as

$$\mathcal{Q}(\Phi) = \big(\xi_1(\varphi_1\chi_1),\ldots,\xi_n(\varphi_n\chi_n)\big),$$

where ξ_i is the value of QoS parameter i for a unique service.

Table 1 shows the initial service rate and expectant service rate for each group in the queue with changed significance. Fractional calculus is utilized to minimize the cost. The decreasing of the fractional value $\nu \in (0,1]$ implies the minimization of the cost function.

4.1 Discrete cloud system

We consider the discrete cloud system of equation (4), by utilizing the formula

$$D_t^\nu \top_\lambda(\wp)(\triangle\Lambda,t) = \sum_{k=1}^{M}\omega(\triangle_k\Lambda)\top_\lambda(\wp)(\triangle_k\Lambda,t), \tag{16}$$

where $\triangle\Lambda$ is the difference operator that the capacity changes in the cloud system and $\top_\lambda(\wp)$ is the fractional Tsallis entropy of order $\lambda \neq 1$. Our aim is to find the entropy solution of the system (16). Our tool is based on the concepts of the Laplace and Mellin transforms. Since $\top_\lambda(\wp)$ is the entropy of the distribution function $\wp$, we may assume that

$$\top_\lambda(\wp)(\triangle\Lambda,t) := f(t)\theta(\triangle\Lambda).$$

Thus the system (16) becomes

$$D_t^\nu f(t)\theta(\triangle\Lambda) = \sum_{k=1}^{M}\omega(\triangle_k\Lambda)f(t)\theta(\triangle\Lambda). \tag{17}$$

Since the fractional derivative is due to the time part, equation (17) can be read as the evolution of a fractional Brownian motion,

$$D_t^\nu f(t) = -\varepsilon f(t), \quad \varepsilon \neq 0, \tag{18}$$

where ε depends on the value of the capacity Λ. A computation implies that the Laplace and Mellin transforms of (17) formally are

$$\Xi_{f(t)}(\rho-1,t)=\frac{1}{\Gamma(1-\rho)}\Xi_{\angle(f(t),u)}(1-\rho,t), \tag{19}$$

where $\angle(f(t),u)$ is the Laplace transform of the function $f(t)$ given by

$$\angle\bigl(f(t),u\bigr)=\int_0^\infty e^{-ut}f(t)\,dt.$$

Therefore, we have the following solution, in view of the Laplace transform:

$$f(u)=\frac{f_0}{\varepsilon+u^\nu}, \tag{20}$$

where f_0 is a constant. Employing (19) in (20) and inverting the result to the time domain, we obtain

$$f(t)=\left(\frac{f_0}{\nu}\right)t^{\nu-1}F^{1\,1}_{1\,2}\left(\varepsilon^{1/\nu}\middle|\begin{matrix}(0,1/\nu)\\(0,1/\nu)(1-\nu,1)\end{matrix}\right), \tag{21}$$

where $F^{1\,1}_{1\,2}$ is the well-known Fox function. In the sequel, we suppose that the value of

$$\omega=\varepsilon=(\triangle\Lambda)^2$$

behaves as a diffusion constant. Hence by utilizing the series expansion of the Fox function (see [41]), the entropy solution of equation (16) can be described as follows:

$$\top_\lambda(\wp)(\triangle\Lambda,t)=(1/\nu)\sum_{n=0}^\infty\frac{(-1)^n}{\Gamma(2n\nu+1)}\bigl[(\triangle\Lambda)^2t\bigr]^{2\nu n},\qquad f_0=1, \tag{22}$$

which is a monotonic decreasing function showing the asymptotic behavior (see [37])

$$\top_\lambda(\wp)(\triangle\Lambda,t)\approx t^{-\nu},\quad t\to\infty.$$

5 Conclusion

We introduced a technique based on a class of fractional differential stochastic equations (fractional Fokker-Planck equations) to discuss the fractional entropy solutions of fractional dynamical systems. We showed that the concept of the transforms (see [42–49]) is very useful to complete our investigation. We applied the relation between Mellin and Laplace transforms. The approximate result is utilized to perform in a cloud computing environment system. We imposed straightforwardly a differential service adapted job scheduling system in the cloud computing setting. Examination and outcomes demonstrated that our method for the job scheduling system cannot only ensure the QoS supplies of the cloud computing service jobs, but it also can create the maximum profits for the system, with minimizing the cost.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All the authors jointly worked on deriving the results and approved the final manuscript.

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