Zhang and Feng *Boundary Value Problems* (2017) 2017:172
DOI 10.1186/s13661-017-0904-8

Boundary Value Problems
a SpringerOpen Journal

RESEARCH

Open Access

Nontrivial convex solutions on a parameter of impulsive differential equation with Monge-Ampère operator

Xuemei Zhang[1*] and Meiqiang Feng[2]

*Correspondence: zxm74@sina.com
[1]Department of Mathematics and Physics, North China Electric Power University, Beijing, 102206, People's Republic of China
Full list of author information is available at the end of the article

Abstract

The authors consider the impulsive differential equation with Monge-Ampère operator in the form of

$$\begin{cases} ((u'(t))^n)' = \lambda n t^{n-1} f(-u(t)), & t \in (0,1), t \neq t_k, k = 1,2,\dots,m, \\ \Delta(u')^n|_{t=t_k} = \lambda I_k(-u(t_k)), & k = 1,2,\dots,m, \\ u'(0) = 0, \quad u(1) = 0, \end{cases}$$

where λ is a nonnegative parameter and $n \geq 1$. We show the existence, uniqueness, and continuity results. Our approach is largely based on the eigenvalue theory and the theory of α-concave operators. The nonexistence result of a nontrivial convex solution is also studied by taking advantage of the internal geometric properties related to the problem.

Keywords: continuity on a parameter; existence of nontrivial convex solutions; Monge-Ampère operator; impulsive differential equation; geometric properties

1 Introduction

In natural sciences, there are various concrete problems involving the Monge-Ampère equation. For example, the Monge-Ampère equation can describe Weingarten curvature, or reflector shape design (see [1]). In recent years, increasing attention has been paid to the study of the Monge-Ampère equation by different methods (see [2–10]).

The typical model of the Monge-Ampère equation is

$$\begin{cases} \det(D^2 u) = \lambda f(-u) & \text{in } B, \\ u = 0 & \text{on } \partial B, \end{cases} \tag{1.1}$$

where $B = \{x \in R^n : |x| < 1\}$ is the unit ball in R^n and $D^2 u = (\frac{\partial^2 u}{\partial x_i \partial x_j})$ is the Hessian of u, λ is a nonnegative parameter and $f: R \to R$ is a continuous function.

The study of problem (1.1) in general domains of R^n may be found in [2, 3]. Kutev [4] investigated the existence of strictly convex radial solutions of problem (1.1) when $f(u) = u^p$. Delanoë [5] treated the existence of convex radial solutions of problem (1.1) for a class of

Springer

more general functions, namely $\lambda \exp f(|x|, u, |\nabla u|)$. Recently, under the case $f(u) = e^{\lambda u}$, Zhang and Wang [6] obtained some interesting results of problem (1.1). They got the local structure of the solutions near a degenerate point by using the Lyapunov-Schmidt reduction method and established the global structure by Leray-Schauder degree theory and bifurcation theory.

In [4], Kutev also pointed out that problem (1.1) can reduce to the following boundary value problem:

$$\begin{cases} ((u'(t))^n)' = \lambda n t^{n-1} f(-u(t)), & t \in (0,1), \\ u'(0) = 0, & u(1) = 0. \end{cases}$$

For the case $f \geq 0$, Hu and Wang [8] proved the existence, multiplicity, and nonexistence of strictly convex solutions for the above problem by using fixed point index theory. However, the corresponding results for impulsive Monge-Ampère type equations have not been investigated until now, even for the unique solution u_λ of the above equation depending continuously on the parameter λ.

At the same time, we notice that a class of differential equations with impulsive effects appeared in biological systems, population dynamics, biotechnology, ecology, industrial robotic, and optimal control; for details and references, see [11–14]. Recently, the existence of solutions to the impulsive differential equations has attracted the attention of many researchers; see Bai et al. [15], Agarwal et al. [16], Liu and Guo [17], Wang and Feng [18], Zhang and Tian [19], Karaca and Tokmak [20], Liu et al. [21], Zeng and Xie [22], and Zhang and Ge [23] and the references cited therein. However, it is not difficult to see that there is almost no paper addressing impulsive differential equations with fully nonlinear operator. It is well known that the Monge-Ampère operator is just fully nonlinear. This motivates us to study an impulsive differential equation with Monge-Ampère operator.

Consider the impulsive differential equation with Monge-Ampère operator

$$\begin{cases} ((u'(t))^n)' = \lambda n t^{n-1} f(-u(t)), & t \in (0,1), t \neq t_k, k = 1,2,\ldots,m, \\ \Delta(u')^n|_{t=t_k} = \lambda I_k(-u(t_k)), & k = 1,2,\ldots,m, \\ u'(0) = 0, & u(1) = 0, \end{cases} \tag{1.2}$$

where λ is a nonnegative parameter and $n \geq 1$, t_k ($k = 1,2,\ldots,m$) (here m is a fixed positive integer) are fixed points with $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots < t_m < t_{m+1} = 1$, $\Delta(u')^n|_{t=t_k} = [u'(t_k^+)]^n - [u'(t_k^-)]^n$, where $u'(t_k^+)$ and $u'(t_k^-)$ represent the right-hand limit and left-hand limit of $u'(t)$ at $t = t_k$. In addition, f and I_k satisfy

(H_1) $f \in C(\mathcal{R}^+, \mathcal{R}^+)$, $I_k \in C(\mathcal{R}^+, \mathcal{R}^+)$ with $f(0) = 0$ and $I_k(0) = 0$, where $\mathcal{R}^+ = [0, +\infty)$, $k = 1,2,\ldots,m$.

Some special cases of problem (1.3) have been investigated. For example, when $n = 1$, problem (1.3) reduces to a second order impulsive boundary value problem (1.2), which has been studied in [24]. The authors obtained many existence results by means of the theory of fixed point index in cones. For other related results on problem (1.2), we refer the reader to the references [25–40].

Here we point out that our problem is new in the sense of impulsive Monge-Ampère type equations introduced here. To the best of our knowledge, the existence of single or multiple positive solutions for impulsive Monge-Ampère type equation (1.2) has not yet been studied, especially for the unique solution u_λ of problem (1.2) depending continuously on the parameter λ. In consequence, our main results of the present work will be a useful contribution to the existing literature on the topic of impulsive Monge-Ampère type equations. The existence, uniqueness, and continuity of positive solutions for the given problem are new, though they are proved by applying the well-known method based on the eigenvalue theory and the theory of α-concave operators.

Remark 1.1 Very recently, Han et al. [41] also considered problem (1.2) under the case $I_k = 0$, $k = 1, 2, \ldots, m$. By the bifurcation theory, the authors investigated the existence of strictly convex or concave solutions of problem (1.2). Notice that differential equations with impulses are characterized by sudden changing of their states. This requires a complete different method from those used in [8, 41] to tackle problem (1.2).

Remark 1.2 On the nonexistence, the arguments that we present here are based in geometric properties of the super-sublinearity of f and I at zero and infinity which was not observed in [8, 41] (see Properties 1.1-1.2 below).

The following geometric Properties 1.1-1.2 will be very important in our arguments.

Property 1.1 If $f_0 = 0$ and $f_\infty = 0$, then there exists $R > 0$ such that

$$\frac{f(R)}{R^n} = \max_{u>0} \frac{f(u)}{u^n}. \tag{1.3}$$

Property 1.2 If $I_0(k) = 0$ and $I_\infty(k) = 0$, then there exists $R_k > 0$ such that

$$\frac{I_k(R_k)}{R_k^n} = \max_{u>0} \frac{I_k(u)}{u^n}, \tag{1.4}$$

where

$$I_0(k) = \lim_{u\to 0} \frac{I_k(u)}{u^n}, \qquad I_\infty(k) = \lim_{u\to\infty} \frac{I_k(u)}{u^n}, \quad k = 1, 2, \ldots, m.$$

Remark 1.3 Some ideas of Properties 1.1-1.2 are from [42].

Remark 1.4 There is almost no result except [8] studying the uniqueness of a nontrivial solution of problem (1.2). However, in [8], they did not obtain that the unique solution $u_\lambda(t)$ of problem (1.2) depends continuously on the parameter λ.

2 Some lemmas

Let $J = [0,1]$ and $J' = J \setminus \{t_1, t_2, \ldots, t_m\}$, $J_0 = [t_0, t_1]$, $J_k = (t_k, t_{k+1}]$, $k = 1, 2, \ldots, m$, and

$$PC^1[0,1] = \left\{v \in C[0,1] : v' \in C(t_k, t_{k+1}), v'\left(t_k^-\right) = v'(t_k), \exists v'\left(t_k^+\right), k = 1, 2, \ldots, m\right\}.$$

Then $PC^1[0,1]$ is a real Banach space with the norm

$$\|v\|_{PC^1} = \max\{\|v\|_\infty, \|v'\|_\infty\},$$

where

$$\|v\|_\infty = \sup_{t\in J}|v(t)|, \qquad \|v'\|_\infty = \sup_{t\in J}|v'(t)|.$$

A function $v \in PC^1[0,1] \cap C^2(J')$ is called a solution of problem (1.3) if it satisfies (1.3). A strictly convex solution of (1.3) is negative on $[0,1)$.

Let $v = -u$. Then problem (1.3) is equivalent to the following problem defined on J:

$$\begin{cases} ((-v'(t))^n)' = \lambda n t^{n-1} f(v(t)), & 0 < t < 1, t \neq t_k, k = 1,2,\dots,m, \\ \Delta(-v')^n|_{t=t_k} = \lambda I_k(v(t_k)), & k = 1,2,\dots,m, \\ v'(0) = 0, \qquad v(1) = 0, \end{cases} \tag{2.1}$$

where $\Delta(-v')^n|_{t=t_k} = [-v'(t_k^+)]^n - [-v'(t_k^-)]^n$, $v'(t_k^+)$ and $v'(t_k^-)$ represent the right-hand limit and left-hand limit of $v'(t)$ at $t = t_k$.

The following lemmas will be used in the proof of our main results.

Lemma 2.1 *Assume that* (H_1) *holds. Then*

(i) *If* $v(t)$ *is a solution of problem* (2.1) *on* J, *then* $u(t) = -v(t)$ *is a solution of problem* (1.3) *on* J;

(ii) *If* $u(t)$ *is a solution of problem* (1.3) *on* J, *then* $v(t) = -u(t)$ *is a solution of problem* (2.1) *on* J.

Therefore, throughout this paper we shall study positive concave classical solutions of problem (2.1).

Lemma 2.2 *Assume that* (H_1) *holds. Then* $v \in PC^1[0,1] \cap C^2(J')$ *is a solution of problem* (2.1) *if and only if* $v \in PC^1[0,1]$ *is a solution of the following equation*:

$$v(t) = \int_t^1 \left(\int_0^\tau \lambda n s^{n-1} f(v(s))\, ds + \lambda \sum_{t_k \le \tau} I_k(v(t_k)) \right)^{\frac{1}{n}} d\tau \tag{2.2}$$

and

$$\min_{t\in[0,\xi]} v(t) \ge \delta \|v\|_{PC^1}, \tag{2.3}$$

where $\xi \in (0,1)$ *and*

$$\delta = 1 - \xi. \tag{2.4}$$

Proof If $0 \le t < t_1$, it is easy to see by integration from 0 to t of problem (2.1) that

$$(-v'(t))^n - (-v'(0))^n = \int_0^t \lambda n r^{n-1} f(v(r))\, dr.$$

If $t_1 \le t < t_2$, then we have

$$\big(-v'(t_1^-)\big)^n - \big(-v'(0)\big)^n = \int_0^{t_1^-} \lambda n r^{n-1} f\big(v(r)\big)\,dr,$$
$$\big(-v'(t)\big)^n - \big(-v'(t_1^+)\big)^n = \int_{t_1^+}^{t} \lambda n r^{n-1} f\big(v(r)\big)\,dr.$$

It follows that

$$\big(-v'(t)\big)^n - \big(-v'(0)\big)^n = \int_0^{t} \lambda n r^{n-1} f\big(v(r)\big)\,dr + I_1\big(v(t_1)\big).$$

If $t_k \le t < t_{k+1}$, we have

$$\big(-v'(t)\big)^n - \big(-v'(0)\big)^n = \int_0^{t} \lambda n r^{n-1} f\big(v(r)\big)\,dr + \sum_{t_k \le t} I_k\big(v(t_k)\big).$$

Then

$$-v'(t) = \left[\int_0^{t} \lambda n r^{n-1} f\big(v(r)\big)\,dr + \sum_{t_k \le t} I_k\big(v(t_k)\big)\right]^{1/n}.$$

Integrating again, we obtain

$$v(t) = \int_t^1 \left[\int_0^{\tau} \lambda n r^{n-1} f\big(v(r)\big)\,dr + \sum_{t_k \le \tau} I_k\big(v(t_k)\big)\right]^{1/n} d\tau.$$

Conversely, if $v \in PC^1[0,1]$ is a solution of (2.2).

Direct differentiation of (2.2) implies

$$v'(t) = -\left(\int_0^{t} \lambda n s^{n-1} f\big(v(s)\big)\,ds + \sum_{t_k \le t} I_k\big(v(t_k)\big)\right)^{1/n}.$$

Evidently,

$$\Delta\big(-v'\big)^n|_{t=t_k} = \lambda I_k\big(v(t_k)\big) \quad (k = 1,2,\ldots,m), \qquad v'(0) = 0, \qquad v(1) = 0.$$

Finally, we show that (2.3) holds. It is clear that $v'(t) = -(\int_0^t \lambda n s^{n-1} f(v(s))\,ds + \sum_{t_k \le t} I_k(v(t_k)))^{1/n}$, which implies that

$$\|v\|_{PC^1} = v(0), \qquad \min_{t\in J} v(t) = v(1).$$

As we assume that $f(v) \ge 0$, we see that any nontrivial solution v of problem (2.1) is concave on J, i.e., $v'' \le 0$, and then we get $v'(t)$ is nonincreasing on J.

So, for every $t \in (0,\xi]$, we have

$$\frac{v(1) - v(0)}{1} \le \frac{v(t) - v(0)}{t},$$

i.e., $v(t) - v(0) \geq t(v(1) - v(0))$, and then

$$v(t) \geq (1-t)v(0) \geq (1-\xi)v(0), \quad \forall t \in [0,\xi].$$

This shows that (2.3) holds. The lemma is proved. □

To establish the existence of positive concave classical solutions in $PC^1[0,1] \cap C^2(J')$ of problem (2.1), we construct a cone K in $PC^1[0,1]$ by

$$K = \left\{ v \in PC^1[0,1] : v \geq 0, \min_{t\in[0,\xi]} v(t) \geq \delta \|v\|_{PC^1} \right\},$$

where δ is defined in (2.4). It is easy to see that K is a closed convex cone of $PC^1[0,1]$.

Remark 2.1 The definition of K is completely different from those of [8, 9].

Remark 2.2 K is a solid normal cone, and

$$K^0 = \left\{ v \in PC^1[0,1] : v > 0, \min_{t\in[0,\xi]} v(t) \geq \delta \|v\|_{PC^1} \right\}.$$

Define $T : K \to PC^1[0,1]$ by

$$(Tv)(t) = \int_t^1 \left(\int_0^\tau ns^{n-1} f\big(v(s)\big)\, ds + \sum_{t_k \leq \tau} I_k\big(v(t_k)\big) \right)^{\frac{1}{n}} d\tau. \tag{2.5}$$

From (2.5) and Lemma 2.2, it is easy to obtain the following result.

Lemma 2.3 *Assume that* (H_1) *holds. Then* $T : K \to K$ *is completely continuous.*

Proof Similar to the proof of Lemma 2.2, we can show that $T : K \to K$. The complete continuity of T is well known. □

Lemma 2.4 *Suppose that* D *is an open subset of an infinite-dimensional real Banach space* E, $\theta \in D$, *and* P *is a cone of* E. *If the operator* $\Gamma : P \cap D \to P$ *is completely continuous with* $\Gamma\theta = \theta$ *and satisfies*

$$\inf_{x\in P\cap\partial D} \Gamma x > 0,$$

then Γ *has a proper element on* $P \cap \partial D$ *associated with a positive eigenvalue. That is, there exist* $x_0 \in P \cap \partial D$ *and* μ_0 *such that* $\Gamma x_0 = \mu_0 x_0$.

Lemma 2.5 *Suppose that* P *is a normal cone of a real Banach space,* $A : P^\circ \to P^\circ$ *is an* α-*concave increasing (or* $-\alpha$-*convex decreasing) operator. Then* A *has exactly one fixed point in* P°.

3 Existence and nonexistence of nontrivial convex solutions on a parameter

In this section, we shall establish the existence and nonexistence results of nontrivial convex solutions on a parameter for problem (1.2). We now state and prove our main results.

Theorem 3.1 *Assume that* (H_1) *holds. If* $0 < f_\infty < +\infty$, $0 < I_\infty(k) < +\infty$ $(k = 1, 2, \ldots, m)$, *then there exists* $\beta_0 > 0$ *such that, for every* $R > \beta_0$, *problem* (1.2) *has a strictly convex solution* $u_R(t)$ *satisfying* $\|u_R\|_{PC^1} = R$ *for any*

$$\lambda = \lambda_R \in [\lambda_0, \bar{\lambda}_0], \tag{3.1}$$

where λ_0 *and* $\bar{\lambda}_0$ *are two positive finite numbers.*

Proof It follows from $0 < f_\infty < +\infty$ and $0 < I_\infty(k) < +\infty$ that there exist $0 < l_1 < l_2$, $\mu > 0$ such that

$$l_1 v^n < f(v) < l_2 v^n, \qquad l_1 v^n < I_k(v) < l_2 v^n, \quad k = 1, 2, \ldots, m, \forall v \geq \mu.$$

Now, we prove that $\beta_0 = \frac{\mu}{\delta}$ is required. Let

$$\Omega_R = \left\{x \in PC^1[0,1] : \|x\|_{PC^1} < R\right\}.$$

Then Ω_R is a bounded open subset of the Banach space $PC^1[0,1]$ and $\theta \in \Omega_R$. Together with Lemma 2.3, we have $T : K \cap \bar{\Omega}_R \to K$ is completely continuous with $T\theta = \theta$.

Noticing $R > \beta_0$, for any $v \in K \cap \partial\Omega_R$, we have

$$v(t) \geq \delta \|v\|_{PC^1} = \delta R, \quad t \in [0, \xi],$$

and then

$$v(t) \geq \delta \|v\|_{PC^1} > \delta \beta_0 = \mu, \quad t \in [0, \xi].$$

Therefore, for any $v \in K \cap \partial\Omega_R$, we have

$$\begin{aligned}
(Tv)(t) &= \int_t^1 \left(\int_0^\tau n s^{n-1} f\big(v(s)\big)\, ds + \sum_{t_k \leq \tau} I_k\big(v(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&\geq \int_t^1 \left(\int_0^\tau n s^{n-1} f\big(v(s)\big)\, ds\right)^{\frac{1}{n}} d\tau \\
&\geq \int_\xi^1 \left(\int_0^\xi n s^{n-1} f\big(v(s)\big)\, ds\right)^{\frac{1}{n}} d\tau \\
&\geq \int_\xi^1 \left(\int_0^\xi n s^{n-1} l_1 v^n(s)\, ds\right)^{\frac{1}{n}} d\tau \\
&\geq \int_\xi^1 \left(\int_0^\xi n s^{n-1} l_1 \delta^n \|v\|_{PC^1}^n\, ds\right)^{\frac{1}{n}} d\tau \\
&= \xi(1-\xi) l_1^{\frac{1}{n}} \delta R,
\end{aligned}$$

which shows that

$$\inf_{v\in K\cap\partial\Omega_R} Tv \geq \xi(1-\xi)l_1^{\frac{1}{n}}\delta R > 0.$$

By Lemma 2.4, for any $R > \beta_0$, the operator T has a proper element $v_R \in K$ associated with the eigenvalue $\mu_R > 0$, further v_R satisfies $\|v_R\|_{PC^1} = R$. Let $\lambda_R = \frac{1}{\mu_R^n}$. Then problem (2.1) has a positive solution v_R associated with λ_R.

From the proof above, for any $R > \beta_0$, there exists a positive solution $v_R \in K \cap \partial\Omega_R$ associated with $\lambda = \lambda_R > 0$. Thus,

$$v_R(t) = \lambda_R^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau ns^{n-1} f\big(v_R(s)\big)\,ds + \sum_{t_k\leq\tau} I_k\big(v_R(t_k)\big)\right)^{\frac{1}{n}} d\tau$$

with $\|v_R\| = R$.

On the one hand,

$$\begin{aligned}
v_R(t) &= \lambda_R^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau ns^{n-1} f\big(v_R(s)\big)\,ds + \sum_{t_k\leq\tau} I_k\big(v_R(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&\leq \lambda_R^{\frac{1}{n}} \int_0^1 \left(\int_0^\tau ns^{n-1} f\big(v_R(s)\big)\,ds + \sum_{t_k\leq\tau} I_k\big(v_R(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&\leq \lambda_R^{\frac{1}{n}} \int_0^1 \left(\int_0^\tau ns^{n-1} l_2 v_R^n(s)\,ds + \sum_{t_k\leq\tau} l_2 v_R^n(s)\right)^{\frac{1}{n}} d\tau \\
&\leq (\lambda_R l_2)^{\frac{1}{n}} \|v_R\|_{PC^1} \int_0^1 \left(\int_0^\tau ns^{n-1}\,ds + m\right)^{\frac{1}{n}} d\tau \\
&\leq (\lambda_R l_2)^{\frac{1}{n}} \|v_R\|_{PC^1} \int_0^1 \big(\tau^n + m\big)^{\frac{1}{n}} d\tau \\
&< (\lambda_R l_2)^{\frac{1}{n}} \|v_R\|_{PC^1} (1+m)^{\frac{1}{n}}, \\
\big|v_R'(t)\big| &= \lambda_R^{\frac{1}{n}} \left(\int_0^t ns^{n-1} f\big(v_R(s)\big)\,ds + \sum_{t_k\leq\tau} I_k\big(v_R(t_k)\big)\right)^{\frac{1}{n}} \\
&\leq \lambda_R^{\frac{1}{n}} \left(\int_0^1 ns^{n-1} f\big(v_R(s)\big)\,ds + \sum_{t_k\leq\tau} I_k\big(v_R(t_k)\big)\right)^{\frac{1}{n}} \\
&\leq \lambda_R^{\frac{1}{n}} \left(\int_0^1 ns^{n-1} l_2 v_R^n(s)\,ds + \sum_{t_k\leq\tau} l_2 v_R^n(s)\right)^{\frac{1}{n}} \\
&\leq (\lambda_R l_2)^{\frac{1}{n}} \|v_R\|_{PC^1} \left(\int_0^1 ns^{n-1}\,ds + m\right)^{\frac{1}{n}} \\
&\leq (\lambda_R l_2)^{\frac{1}{n}} \|v_R\|_{PC^1} (1+m)^{\frac{1}{n}},
\end{aligned}$$

which implies that

$$\|v_R\|_{PC^1} = R \leq (\lambda_R l_2)^{\frac{1}{n}} \|v_R\|_{PC^1} (1+m)^{\frac{1}{n}},$$

and hence,

$$\lambda_R \geq \frac{1}{l_2(1+m)} = \lambda_0.$$

On the other hand,

$$\begin{aligned}
v_R(t) &= \lambda_R^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau ns^{n-1} f\bigl(v_R(s)\bigr)\,ds + \sum_{t_k \leq \tau} I_k\bigl(v_R(t_k)\bigr) \right)^{\frac{1}{n}} d\tau \\
&\geq \lambda_R^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau ns^{n-1} f\bigl(v_R(s)\bigr)\,ds \right)^{\frac{1}{n}} d\tau \\
&\geq \lambda_R^{\frac{1}{n}} \int_\xi^1 \left(\int_0^\xi ns^{n-1} f\bigl(v_R(s)\bigr)\,ds \right)^{\frac{1}{n}} d\tau \\
&\geq \lambda_R^{\frac{1}{n}} \int_\xi^1 \left(\int_0^\xi ns^{n-1} l_1 v_R^n(s)\,ds \right)^{\frac{1}{n}} d\tau \\
&\geq \lambda_R^{\frac{1}{n}} \int_\xi^1 \left(\int_0^\xi ns^{n-1} l_1 \bigl(\delta \|v_R\|_{PC^1}\bigr)^n\,ds \right)^{\frac{1}{n}} d\tau \\
&= (\lambda_R l_1)^{\frac{1}{n}} \delta \|v_R\|_{PC^1} \xi(1-\xi),
\end{aligned}$$

which shows that

$$\|v_R\|_{PC^1} = R \geq (\lambda_R l_1)^{\frac{1}{n}} \delta \|v_R\|_{PC^1} \xi(1-\xi),$$

and hence,

$$\lambda_R \leq \frac{1}{(l_1\delta(1-\xi)\xi)^n} = \bar{\lambda}_0.$$

In conclusion, $\lambda_R \in [\lambda_0, \bar{\lambda}_0]$. It follows from Lemma 2.1 that Theorem 3.1 holds. The proof is complete. □

Theorem 3.2 *Assume that* (H_1) *holds. If* $0 < f_0 < +\infty$, $0 < I_0(k) < +\infty$ $(k = 1, 2, \ldots, m)$, *then there exists* $\beta_0^* > 0$ *such that, for every* $0 < r_0 < \beta_0^*$, *problem* (1.2) *has a strictly convex solution* $u_{r_0}(t)$ *satisfying* $\|u_{r_0}\|_{PC^1} = r_0$ *associated with*

$$\lambda = \lambda_{r_0} \in \bigl[\lambda_0^*, \bar{\lambda}_0^*\bigr],$$

where λ_0^* *and* $\bar{\lambda}_0^*$ *are two positive finite numbers.*

Proof The proof is similar to that of Theorem 4.1, we omit it here. □

Theorem 3.3 *Assume that* (H_1) *holds. If* $f_\infty = +\infty$, $I_\infty(k) = +\infty$ $(k = 1, 2, \ldots, m)$, *then there exists* $\bar{\beta}_0 > 0$ *such that, for every* $r_* > \bar{\beta}_0$, *problem* (1.2) *has a strictly convex solution* $u_{r_*}(t)$ *satisfying* $\|u_{r_*}\|_{PC^1} = r_*$ *for any*

$$\lambda = \lambda_{r_*} \in (0, \lambda_*], \tag{3.2}$$

where λ_* *is a positive finite number.*

Proof It follows from $f_\infty = +\infty$ and $I_\infty(k) = +\infty$ that there exist $l_* > 0$, $\mu^* > 0$ such that

$$f(v) > l_* v^n, \qquad I_k(v) > l_* v^n, \quad k = 1,2,\ldots,m, \forall v \geq \mu^*.$$

Now, we prove that $\bar{\beta}_0 = \frac{\mu^*}{\delta}$ is required. Let

$$\Omega_r = \{v \in PC^1[0,1] : \|v\|_{PC^1} < r\}.$$

Then Ω_r is a bounded open subset of the Banach space $PC^1[0,1]$ and $0 \in \Omega_r$. Together with Lemma 2.3, we have $T : K \cap \bar{\Omega}_r \to K$ is completely continuous with $T\theta = \theta$.

Noticing $r > \bar{\beta}_0$, for any $v \in K \cap \partial\Omega_r$, we have

$$v(t) \geq \delta \|v\|_{PC^1} = \delta r, \quad t \in [0,\xi],$$

and then

$$v(t) \geq \delta \|v\|_{PC^1} > \delta \bar{\beta}_0 = \mu^*, \quad t \in [0,\xi].$$

Therefore, for any $v \in K \cap \partial\Omega_r$, we have

$$\begin{aligned}
(Tv)(t) &= \int_t^1 \left(\int_0^\tau n s^{n-1} f(v(s))\, ds + \sum_{t_k \leq \tau} I_k(v(t_k)) \right)^{\frac{1}{n}} d\tau \\
&\geq \int_t^1 \left(\int_0^\tau n s^{n-1} f(v(s))\, ds \right)^{\frac{1}{n}} d\tau \\
&\geq \int_\xi^1 \left(\int_0^\xi n s^{n-1} f(v(s))\, ds \right)^{\frac{1}{n}} d\tau \\
&\geq \int_\xi^1 \left(\int_0^\xi n s^{n-1} l_* v^n(s)\, ds \right)^{\frac{1}{n}} d\tau \\
&\geq \int_\xi^1 \left(\int_0^\xi n s^{n-1} l_* \left(\delta \|v\|_{PC^1}\right)^n(s)\, ds \right)^{\frac{1}{n}} d\tau \\
&\geq l_*^{\frac{1}{n}} \delta \xi (1-\xi) r,
\end{aligned}$$

which shows that

$$\inf_{v \in K \cap \partial\Omega_r} Tv \geq l_*^{\frac{1}{n}} \delta \xi (1-\xi) r > 0.$$

By Lemma 2.4, for any $r > \bar{\beta}_0$, the operator T has a proper element $v_r \in K$ associated with the eigenvalue $\mu_r > 0$, further v_r satisfies $\|v_r\|_{PC^1} = r$. Let $\lambda_r = \frac{1}{\mu_r^n}$ and follow the proof of Theorem 3.1, we complete the proof of Theorem 3.3. □

Theorem 3.4 *Assume that* (H_1) *holds. If* $f_0 = +\infty$, $I_0(k) = +\infty$ $(k = 1,2,\ldots,m)$, *then there exists* $\beta_1 > 0$ *such that, for any* $0 < r^* < \beta_1$, *problem* (1.2) *has a strictly convex solution* $u_{r^*}(t)$ *satisfying* $\|u_{r^*}\|_{PC^1} = r^*$ *for any*

$$\lambda = \lambda_{r^*} \in (0, \lambda^*],$$

where λ^* *is a positive finite number.*

Proof The proof is similar to that of Theorem 3.3, we omit it here. □

For ease of exposition, we set

$$m_f(r_{**}) = \min\left\{\frac{f(u)}{r_{**}^n} : u \in [\delta r_{**}, r_{**}]\right\},$$
$$m_{I_k}(r_{**}) = \min\left\{\frac{I_k(u)}{r_{**}^n} : u \in [\delta r_{**}, r_{**}]\right\}, \quad k = 1, 2, \ldots, m,$$

where δ is defined in (2.4).

Theorem 3.5 *Assume that* (H_1) *holds. If there exist* $r^{**} > 0$ *and* $\beta_{r^{**}} > 0$ *such that* $m_f(r^{**}) \geq \beta_{r^{**}}$ *and* $m_{I_k}(r^{**}) \geq \beta_{r^{**}}$ $(k = 1, 2, \ldots, m)$, *then problem* (1.2) *has a strictly convex solution* $u_{r^{**}}(t)$ *satisfying* $\|u_{r^{**}}\|_{PC^1} = r^{**}$ *for any*

$$\lambda = \lambda_{r^{**}} \in (0, \lambda^{**}], \tag{3.3}$$

where λ^{**} *is a positive finite number.*

Proof In fact, for any $v \in K \cap \partial\Omega_{r_{**}}$, we have $\delta r_{**} \leq v(t) \leq r_{**}$, $t \in [0, \xi]$.

Noticing that $m_f(r_{**}) \geq \beta_{r_{**}} > 0$ and $m_{I_k}(r^{**}) \geq \beta_{r^{**}} > 0$ $(k = 1, 2, \ldots, m)$, we have

$$f\big(v(t)\big) \geq m_f(r_{**})r_{**}^n \geq r_{**}^n\beta_{r_{**}} \geq \beta_{r_{**}}v^n(t), \quad \forall t \in [0, \xi], v \in [\delta r_{**}, r_{**}],$$

and

$$I_k(v) \geq m_{I_k}\big(r^{**}\big)r_{**}^n \geq r_{**}^n\beta_{r_{**}} \geq \beta_{r_{**}}v^n, \quad k = 1, 2, \ldots, m, v \in [\delta r_{**}, r_{**}].$$

The following proof is similar to that of Theorem 3.3. This finishes the proof of Theorem 3.5. □

Theorem 3.6 *Assume that* (H_1) *holds. If* $f_0 = f_\infty = 0$ *and* $I_0(k) = I_\infty(k) = 0$ $(k = 1, 2, \ldots, m)$, *then there exists* $\underline{\lambda} > 0$ *such that problem* (1.2) *has no strictly convex solutions for* $\lambda \in (0, \underline{\lambda})$.

Proof It follows from $f_0 = f_\infty = 0$ and $I_0(k) = I_\infty(k) = 0$ $(k = 1, 2, \ldots, m)$, (1.3) and (1.4) that there exist $\bar{v}_0 > 0$ and $v_k > 0$ such that

$$\frac{f(\bar{v}_0)}{\bar{v}_0^n} = \max_{v>0}\frac{f(v)}{v^n}, \qquad \frac{I_k(\bar{v}_k)}{v_k^n} = \max_{v>0}\frac{I_k(v)}{v^n}, \quad k = 1, 2, \ldots, m.$$

Let

$$M = \max\left\{\frac{f(\bar{v}_0)}{\bar{v}_0^n}, \frac{I_1(\bar{v}_1)}{v_1^n}, \frac{I_2(\bar{v}_2)}{v_2^n}, \ldots, \frac{I_m(\bar{v}_m)}{v_m^n}\right\} + 1.$$

Then $M > 0$ and

$$f(v) \leq Mv^n, \qquad I_k(v) \leq Mv^n, \quad k = 1, 2, \ldots, m, v > 0. \tag{3.4}$$

Assume that $v(t)$ is a strictly concave solution of problem (2.1). We will show that this leads to a contradiction for $\lambda < \underline{\lambda}$, where $\underline{\lambda} = ((M(1+m))^{\frac{1}{n}})^{-1}$. Let $\mu = \frac{1}{\lambda^{\frac{1}{n}}}$. Since $(Tv)(t) = \mu v(t)$ for $t \in J$, it follows from (2.5) that

$$\begin{aligned}
v(t) &= \lambda^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau ns^{n-1} f\big(v(s)\big)\, ds + \sum_{t_k \le \tau} I_k\big(v(t_k)\big) \right)^{\frac{1}{n}} d\tau \\
&\le \lambda^{\frac{1}{n}} \int_0^1 \left(\int_0^\tau ns^{n-1} f\big(v(s)\big)\, ds + \sum_{t_k \le \tau} I_k\big(v(t_k)\big) \right)^{\frac{1}{n}} d\tau \\
&\le \lambda^{\frac{1}{n}} \int_0^1 \left(\int_0^\tau ns^{n-1} M v^n(s)\, ds + \sum_{t_k \le \tau} M v^n(s) \right)^{\frac{1}{n}} d\tau \\
&\le \lambda^{\frac{1}{n}} M^{\frac{1}{n}} \|v\|_{PC^1} \int_0^1 \left(\int_0^\tau ns^{n-1}\, ds + m \right)^{\frac{1}{n}} d\tau \\
&\le \lambda^{\frac{1}{n}} M^{\frac{1}{n}} \|v\|_{PC^1} \int_0^1 \left(\tau^n + m\right)^{\frac{1}{n}} d\tau \\
&\le \lambda^{\frac{1}{n}} M^{\frac{1}{n}} \|v\|_{PC^1} (1+m)^{\frac{1}{n}}, \\
\left|v'(t)\right| &= \lambda^{\frac{1}{n}} \left(\int_0^t ns^{n-1} f\big(v(s)\big)\, ds + \sum_{t_k \le \tau} I_k\big(v(t_k)\big) \right)^{\frac{1}{n}} \\
&\le \lambda^{\frac{1}{n}} \left(\int_0^1 ns^{n-1} f\big(v(s)\big)\, ds + \sum_{t_k \le \tau} I_k\big(v(t_k)\big) \right)^{\frac{1}{n}} \\
&\le \lambda^{\frac{1}{n}} \left(\int_0^1 ns^{n-1} M v^n(s)\, ds + \sum_{t_k \le \tau} M v^n(s) \right)^{\frac{1}{n}} \\
&\le \lambda^{\frac{1}{n}} M^{\frac{1}{n}} \|v\|_{PC^1} \left(\int_0^1 ns^{n-1}\, ds + m \right)^{\frac{1}{n}} \\
&\le \lambda^{\frac{1}{n}} M^{\frac{1}{n}} \|v\|_{PC^1} (1+m)^{\frac{1}{n}},
\end{aligned}$$

which shows that

$$\begin{aligned}
\|v\|_{PC^1} &\le \lambda^{\frac{1}{n}} M^{\frac{1}{n}} \|v\|_{PC^1} (1+m)^{\frac{1}{n}} \\
&< \underline{\lambda}^{\frac{1}{n}} M^{\frac{1}{n}} \|v\|_{PC^1} (1+m)^{\frac{1}{n}} \\
&= \|v\|_{PC^1},
\end{aligned}$$

which is a contradiction. This finishes the proof. □

Remark 3.1 The method to study the existence and nonexistence results of nontrivial convex solutions is completely different from those of Hu and Wang [8] and Han et al. [41].

Remark 3.2 Some ideas of the proof of Theorem 3.6 come from Theorems 1-2 in [42]. From the proof of the main results in [42], it is easy to see that $f(u) > 0$ for $u > 0$ is an important condition, although we consider the nonexistence of a nontrivial convex solution without using it; for details, see Theorem 3.6. Moreover, we introduce a new notation

$$M = \max\left\{\frac{f(\bar{v}_0)}{\bar{v}_0^n}, \frac{I_1(\bar{v}_1)}{v_1^n}, \frac{I_2(\bar{v}_2)}{v_2^n}, \dots, \frac{I_m(\bar{v}_m)}{v_m^n}\right\} + 1.$$

4 Uniqueness and continuity of nontrivial convex solutions on a parameter

In the previous section, we have established some existence and nonexistence criteria of nontrivial convex solutions for problem (1.2). Next we consider the uniqueness and continuity of nontrivial convex solutions on a parameter for problem (1.2).

Theorem 4.1 *Suppose that* $f(u), I_k(u) : [0,+\infty) \to [0,+\infty)$ *are nondecreasing functions with* $f(u) > 0$, $I_k(u) > 0$ $(k = 1,2,\dots,m)$ *for* $u > 0$ *and satisfy* $f(\rho u) \geq \rho^{n\alpha} f(u)$, $I_k(\rho u) \geq \rho^{n\alpha} I_k(u)$ $(k = 1,2,\dots,m)$ *for any* $0 < \rho < 1$, *where* $0 \leq \alpha < 1$. *Then, for any* $\lambda \in (0,\infty)$, *problem* (1.2) *has a unique nontrivial convex solution* $u_\lambda(t)$. *Furthermore, such a solution* $u_\lambda(t)$ *satisfies the following properties*:

(i) $u_\lambda(t)$ *is strongly decreasing in* λ. *That is,* $\lambda_1 > \lambda_2 > 0$ *implies* $u_{\lambda_1}(t) \ll u_{\lambda_2}(t)$ *for* $t \in J$.

(ii) $\lim_{\lambda\to 0^+} \|u_\lambda\|_{PC^1} = 0$, $\lim_{\lambda\to+\infty} \|u_\lambda\|_{PC^1} = +\infty$.

(iii) $u_\lambda(t)$ *is continuous with respect to* λ. *That is,* $\lambda \to \lambda_0 > 0$ *implies* $\|u_\lambda - u_{\lambda_0}\|_{PC^1} \to 0$.

Proof Set $\Psi = \lambda^{\frac{1}{n}} T$, and T be the same as in (2.5).

Let

$$K_1 = \{y(t) \in PC^1[0,1] : y(t) \geq 0\}.$$

It is easy to see that K_1 is a normal solid cone of $PC^1[0,1]$, and its interior $K_1^0 = \{y(t) \in PC^1[0,1] : y(t) > 0\}$. Being similar to Lemma 2.3, the operator Ψ maps K_1 into K_1. In view of $f(u) > 0$, $I_k(u) > 0$ $(k = 1,2,\dots,m)$ for $u > 0$, it is easy to see that $\Psi : K_1^0 \to K_1^0$. We assert that $\Psi : K_1^0 \to K_1^0$ is an α-concave increasing operator. Indeed

$$\begin{aligned}
\Psi(\rho v) &= \lambda^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau n s^{n-1} f\big(\rho v(s)\big)\,ds + \sum_{t_k \leq \tau} I_k\big(\rho v(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&\geq \lambda^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau n s^{n-1} \rho^{n\alpha} f\big(v(s)\big)\,ds + \sum_{t_k \leq \tau} \rho^{n\alpha} I_k\big(v(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&\geq \rho^\alpha \lambda^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau n s^{n-1} f\big(v(s)\big)\,ds + \sum_{t_k \leq \tau} I_k\big(v(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&= \rho^\alpha \Psi(v), \quad \forall 0 < \rho < 1,
\end{aligned}$$

where $0 \leq \alpha < 1$. Since $f(u)$ and $I_k(u)$ $(k = 1,2,\dots,m)$ are nondecreasing, then

$$\begin{aligned}
(\Psi v_*)(t) &= \lambda^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau n s^{n-1} f\big(v_*(s)\big)\,ds + \sum_{t_k \leq \tau} I_k\big(v_*(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&\leq \lambda^{\frac{1}{n}} \int_t^1 \left(\int_0^\tau n s^{n-1} f\big(v_{**}(s)\big)\,ds + \sum_{t_k \leq \tau} I_k\big(v_{**}(t_k)\big)\right)^{\frac{1}{n}} d\tau \\
&= (\Psi v_{**})(t) \quad \text{for } v_* \leq v_{**}, v_*, v_{**} \in PC^1[0,1].
\end{aligned}$$

In view of Lemma 2.5, Ψ has a unique fixed point $v_\lambda \in K_1^0$. This shows that problem (2.1) has a unique concave positive solution $v_\lambda(t)$. It follows from Lemma 2.1 that problem (1.3) has a unique nontrivial convex solution $u_\lambda(t)$.

Next, we give the proof for (i)-(iii). Let $\gamma = \frac{1}{\lambda^{\frac{1}{n}}}$, and denote $\lambda^{\frac{1}{n}} T v_\lambda = v_\lambda$ by $T v_\gamma = \gamma v_\gamma$. Assume $0 < \gamma_1 < \gamma_2$. Then $v_{\gamma_1} \geq v_{\gamma_2}$. Indeed, set

$$\bar{\eta} = \sup\{\eta : v_{\gamma_1} \geq \eta v_{\gamma_2}\}. \tag{4.1}$$

We assert $\bar{\eta} \geq 1$. If it is not true, then $0 < \bar{\eta} < 1$, and further

$$\gamma_1 v_{\gamma_1} = T v_{\gamma_1} \geq T(\bar{\eta} v_{\gamma_2}) \geq \bar{\eta}^\alpha T v_{\gamma_2} = \bar{\eta}^\alpha \gamma_2 v_{\gamma_2},$$

which imply

$$v_{\gamma_1} \geq \bar{\eta}^\alpha \frac{\gamma_2}{\gamma_1} v_{\gamma_2} \gg \bar{\eta}^\alpha v_{\gamma_2} \gg \bar{\eta} v_{\gamma_2}.$$

This is a contradiction to (4.1).

In view of the discussion above, we have

$$v_{\gamma_1} = \frac{1}{\gamma_1} T v_{\gamma_1} \geq \frac{1}{\gamma_1} T v_{\gamma_2} = \frac{\gamma_2}{\gamma_1} v_{\gamma_2} \gg v_{\gamma_2}. \tag{4.2}$$

Hence, $v_\gamma(t)$ is strongly decreasing in γ. Namely $v_\lambda(t)$ is strongly increasing in $\lambda^{\frac{1}{n}}$. By Lemma 2.1, (i) is proved.

Set $\gamma_2 = \gamma$ and fix γ_1 in (4.2), we have $v_{\gamma_1} \geq \frac{\gamma}{\gamma_1} v_\gamma$, for $\gamma > \gamma_1$. Further

$$\|v_\gamma\|_{PC^1} \leq \frac{\gamma_1 N_1}{\gamma} \|v_{\gamma_1}\|_{PC^1}, \tag{4.3}$$

where $N_1 > 0$ is a normal constant. Note that $\gamma = \frac{1}{\lambda^{\frac{1}{n}}}$, we have $\lim_{\lambda \to 0^+} \|v_\lambda\|_{PC^1} = 0$.

And then, it follows from Lemma 2.1 that $\lim_{\lambda \to 0^+} \|u_\lambda\|_{PC^1} = 0$.

Let $\gamma_1 = \gamma$, and fix γ_2, again by (4.2) and normality of K, we have $\lim_{\lambda \to +\infty} \|v_\lambda\|_{PC^1} = +\infty$.

And then, it follows from Lemma 2.1 that $\lim_{\lambda \to +\infty} \|u_\lambda\|_{PC^1} = +\infty$.

This gives the proof of (ii).

Next, we show the continuity of $u_\gamma(t)$. For given $\gamma_0 > 0$, by (i),

$$v_\gamma \ll v_{\gamma_0} \quad \forall \gamma > \gamma_0. \tag{4.4}$$

Let

$$l_\gamma = \sup\{v > 0 \mid v_\gamma \geq v v_{\gamma_0}, \gamma > \gamma_0\}.$$

Obviously, $0 < l_\gamma < 1$ and $v_\gamma \geq l_\gamma v_{\gamma_0}$. So, we have

$$\gamma v_\gamma = T v_\gamma \geq T(l_\gamma v_{\gamma_0}) \geq l_\gamma^\alpha T v_{\gamma_0} = l_\gamma^\alpha \gamma_0 v_{\gamma_0},$$

and further

$$v_\gamma \geq \frac{\gamma_0}{\gamma} l_\gamma^\alpha v_{\gamma_0}.$$

By the definition of l_γ,

$$\frac{\gamma_0}{\gamma} l_\gamma^\alpha \leq l_\gamma \quad \text{or} \quad l_\gamma \geq \left(\frac{\gamma_0}{\gamma}\right)^{\frac{1}{1-\alpha}}.$$

Again, by the definition of l_γ, we have

$$v_\gamma \geq \left(\frac{\gamma_0}{\gamma}\right)^{\frac{1}{1-\alpha}} v_{\gamma_0}, \quad \forall \gamma > \gamma_0. \tag{4.5}$$

Notice that K_1 is a normal cone. In view of (4.4) and (4.5), we obtain

$$\|v_{\gamma_0} - v_\gamma\|_{PC^1} \leq N_2 \left[1 - \left(\frac{\gamma_0}{\gamma}\right)^{\frac{1}{1-\alpha}}\right] \|v_{\gamma_0}\|_{PC^1} \to 0, \quad \gamma \to \gamma_0 + 0,$$

where $N_2 > 0$ is a normal constant.

In the same way, we can prove

$$\|v_\gamma - v_{\gamma_0}\|_{PC^1} \to 0, \quad \gamma \to \gamma_0 - 0.$$

Hence, v_γ is continuous at $\gamma = \gamma_0$.

Therefore, by Lemma 2.1, we have

$$\|u_{\gamma_0} - u_\gamma\|_{PC^1} = \|v_\gamma - v_{\gamma_0}\|_{PC^1} \to 0, \quad \gamma \to \gamma_0 + 0 \ (\gamma \to \gamma_0 - 0).$$

Consequently, (iii) holds. The proof is complete. □

Remark 4.1 Some ideas of the proof of Theorem 4.1 come from Theorem 2.2.7 in [43] and Theorem 6 in [44].

Remark 4.2 In Theorem 4.1, even though we do not assume that T is completely continuous, even continuous, we can assert that u_λ depends continuously on λ.

Remark 4.3 If we replace K_1, K_1^0 by K, K^0, respectively, then Theorem 4.1 also holds.

Remark 4.4 The function f and I_k ($k = 1, 2, \ldots, m$) satisfying the conditions of Theorem 4.1 can be easily found. For example,

$$f(u) = u^{n\alpha_1} + u^{n\alpha_2} + \cdots + u^{n\alpha_s},$$
$$I_k(u) = u^{n\alpha_1} + u^{n\alpha_2} + \cdots + u^{n\alpha_s}, \quad k = 1, 2, \ldots, m,$$

where $\alpha_j > 0$, $\sup_j \alpha_j < 1$, s is a positive integer.

5 Conclusion

Using the eigenvalue theory, we show the existence of a strictly convex solution for problem (1.2), which is a new problem in the sense of impulsive Monge-Ampère type equations introduced here. Further, we prove that problem (1.2) has no strictly convex solution for sufficiently small λ by means of the internal geometric properties related to the problem. Finally, by applying the theory of α-concave operators, we prove that the unique solution $u_\lambda(t)$ of problem (1.2) is strongly increasing and depends continuously on the parameter λ. In consequence, our main results of the present work will be a useful contribution to the existing literature on the topic of impulsive Monge-Ampère type equations.

Acknowledgements
This work is sponsored by the National Natural Science Foundation of China (11301178), the Beijing Natural Science Foundation of China (1163007), and the Scientific Research Project of Construction for Scientific and Technological Innovation Service Capacity (KM201611232019). The authors are grateful to anonymous referees for their constructive comments and suggestions which have greatly improved this paper.

Funding
This work is sponsored by the National Natural Science Foundation of China (11301178), the Beijing Natural Science Foundation (1163007) and the Scientific Research Project of Construction for Scientific and Technological Innovation Service Capacity (KM201611232019).

Availability of data and materials
Not applicable.

Ethics approval and consent to participate
Not applicable.

Competing interests
The authors declare that there is no conflict of interest regarding the publication of this manuscript. The authors declare that they have no competing interests.

Consent for publication
Not applicable.

Authors' contributions
All results belong to XZ and MF. All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics and Physics, North China Electric Power University, Beijing, 102206, People's Republic of China. [2]School of Applied Science, Beijing Information Science & Technology University, Beijing, 100192, People's Republic of China.

Publisher's Note

Received: 14 August 2017 Accepted: 12 November 2017 Published online: 17 November 2017

References

1. Trudinger, N, Wang, X: The Monge-Ampère equation and its geometric applications. Handb. Geom. Anal. **1**, 467-524 (2008)
2. Caffarelli, L, Nirenberg, L, Spruck, J: Dirichlet problem for nonlinear second order elliptic equations I, Monge-Ampère equations. Commun. Pure Appl. Math. **37**, 369-402 (1984)
3. Gilbarg, D, Trudinger, N: Elliptic Partial Differential Equations of Second Order. Springer, Berlin (2001)
4. Kutev, N: Nontrivial solutions for the equations of Monge-Ampère type. J. Math. Anal. Appl. **132**, 424-433 (1988)
5. Delanoë, P: Radially symmetric boundary value problems for real and complex elliptic Monge-Ampère equations. J. Differ. Equ. **58**, 318-344 (1985)
6. Zhang, Z, Wang, K: Existence and non-existence of solutions for a class of Monge-Ampère equations. J. Differ. Equ. **246**, 2849-2875 (2009)
7. Qi, Z, Zhang, Z: On a power-type coupled system of Monge-Ampère equations. Topol. Methods Nonlinear Anal. **46**, 717-730 (2015)
8. Hu, S, Wang, H: Convex solutions of boundary value problems arising from Monge-Ampère equations. Discrete Contin. Dyn. Syst. **16** 705-720 (2006)
9. Wang, H: Convex solutions of boundary value problems. J. Math. Anal. Appl. **318**, 246-252 (2006)
10. Mohammed, A: Existence and estimates of solutions to a singular Dirichlet problem for the Monge-Ampére equation. J. Math. Anal. Appl. **340**, 1226-1234 (2008)

11. Nieto, JJ, O'Regan, D: Variational approach to impulsive differential equations. Nonlinear Anal., Real World Appl. **10**, 680-690 (2009)
12. Liu, X, Willms, A: Impulsive controllability of linear dynamical systems with applications to maneuvers of spacecraft. Math. Probl. Eng. **2**, 277-299 (1996)
13. Pasquero, S: Ideality criterion for unilateral constraints in time-dependent impulsive mechanics. J. Math. Phys. **46**, 112904 (2005)
14. Cushing, JM: Periodic time-dependent predator-prey system. SIAM J. Appl. Math. **32**, 82-95 (1977)
15. Bai, Z, Dong, X, Yin, C: Existence results for impulsive nonlinear fractional differential equation with mixed boundary conditions. Bound. Value Probl. **2016**, 63 (2016)
16. Agarwal, RP, Franco, D, O'Regan, D: Singular boundary value problems for first and second order impulsive differential equations. Aequ. Math. **69**, 83-96 (2005)
17. Liu, X, Guo, D: Method of upper and lower solutions for second-order impulsive integro-differential equations in a Banach space. Comput. Math. Appl. **38**, 213-223 (1999)
18. Wang, M, Feng, M: Infinitely many singularities and denumerably many positive solutions for a second-order impulsive Neumann boundary value problem. Bound. Value Probl. **2017**, 50 (2017)
19. Zhang, X, Tian, Y: Sharp conditions for the existence of positive solutions for a second-order singular impulsive differential equation. Appl. Anal. 1-13 (2017)
20. Karaca, IY, Tokmak, F: Existence of solutions for nonlinear impulsive fractional differential equations with *p*-Laplacian operator. Math. Probl. Eng. **2014**, 692703 (2014)
21. Liu, Z, Lu, L, Szántó, I: Existence of solutions for fractional impulsive differential equations with *p*-Laplacian operator. Acta Math. Hung. **141**, 203-219 (2013)
22. Zeng, Y, Xie, J: Three solutions to impulsive differential equations involving *p*-Laplacian. Adv. Differ. Equ. **2015**, 95 (2015)
23. Zhang, X, Ge, W: Impulsive boundary value problems involving the one-dimensional *p*-Laplacian. Nonlinear Anal. **70**, 1692-1701 (2009)
24. Lin, X, Jiang, D: Multiple positive solutions of Dirichlet boundary value problems for second order impulsive differential equations. J. Math. Anal. Appl. **321**, 501-514 (2006)
25. Liu, Y, O'Regan, D: Multiplicity results using bifurcation techniques for a class of boundary value problems of impulsive differential equations. Commun. Nonlinear Sci. Numer. Simul. **16**, 1769-1775 (2011)
26. Ma, R, Yang, B, Wang, Z: Positive periodic solutions of first-order delay differential equations with impulses. Appl. Math. Comput. **219**, 6074-6083 (2013)
27. Hao, X, Liu, L, Wu, Y: Positive solutions for second order impulsive differential equations with integral boundary conditions. Commun. Nonlinear Sci. Numer. Simul. **16**, 101-111 (2011)
28. Feng, M, Xie, D: Multiple positive solutions of multi-point boundary value problem for second-order impulsive differential equations. J. Comput. Appl. Math. **223**, 438-448 (2009)
29. Zhang, X, Feng, M, Ge, W: Existence of solutions of boundary value problems with integral boundary conditions for second-order impulsive integro-differential equations in Banach spaces. J. Comput. Appl. Math. **233**, 1915-1926 (2010)
30. Liu, L, Liu, Z, Wu, Y: Infinite boundary value problems for *n*th-order nonlinear impulsive integro-differential equations in Banach spaces. Nonlinear Anal. **67**, 2670-2679 (2007)
31. Yan, J: Existence of positive periodic solutions of impulsive functional differential equations with two parameters. J. Math. Anal. Appl. **327**, 854-868 (2007)
32. Liu, X, Guo, D: Method of upper and lower solutions for second-order impulsive integro-differential equations in a Banach space. Comput. Math. Appl. **38**, 213-223 (1999)
33. Tian, Y, Bai, Z: Existence results for the three-point impulsive boundary value problem involving fractional differential equations. Comput. Math. Appl. **59**, 2601-2609 (2010)
34. Hao, X, Liu, L, Wu, Y: Positive solutions for *n*th-order singular nonlocal boundary value problems. Bound. Value Probl. **2007**, 074517 (2007)
35. Zhou, J, Li, Y: Existence and multiplicity of solutions for some Dirichlet problems with impulsive effects. Nonlinear Anal. **71**, 2856-2865 (2009)
36. Bai, L, Nieto, J, Wang, X: Variational approach to non-instantaneous impulsive nonlinear differential equations. J. Nonlinear Sci. Appl. **10**, 2440-2448 (2017)
37. Xiao, J, Nieto, J, Luo, Z: Multiplicity of solutions for nonlinear second order impulsive differential equations with linear derivative dependence via variational methods. Commun. Nonlinear Sci. Numer. Simul. **17**, 426-432 (2012)
38. Tian, Y, Ge, W: Variational methods to Sturm-Liouville boundary value problem for impulsive differential equations. Nonlinear Anal. TMA **72**, 277-287 (2010)
39. Liu, L, Hu, L, Wu, Y: Positive solutions of two-point boundary value problems for systems of nonlinear second order singular and impulsive differential equations. Nonlinear Anal. **69**, 3774-3789 (2008)
40. Zhang, X, Feng, M: Transformation techniques and fixed point theories to establish the positive solutions of second order impulsive differential equations. J. Comput. Appl. Math. **271**, 117-129 (2014)
41. Han, X, Ma, R, Dai, G: Eigenvalue, bifurcation and convex solutions for Monge-Ampère equations. Topol. Methods Nonlinear Anal. **45**, 135-164 (2015)
42. Sánchez, J: Multiple positive solutions of singular eigenvalue type problems involving the one-dimensional *p*-Laplacian. J. Math. Anal. Appl. **292**, 401-414 (2004)
43. Guo, D, Lakshmikantham, V: Nonlinear Problems in Abstract Cones. Academic Press, New York (1988)
44. Liu, X, Li, W: Existence and uniqueness of positive periodic solutions of functional differential equations. J. Math. Anal. Appl. **293**, 28-39 (2004)