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Stochastic quasilinear viscoelastic wave equation with nonlinear damping and source terms

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Abstract
The goal of this study is to investigate an initial boundary value problem for the stochastic quasilinear viscoelastic wave equation involving the nonlinear damping $|u_t|^{q-2}u_t$ and a source term of the type $|u|^{p-2}u$ driven by additive noise. By an appropriate energy inequality, we prove that finite time blow-up is possible for equation (1.1) below if $p > \{q, \rho + 2\}$ and the initial data are large enough (that is, if the initial energy is sufficiently negative). Also, we show that if $q \geq p$, the local solution can be extended for all time and is thus global.

MSC: 60H15; 35L05; 35L70

Keywords: stochastic quasilinear viscoelastic wave equation; blow-up of solution; global existence

1 Introduction

In this paper, we are concerned with the following stochastic viscoelastic wave equation:

$$
\begin{aligned}
&|u_t|^{\rho} u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t h(t-\tau)\Delta u(\tau)\,d\tau + |u_t|^{q-2}u_t \\
&\quad = |u|^{p-2}u + \epsilon\sigma(x,t)\partial_t W(x,t) \quad \text{in } D \times (0,T), \\
&u = 0 \quad \text{on } \partial D \times (0,T), \\
&u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) \quad \text{in } \bar{D},
\end{aligned}
\tag{1.1}
$$

where D is a bounded domain in $\mathbb{R}^n$ with smooth boundary ∂D, with given positive constants $\rho > 0$, $q \geq 2$, and $p \geq 2$. The function $h : \mathbb{R}^+ \to \mathbb{R}^+$ in the viscoelastic term is a positive relaxation function satisfying some conditions to be specified later. $W(x,t)$ is an infinite dimensional Wiener process, $\sigma(x,t)$ is $L^2(D)$-valued progressively measurable, and ϵ is a given positive constant which measures the strength of noise.

System (1.1) without the stochastic term is a model for quasilinear viscoelastic wave equation with nonlinear damping and source terms. Various forms of the deterministic system (1.1) have been considered by many authors, and several results considering existence, nonexistence, and asymptotic behavior have been established in [1–5], and the references therein. For example, Liu [3] considered the following quasilinear viscoelastic

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wave equation problem:

$$\begin{aligned}
&|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-\tau)\Delta u(\tau)\,d\tau = b|u|^{p-2}u \quad \text{in } D \times (0,\infty), \\
&u = 0 \quad \text{on } \partial D \times (0,\infty), \\
&u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) \quad \text{in } \Omega,
\end{aligned}$$

where D is a bounded domain in $\mathbb{R}^n$ $(n \geq 1)$ with a smooth boundary ∂D, and ρ, $b > 0$, $p > 2$ are constants. The author investigated the general solution and blow-up solutions for this problem. Also, Song [4] studied the nonlinear quasilinear viscoelastic wave equation problem

$$\begin{aligned}
&|u_t|^\rho u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)\,d\tau + |u_t|^{m-2}u_t = |u|^{p-2}u \quad \text{in } D \times [0,T], \\
&u = 0 \quad \text{on } \partial D \times [0,T], \\
&u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) \quad \text{in } D,
\end{aligned}$$

where D is a bounded domain of $\mathbb{R}^n$ $(n \geq 1)$ with a smooth boundary ∂D, $m > 2$, $g : \mathbb{R}^+ \to \mathbb{R}^+$ is a positive nonincreasing function, and

$$\begin{aligned}
&2 < p < \infty \quad \text{if } n = 1,2, \qquad 2 < p < 2(n-1)/(n-2) \quad \text{if } n \geq 3, \\
&2 < \rho < \infty \quad \text{if } n = 1,2, \qquad 2 < \rho \leq n/(n-2) \quad \text{if } n \geq 3.
\end{aligned}$$

He proved the global nonexistence of positive initial energy solutions for a quasilinear viscoelastic wave equation.

Under the consideration of random environment, there are many studies on the stochastic wave equation with global existence and invariant measures for linear and nonlinear damping (see the references in [6–25]).

Wei and Jiang [26] and Gao, Guo and Liang [24] considered the following nonlinear stochastic viscoelastic wave equation:

$$\begin{aligned}
&u_{tt} - \Delta u + \int_0^t h(t-\tau)\Delta u(\tau)\,d\tau + u_t \\
&\quad = |u|^{p-2}u + \epsilon\sigma(u,\nabla u,x,t)\partial_t W(x,t) \quad \text{in } D \times (0,T), \\
&u = 0 \quad \text{on } \partial D \times (0,T), \\
&u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) \quad \text{in } \bar{D}.
\end{aligned}$$

They investigated the global existence and the energy decay estimate of a solution and showed that the solution blows up with positive probability or it is explosive in L^2 sense under some conditions.

Moreover, Cheng et al. [23] proved the existence of a global solution and blow-up solutions with positive probability for the nonlinear stochastic viscoelastic wave equation with linear damping (see [18, 22, 26]).

Recently, Cheng et al. [23] studied the stochastic viscoelastic wave equation with nonlinear damping and source terms

$$\begin{aligned}
&u_{tt} - \Delta u + \int_0^t h(t-\tau)\Delta u(\tau)\,d\tau + |u_t|^{q-2}u_t \\
&\quad = |u|^{p-2}u + \epsilon\sigma(x,t)\partial_t W(x,t) \quad \text{in } D \times (0,T), \\
&u = 0 \quad \text{on } \partial D \times (0,T), \\
&u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) \quad \text{in } \bar{D},
\end{aligned}$$

where D is a bounded domain in $\mathbb{R}^n$ with smooth boundary ∂D, $q \geq 2, p \geq 2$, ϵ is a given positive constant which measures the strength of noise; $W(x,t)$ is an infinite dimensional Wiener process; $\sigma(x,t,w)$ is $L^2(D)$-valued progressively measurable; and h is a positive relaxation function. The authors studied the global solution of stochastic viscoelastic wave equations with nonlinear damping and source terms.

The previous work in Cheng et al. [23] established that the solution blows up with positive probability or it is explosive in energy sense for $p > q$. Motivated by this work, we prove that the stochastic quasilinear viscoelastic wave equation (1.1) can blow up with positive probability or it is explosive in energy sense for $p > \{q, \rho + 2\}$ and obtain the existence of global solution by the Borel-Cantelli lemma. To the best of our knowledge, there have been no results for the blow-up of solutions of stochastic quasilinear viscoelastic wave equation with positive probability.

This paper is organized as follows. In Section 2, we present some assumptions, definitions, and lemmas needed for our work. The result for the local existence and a pointwise unique solution of equation (1.1) are given too. In Section 3, we show Lemmas 3.1 and 3.2. With those lemmas, we prove our main result for $p > \{q, \rho + 2\}$. In Section 4, we obtain global existence of equation (1.1).

2 Preliminaries

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ-algebra $\mathbf{B}(X)$, and let $(\Omega, \mathfrak{F}, P)$ be a probability space. We set $H = L^2(D)$ with the inner product and norm denoted by $(\cdot,\cdot)$ and $\|\cdot\|$, respectively. We denote by $\|\cdot\|_q$ the $L^q(D)$ norm for $0 \leq q \leq \infty$ and by $\|\nabla \cdot\|$ the Dirichlet norm in $V = H_0^1(D)$ which is equivalent to $H^1(D)$ norm.

First, we introduce the following hypotheses:

(H1) We assume that p, q, ρ satisfy

$$\begin{aligned}
&q \geq 2, \qquad p > 2, \qquad \max\{p,q\} \leq \frac{2(n-1)}{n-2} \quad \text{if } n \geq 3; \\
&q \geq 2, \qquad p > 2 \quad \text{if } n = 1,2; \\
&0 \leq \rho \leq \frac{2}{n-2} \quad \text{if } n \geq 3, \qquad 0 < \rho < \infty \quad \text{if } n = 1,2.
\end{aligned} \tag{2.1}$$

(H2) We assume that $h : \mathbb{R}^+ \to \mathbb{R}^+$ is a bounded nonincreasing C^1 function satisfying

$$h(s) > 0, \quad 1 - \int_0^\infty h(s)\,ds = l > 0,$$

and there exist positive constants ξ_1 and ξ_2 such that

$$-\xi_1 h(t) \le h'(t) \le -\xi_2 h(t), \quad t \ge 0. \tag{2.2}$$

(H3) $\sigma(x,t)$ is $H_0^1(D) \cap L^\infty(D)$-valued progressively measurable such that

$$E\int_0^T \left(\|\nabla\sigma(t)\|^2 + \|\sigma(t)\|_\infty^2\right) dt < \infty. \tag{2.3}$$

Lemma 2.1 ([8]) *For all $u, v \in H^1(\mathbb{R}^n)$ and $0 < p \le \frac{2}{n-2}$ $(n \ge 3)$ or $p > 0$ $(n = 1, 2)$, there exists a constant $c_1 = c_1(n,p) > 0$ such that*

$$\|u\|_{L^{2(p-1)}} \le c_1 \|u\|_{H^1}, \qquad \|u^p v\| \le c_1^{p+1} \|u\|_{H^1}^p \|v\|_{H^1}. \tag{2.4}$$

In this paper, $E(\cdot)$ stands for expectation with respect to probability measure P, and $W(x,t)(t \ge 0)$ is a V-valued Q-Wiener process on the probability space with the covariance operator Q satisfying $\mathrm{Tr}(Q) < \infty$. A complete orthonormal system $\{e_k\}_{k=1}^\infty$ in V with $c_0 := \sup_{k\ge 1} \|e_k\|_\infty < \infty$ and a bounded sequence of nonnegative real members $\{\lambda_k\}_{k=1}^\infty$ satisfy that

$$Qe_k = \lambda_k e_k, \quad k = 1, 2, \ldots.$$

To simplify the computations, we assume that the covariance operator Q and Laplacian $-\triangle$ with a homogeneous Dirichlet boundary condition have a common set of eigenfunctions, that is,

$$-\triangle e_k = \mu_k e_k, \quad x \in D,$$
$$e_k = 0, \quad x \in \partial D,$$

and then, for any $t \in [0, T]$, $W(x,t)$ has an expansion

$$W(x,t) = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\beta_k(t) e_k(t), \tag{2.5}$$

where $\{\beta_k(t)\}_{k=1}^\infty$ are real-valued Brownian motions mutually independent of $(\Omega, \mathfrak{F}, P)$. Let $\mathcal{H}$ be the set of $L_2^0 = L^2(Q^{1/2}V, V)$-valued processes with the norm

$$\|\Phi(t)\|_{\mathcal{H}} = \left(E\int_0^t \|\Phi(s)\|_{L_2^0}^2 \, ds\right)^{1/2} = \left(E\int_0^t \mathrm{Tr}\big(\Phi(s)Q\Phi^*(s)\big)\, ds\right)^{1/2} < \infty, \tag{2.6}$$

where $\Phi^*(s)$ denotes the adjoint operator of $\Phi(s)$. For any $\Phi^*(t) \in \mathcal{H}$, we can define the stochastic integral with respect to the Q-Wiener process as $\int_0^t \Phi(s)\, dW(s)$, which is martingale. For more details about the finite dimension Winner process and the stochastic integral, see [22].

Definition 2.1 Assume that $(u_0, u_1) \in H_0^1(D) \times L^2(D)$ and $E\int_0^T \|\sigma(t)\|^2 \, dt < \infty$. u is said to be the solution of (1.1) on the interval $[0, T]$ if (u, u_t) is $H_0^1(D) \times L^2(D)$-valued progressively measurable, $(u,u_t) \in L^2(\Omega; C([0,T]; H_0^1(D) \times L^2(D)))$, $u_t \in L^q((0,T) \times D)$, and such that (1.1) holds in the sense of distributions over $(0,T) \times D$ for almost all w.

By combining the arguments of [20, 23, 24], we get the existence result.

Theorem 2.1 ([20, 24]) *Assume that* (H1)-(H3) *hold. Then, for the initial data* $(u_0,u_1) \in (H^2(D) \cap H_0^1(D)) \times H_0^1(D)$, *problem* (1.1) *has a pointwise unique solution* u *such that*

$$u \in L^2\bigl(\Omega; L^\infty\bigl(0,T; H^2(D) \cap H_0^1(D)\bigr)\bigr) \cap L^2\bigl(\Omega; C\bigl([0,T]; H_0^1(D)\bigr)\bigr)$$

and

$$u_t \in L^2\bigl(\Omega; L^\infty\bigl(0,T; H_0^1(D)\bigr)\bigr) \cap L^2\bigl(\Omega; C\bigl([0,T]; L^2(D)\bigr)\bigr).$$

3 Blow-up result

In this section, we prove our main result for $p > q$. For this purpose, we give defined restrictions on $\sigma(x,t)$ and the relaxation function h such that

$$E\int_0^\infty \int_D \sigma^2(x,t)\,dx\,dt < \infty, \qquad \int_0^\infty h(s)\,ds < \frac{p(p-2)}{(p-1)^2}. \tag{3.1}$$

Now, we define an energy function

$$\begin{aligned} F(t) = {}& \frac{1}{\rho+2}\bigl\|u_t(t)\bigr\|_{\rho+2}^{\rho+2} + \frac{1}{2}\bigl\|\nabla u_t(t)\bigr\|^2 \\ &+ \frac{1}{2}\biggl(1 - \int_0^t h(s)\,ds\biggr)\bigl\|\nabla u(t)\bigr\|^2 + \frac{1}{2}(h \circ \nabla u)(t) - \frac{1}{p}\bigl\|u(t)\bigr\|_p^p, \end{aligned} \tag{3.2}$$

where

$$(h \circ \nabla u)(t) = \int_0^t h(t-s)\bigl\|\nabla u(t) - \nabla u(s)\bigr\|^2\,ds.$$

For each N, stopping time τ_N is given as

$$\tau_N = \inf\bigl\{t > 0 : \bigl\|\nabla u(t)\bigr\|^2 \ge N\bigr\},$$

where τ_N is increasing in N, and $\tau_\infty = \lim_{N\to\infty}\tau_N$. In order to prove our blow-up result, we rewrite (1.1) as an equivalent Itô's system

$$\begin{aligned} &du = v\,dt, \\ &d\biggl(\frac{1}{\rho+1}|v|^\rho v - \Delta v\biggr) = \biggl(\Delta u - \int_0^t h(t-s)\Delta u(s)\,ds - |v|^{q-2}v + |u|^{p-2}u\biggr)\,dt \\ &\qquad\qquad + \epsilon\sigma(x,t)\,dW_t(x,t), \quad (x,t) \in D \times (0,T), \\ &u(x,t) = 0, \quad (x,t) \in \partial D \times (0,T), \\ &u(x,0) = u_0(x), \qquad v(x,0) = v_0(x) = u_1(x), \quad x \in D, \end{aligned} \tag{3.3}$$

where $(u_0, u_1) \in H_0^1(D) \times L^2(D)$. Then the energy function $F(t)$ becomes

$$\begin{aligned} F(t) &= \frac{1}{\rho+2}\|v(t)\|_{\rho+2}^{\rho+2} + \frac{1}{2}\|\nabla v(t)\|^2 \\ &\quad + \frac{1}{2}\left(1 - \int_0^t h(s)\,ds\right)\|\nabla u(t)\|^2 + \frac{1}{2}(h \circ \nabla u)(t) - \frac{1}{p}\|u(t)\|_p^p. \end{aligned} \tag{3.4}$$

Lemma 3.1 *Let (u, v) be a solution of Eq.* (3.3) *with the initial data $(u_0, v_0) \in H_0^1(D) \times L^2(D)$. Then we have*

$$\begin{aligned} \frac{d}{dt}EF(t) &= -E\|v(t)\|_q^q + \frac{\epsilon^2}{2}\sum_{j=1}^{\infty} E\int_D \lambda_j e_j^2(x)\sigma^2(x,t)\,dx \\ &\quad - E\big(-h' \circ \nabla u\big)(t) - \frac{1}{2}h(t)E\|\nabla u(t)\|^2 \\ &\le -E\|v(t)\|_q^q + \frac{\epsilon^2}{2}\sum_{j=1}^{\infty} E\int_D \lambda_j e_j^2(x)\sigma^2(x,t)\,dx, \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} &E\left(u(t), \frac{1}{\rho+1}|v(t)|^{\rho}v(t) - \Delta v(t)\right) \\ &\quad = \left(u_0, \frac{1}{\rho+1}|u_1|^{\rho}u_1 - \Delta u_1\right) - \int_0^t E\|\nabla u(s)\|^2\,ds \\ &\quad - \int_0^t E\big(u(s), |v(s)|^{q-2}v(s)\big)\,ds + \int_0^t E\|u(s)\|_p^p\,ds \\ &\quad + E\int_0^t \int_0^s h(s-\tau)\big(\nabla u(\tau), \nabla u(s)\big)\,d\tau\,ds \\ &\quad + \frac{1}{\rho+1}E\int_0^t \|v(s)\|_{\rho+2}^{\rho+2}\,ds + E\int_0^t \|\nabla v(s)\|^2\,ds. \end{aligned} \tag{3.6}$$

Proof By multiplying Eq. (3.3) by $v(t)$ and using Itô's formula, we deduce (3.5). Also, multiplying Eq. (3.3) by $u(t)$ and integrating by parts over $(0, T)$, we arrive at (3.6) (see [24]). □

Let

$$G(t) = \frac{\epsilon^2}{2}\sum_{j=1}^{\infty} E\int_0^t \int_D \lambda_j e_j^2(x)\sigma^2(x,s)\,dx\,ds. \tag{3.7}$$

Due to (3.1), we deduce

$$\begin{aligned} G(\infty) &= \frac{\epsilon^2}{2}\sum_{j=1}^{\infty} E\int_0^{\infty} \int_D \lambda_j e_j^2(x)\sigma^2(x,s)\,dx\,ds \\ &\le \frac{\epsilon^2}{2}\operatorname{Tr}(Q)c_0^2 E\int_0^{\infty} \int_D \sigma^2(x,s)\,dx\,ds = E_1 < \infty. \end{aligned} \tag{3.8}$$

We set

$$H(t) = G(t) - E\big[F(t)\big].$$

Then (3.5) implies that

$$H'(t) = G'(t) - \frac{d}{dt}E\big[F(t)\big] \geq E\big\|v(t)\big\|_q^q \geq 0. \tag{3.9}$$

Lemma 3.2 *Let* (u, v) *be a solution of Eq.* (3.3). *Then there exists a positive constant* C *such that*

$$\begin{aligned} E\big\|u(t)\big\|_p^s &\leq C\big[G(t) - H(t) - E\big\|v(t)\big\|_{\rho+2}^{\rho+2} \\ &\quad - E\big\|\nabla v(t)\big\|^2 + E\big\|u(t)\big\|_p^p - E(h \circ \nabla u)(t)\big], \quad 2 \leq s \leq p. \end{aligned} \tag{3.10}$$

Proof If $\|u\|_p \leq 1$, then $\|u\|_p^s \leq \|u\|_p^2 \leq C\|\nabla u\|^2$ by the Sobolev embedding theorem. If $\|u\|_p \geq 1$, then $\|u\|_p^s \leq \|u\|_p^p$. Thus there exists a constant $C > 0$ such that $E\|u\|_p^s \leq C(E\|\nabla u\|^2 + E\|u\|_p^p)$. Therefore, combining with the definition of energy function, we get (3.10). □

Theorem 3.1 *Assume that* (H1)-(H3) *and* (3.1) *hold*. *Let* (u, v) *be a solution of Eq.* (3.3) *with the initial data* $(u_0, v_0) \in H_0^1(D) \times L^2(D)$ *satisfying*

$$F(0) \leq -(1 + \beta)E_1, \tag{3.11}$$

where $\beta > 0$ *is an arbitrary constant and* E_1 *is defined in* (3.8). *If* $p > \{q, \rho + 2\}$, *then the solution* (u, v) *and the lifespan* τ_∞ *defined above are either*

(1) $P(\tau_\infty < \infty) > 0$, *that is,* $\|\nabla u(t)\|$ *blows up in finite time with positive probability, or*
(2) *there exists a positive time* $T^* \in [0, T_0]$ *such that*

$$\lim_{t \to T^*} E\big[F(t)\big] = +\infty, \tag{3.12}$$

where

$$\begin{aligned} T_0 &= \frac{1 - \alpha}{\alpha K L^{\alpha/(1-\alpha)}(0)}, \\ L(0) &= H^{1-\alpha}(0) + \delta E\left(u_0, \frac{1}{\rho + 1}|u_1|^\rho u_1 - \Delta u_1\right) > 0, \end{aligned} \tag{3.13}$$

and α, K *are given later.*

Proof For the lifespan τ_∞ of the solution $\{u(t) : t > 0\}$ of Eq. (3.3) with $H_0^1(D)$ norm, we treat the case when $P(\tau_\infty = +\infty) < 1$. Then, for sufficiently large $T > 0$, by (3.9) and (3.11), we obtain

$$0 < (1 + \beta)E_1 \leq -F(0) = H(0) \leq H(t) \leq G(t) + \frac{1}{p}E\big\|u(t)\big\|_p^p \leq E_1 + \frac{1}{p}E\big\|u(t)\big\|_p^p. \tag{3.14}$$

Define

$$L(t) = H^{1-\alpha}(t) + \delta E\left(u(t), \frac{1}{\rho+1}\left|v(t)\right|^{\rho} v(t) - \Delta v(t)\right),$$

where

$$0 < \alpha < \min\left\{\frac{1}{2}, \frac{p-2}{2p}, \frac{p-q}{pq}, \frac{1}{\rho+2} - \frac{1}{p}\right\} \tag{3.15}$$

and δ is a very small constant to be determined later.

Using (3.6) and (3.9), we deduce

$$\begin{aligned}
L'(t) &= (1-\alpha)H^{-\alpha}(t)H'(t) + \delta\Big[-E\|\nabla u(t)\|^2 - E\big(u(t), |v(t)|^{q-2}v(t)\big) \\
&\quad + E\|u(t)\|_p^p + E\int_0^t h(t-\tau)\big(\nabla u(\tau), \nabla u(t)\big)\,d\tau \\
&\quad + \frac{1}{\rho+1}E\|v(t)\|_{\rho+2}^{\rho+2} + E\|\nabla v(t)\|^2\Big] \\
&\geq (1-\alpha)H^{-\alpha}(t)E\|v(t)\|_q^q + \delta p\big[H(t) - G(t) + EF(t)\big] \\
&\quad - \delta E\|\nabla u(t)\|^2 - \delta E\big(u(t), |v(t)|^{q-2}v(t)\big) + \delta E\|u(t)\|_p^p \\
&\quad + \delta E\int_0^t h(t-\tau)\big(\nabla u(\tau), \nabla u(t)\big)\,d\tau + \frac{\delta}{\rho+1}E\|v(t)\|_{\rho+2}^{\rho+2} + \delta E\|\nabla v(t)\|^2 \\
&\geq (1-\alpha)H^{-\alpha}(t)E\|v(t)\|_q^q + \delta pH(t) \\
&\quad + \delta\left(\frac{p}{\rho+2} + \frac{1}{\rho+1}\right)E\|v(t)\|_{\rho+2}^{\rho+2} \\
&\quad + \delta\left(\frac{p}{2} - 1\right)E\|\nabla u(t)\|^2 \\
&\quad + \delta\left(\frac{p}{2} + 1\right)E\|\nabla v(t)\|^2 - \delta E\big(u(t), |v(t)|^{q-2}v(t)\big) \\
&\quad + \delta E\int_0^t h(t-\tau)\big(\nabla u(\tau), \nabla u(t)\big)\,d\tau \\
&\quad + \frac{\delta p}{2}E(h\circ\nabla u)(t) - \frac{\delta p}{2}E\int_0^t h(\tau)\,d\tau\,\|\nabla u(t)\|^2 - \delta pG(t).
\end{aligned} \tag{3.16}$$

On the other hand, we have

$$\begin{aligned}
&\delta E\int_0^t h(t-\tau)\big(\nabla u(\tau), \nabla u(t)\big)\,d\tau \\
&\quad = \delta E\int_0^t h(t-\tau)\big(\nabla u(\tau) - \nabla u(t), \nabla u(t)\big)\,d\tau \\
&\qquad + \delta E\int_0^t h(\tau)\,d\tau\,\|\nabla u(t)\|^2,
\end{aligned} \tag{3.17}$$

and by Hölder's inequality, we get

$$\begin{aligned} &\delta E\int_0^t h(t-\tau)\big(\nabla u(\tau)-\nabla u(t),\nabla u(t)\big)\,d\tau \\ &\quad\geq -\frac{\delta p}{2}E(h\circ\nabla u)(t)-\frac{\delta}{2p}E\int_0^t h(\tau)\,d\tau\,\big\|\nabla u(t)\big\|^2. \end{aligned} \tag{3.18}$$

Inserting (3.17) and (3.18) into (3.16), we obtain

$$\begin{aligned} L'(t) \geq{}& (1-\alpha)H^{-\alpha}(t)E\big\|v(t)\big\|_q^q+\delta pH(t) \\ &+\delta\left(\frac{p}{\rho+2}+\frac{1}{\rho+1}\right)E\big\|v(t)\big\|_{\rho+2}^{\rho+2} \\ &+\delta\left(\frac{p}{2}-1\right)E\big\|\nabla u(t)\big\|^2 \\ &+\delta\left(\frac{p}{2}+1\right)E\big\|\nabla v(t)\big\|^2-\delta E\big(u(t),\big|v(t)\big|^{q-2}v(t)\big) \\ &-\delta pG(t)+\delta\left(1-\frac{p^2+1}{2p}\right)E\int_0^t h(\tau)\,d\tau\,\big\|\nabla u(t)\big\|^2. \end{aligned} \tag{3.19}$$

For $q<p$, by $E\|u(t)\|_q^q\leq cE\|u(t)\|_p^q$ and Hölder's inequality, we deduce the following estimate (see [23]):

$$\begin{aligned} E\big(u(t),\big|v(t)\big|^{q-2}v(t)\big) &\leq \big(E\big\|v(t)\big\|_q^q\big)^{\frac{q-1}{q}}\big(E\big\|u(t)\big\|_q^q\big)^{\frac{1}{q}} \\ &\leq C\big(E\big\|v(t)\big\|_q^q\big)^{\frac{q-1}{q}}\big(E\big\|u(t)\big\|_p^q\big)^{\frac{1}{q}} \\ &\leq C\big(E\big\|v(t)\big\|_q^q\big)^{\frac{q-1}{q}}\big(E\big\|u(t)\big\|_p^p\big)^{\frac{1}{p}} \\ &\leq C\big(E\big\|v(t)\big\|_q^q\big)^{\frac{q-1}{q}}\big(E\big\|u(t)\big\|_p^p\big)^{\frac{1}{q}}\big(E\big\|u(t)\big\|_p^p\big)^{\frac{1}{p}-\frac{1}{q}} \end{aligned} \tag{3.20}$$

and Young's inequality

$$\big(E\big\|v(t)\big\|_q^q\big)^{\frac{q-1}{q}}\big(E\big\|u(t)\big\|_p^p\big)^{\frac{1}{q}} \leq \frac{q-1}{q}\mu E\big\|v(t)\big\|_q^q+\frac{\mu^{1-q}}{q}E\big\|u(t)\big\|_p^p, \tag{3.21}$$

where μ is a constant to be determined later. In view of (3.14), we get

$$E\big\|u(t)\big\|_p^p\geq p\big(H(t)-G(t)\big)\geq\widetilde{\rho}H(t), \tag{3.22}$$

where $\widetilde{\rho}=p\beta/(1+\beta)$. With the assumption of $H(0)>1$, (3.21), (3.22), and (3.15) imply that

$$\big(E\big\|u(t)\big\|_p^p\big)^{\frac{1}{p}-\frac{1}{q}}\leq\widetilde{\rho}^{\frac{1}{p}-\frac{1}{q}}H(t)^{\frac{1}{p}-\frac{1}{q}}\leq\widetilde{\rho}^{\frac{1}{p}-\frac{1}{q}}H^{-\alpha}(t)\leq\widetilde{\rho}^{\frac{1}{p}-\frac{1}{q}}H^{-\alpha}(0). \tag{3.23}$$

Combining with (3.20), (3.21), and (3.23), we arrive at

$$\big|E\big(u(t),\big|v(t)\big|^{q-2}v(t)\big)\big|\leq a_1\frac{q-1}{q}\mu E\big\|v(t)\big\|_q^qH^{-\alpha}(t)+a_1\frac{\mu^{1-q}}{q}E\big\|u(t)\big\|_p^pH^{-\alpha}(t), \tag{3.24}$$

where $a_1 = C\widetilde{\rho}^{\frac{1}{p}-\frac{1}{q}}$. Hence, substituting (3.24) for (3.19), we get

$$\begin{aligned}
L'(t) \geq{} & \left(1-\alpha-a_1\frac{q-1}{q}\mu\delta\right)H^{-\alpha}(t)E\|v(t)\|_q^q+\delta pH(t) \\
& +\delta\left(\frac{p}{\rho+2}+\frac{1}{\rho+1}\right)E\|v(t)\|_{\rho+2}^{\rho+2} \\
& +\delta\left(\frac{p}{2}-1\right)E\|\nabla u(t)\|^2 \\
& +\delta\left(\frac{p}{2}+1\right)E\|\nabla v(t)\|^2-\delta pG(t) \\
& +\delta\left(1-\frac{p^2+1}{2p}\right)\int_0^t h(\tau)\,d\tau E\|\nabla u(t)\|^2 \\
& -\delta a_1\frac{\mu^{1-q}}{q}E\|u(t)\|_p^p H^{-\alpha}(0).
\end{aligned} \tag{3.25}$$

Using Lemma 3.2 with $s=p$ and (3.25), we have

$$\begin{aligned}
L'(t) \geq{} & \left(1-\alpha-a_1\frac{q-1}{q}\mu\delta\right)H^{-\alpha}(t)E\|v(t)\|_q^q+\delta pH(t) \\
& +\delta\left(\frac{p}{\rho+2}+\frac{1}{\rho+1}\right)E\|v(t)\|_{\rho+2}^{\rho+2} \\
& +\delta\left(\frac{p}{2}-1\right)E\|\nabla u(t)\|^2 \\
& +\delta\left(\frac{p}{2}+1\right)E\|\nabla v(t)\|^2-\delta pG(t) \\
& +\delta\left(1-\frac{p^2+1}{2p}\right)E\int_0^t h(\tau)\,d\tau\|\nabla u(t)\|^2 \\
& -\delta a_2\mu^{1-q}\bigl[G(t)-H(t)-E\|v(t)\|_{\rho+2}^{\rho+2} \\
& -E\|\nabla v(t)\|^2+E\|u(t)\|_p^p-E(h\circ\nabla u)(t)\bigr] \\
\geq{} & \left(1-\alpha-a_1\frac{q-1}{q}\mu\delta\right)H^{-\alpha}(t)E\|v(t)\|_q^q \\
& +\delta\bigl(p+a_2\mu^{1-q}\bigr)H(t)-\delta\bigl(p+a_2\mu^{1-q}\bigr)G(t) \\
& +\delta\left(\frac{p}{\rho+2}+\frac{1}{\rho+1}+a_2\mu^{1-q}\right)E\|v(t)\|_{\rho+2}^{\rho+2} \\
& +\delta\left(\frac{p}{2}+1+a_2\mu^{1-q}\right)E\|\nabla v(t)\|^2-\delta a_2\mu^{1-q}E\|u(t)\|_p^p \\
& +\delta a_2\mu^{1-q}E(h\circ\nabla u)(t) \\
& +\delta\left[\frac{p}{2}-1+\left(1-\frac{p^2+1}{2p}\right)\int_0^t h(\tau)\,d\tau\right]E\|\nabla u(t)\|^2,
\end{aligned} \tag{3.26}$$

where $a_2 = Ca_1H^{-\alpha}(0)/q$.

Note that

$$
\begin{aligned}
H(t) &\geq G(t) + \frac{1}{p} E \| u(t) \|_p^p - \frac{1}{\rho+2} E \| v(t) \|_{\rho+2}^{\rho+2} \\
&\quad - \frac{1}{2} E \| \nabla v(t) \|^2 - \frac{1}{2} \| \nabla u(t) \|^2 - \frac{1}{2} E(h \circ \nabla u)(t)
\end{aligned}
\tag{3.27}
$$

with

$$
a_3 = \frac{p}{2} - 1 + \left(1 - \frac{p^2+1}{2p}\right) \int_0^t h(\tau)\, d\tau > 0,
\tag{3.28}
$$

we write $p = 2a_4 + (p - 2a_4)$ with $a_4 = \min\{a_1, a_3\}$, then estimate (3.26) yields

$$
\begin{aligned}
L'(t) &\geq \left(1 - \alpha - a_1 \frac{q-1}{q} \mu\delta\right) H^{-\alpha}(t) E \| v(t) \|_q^q \\
&\quad + \delta\bigl(p - 2a_4 + a_2\mu^{1-q}\bigr) H(t) - \delta\bigl(p - 2a_4 + a_2\mu^{1-q}\bigr) G(t) \\
&\quad + \delta\left(\frac{p}{\rho+2} - \frac{2a_4}{\rho+2} + \frac{1}{\rho+1} + a_2\mu^{1-q}\right) E \| v(t) \|_{\rho+2}^{\rho+2} \\
&\quad + \delta\left(\frac{p}{2} - a_4 + 1 + a_2\mu^{1-q}\right) E \| \nabla v(t) \|^2 \\
&\quad + \delta\left(-a_2\mu^{1-q} + \frac{2a_4}{p}\right) E \| u(t) \|_p^p \\
&\quad + \delta\bigl(a_2\mu^{1-q} - a_4\bigr) E(h \circ \nabla u)(t) + \delta(a_3 - a_4) E \| \nabla u(t) \|^2.
\end{aligned}
\tag{3.29}
$$

From (3.8) and (3.14), we deduce

$$
\begin{aligned}
\bigl(p - 2a_4 + a_2\mu^{1-q}\bigr) G(t) &\leq \bigl(p - 2a_4 + a_2\mu^{1-q}\bigr) E_1 \\
&\leq \frac{p - 2a_4 + a_2\mu^{1-q}}{1+\beta} H(t).
\end{aligned}
\tag{3.30}
$$

Substituting (3.30) with (3.29), we get

$$
\begin{aligned}
L'(t) &\geq \left(1 - \alpha - a_1 \frac{q-1}{q} \mu\delta\right) H^{-\alpha}(t) E \| v(t) \|_q^q \\
&\quad + \delta\bigl(p - 2a_4 + a_2\mu^{1-q}\bigr) \frac{\beta}{1+\beta} H(t) \\
&\quad + \delta\left(\frac{p}{\rho+2} - \frac{2a_4}{\rho+2} + \frac{1}{\rho+1} + a_2\mu^{1-q}\right) E \| v(t) \|_{\rho+2}^{\rho+2} \\
&\quad + \delta\left(\frac{p}{2} - a_4 + 1 + a_2\mu^{1-q}\right) E \| \nabla v(t) \|^2 \\
&\quad + \delta\left(-a_2\mu^{1-q} + \frac{2a_4}{p}\right) E \| u(t) \|_p^p \\
&\quad + \delta\bigl(a_2\mu^{1-q} - a_4\bigr) E(h \circ \nabla u)(t) + \delta(a_3 - a_4) E \| \nabla u(t) \|^2.
\end{aligned}
\tag{3.31}
$$

Next, we can choose μ large enough so that (3.31) becomes

$$L'(t) \geq \left(1-\alpha-a_1\frac{q-1}{q}\mu\delta\right)H^{-\alpha}(t)E\|v(t)\|_q^q+\delta\gamma\big(H(t)+E\|v(t)\|_{\rho+2}^{\rho+2}+E\|\nabla v(t)\|^2$$
$$+E\|u(t)\|_p^p+E(h\circ\nabla u)(t)+E\|\nabla u(t)\|^2\big), \tag{3.32}$$

where

$$\gamma=\min\left\{\left(p-2a_4+a_2\mu^{1-q}\right)\frac{\beta}{1+\beta},\frac{p}{\rho+2}-\frac{2a_4}{\rho+2}+\frac{1}{\rho+1}+a_2\mu^{1-q},\right.$$
$$\left.\frac{p}{2}-a_4+1+a_2\mu^{1-q},-a_2\mu^{1-q}+\frac{2a_4}{p},a_2\mu^{1-q}-a_4,a_3-a_4\right\}>0.$$

Once μ is fixed, we pick δ small enough so that

$$1-\alpha-a_1\frac{q-1}{q}\mu\delta>0.$$

Using this, (3.32) takes the form

$$L'(t)\geq\delta\gamma\big(H(t)+E\|v(t)\|_{\rho+2}^{\rho+2}+E\|\nabla v(t)\|^2$$
$$+E\|u(t)\|_p^p+E(h\circ\nabla u)(t)+E\|\nabla u(t)\|^2\big). \tag{3.33}$$

Thus, we see that

$$L(t)\geq L(0)=H^{1-\alpha}(0)+\delta E\left(u_0,\frac{1}{\rho+1}|u_1|^\rho u_1-\Delta u_1\right)>0,\quad \forall t\geq 0. \tag{3.34}$$

Consequently, we get

$$L(t)\geq L(0)>0,\quad \forall t\geq 0. \tag{3.35}$$

Since

$$\left|E\int_D |v(t)|^\rho v(t)u(t)\,dx\right|\leq E\|v(t)\|_{\rho+2}^{\rho+1}E\|u(t)\|_{\rho+2}$$
$$\leq CE\|v(t)\|_{\rho+2}^{\rho+1}E\|u(t)\|_p,$$

we have

$$\left|E\int_D |v(t)|^\rho v(t)u(t)\,dx\right|^{\frac{1}{1-\alpha}}\leq E\|v(t)\|_{\rho+2}^{\frac{\rho+1}{1-\alpha}}E\|u(t)\|_{\rho+2}^{\frac{1}{1-\alpha}}$$
$$\leq C\left[\left(E\|v(t)\|_{\rho+2}^{\rho+2}\right)^{\frac{(\rho+1)}{(\rho+2)(1-\alpha)}\zeta}+\left(E\|u(t)\|_p^2\right)^{\frac{\theta}{2(1-\alpha)}}\right], \tag{3.36}$$

where $\frac{1}{\zeta}+\frac{1}{\theta}=1$. By choosing $\zeta=\frac{(1-\alpha)(\rho+2)}{\rho+1}(>1)$, we have $\frac{\theta}{2(1-\alpha)}=\frac{\rho+2}{2[(1-\alpha)(\rho+2)-(\rho+1)]}<\frac{p}{2}$. And with (3.15), (3.36) becomes

$$\left|E\int_D |v(t)|^\rho v(t)u(t)\,dx\right|^{\frac{1}{1-\alpha}}\leq C\left\{E\|v(t)\|_{\rho+2}^{\rho+2}+\left(E\|u(t)\|_p^2\right)^{\frac{\rho+2}{2[(1-\alpha)(\rho+2)-(\rho+1)]}}\right\}.$$

Using Lemma 3.2 with $s = \frac{\rho+2}{2[(1-\alpha)(\rho+2)-(\rho+1)]}$, we obtain

$$\left|E\int_D \left|v(t)\right|^\rho v(t)u(t)\,dx\right|^{\frac{1}{1-\alpha}} \le C\bigl[H(t) + E\bigl\|v(t)\bigr\|_{\rho+2}^{\rho+2} + E\bigl\|\nabla v(t)\bigr\|^2 + E\bigl\|u(t)\bigr\|_p^p + E(h\circ\nabla u)(t) + E\bigl\|\nabla u(t)\bigr\|^2\bigr].$$

Therefore, we deduce, for all $t \ge 0$,

$$\begin{aligned} L^{\frac{1}{1-\alpha}}(t) &= \left(H^{1-\alpha}(t) + \frac{\delta}{\rho+1}E\int_D \left|v(t)\right|^\rho v(t)u(t)\,dx + \delta E\int_D \nabla v(t)\cdot\nabla u(t)\,dx\right)^{\frac{1}{1-\alpha}} \\ &\le C\bigl[H(t) + E\bigl\|v(t)\bigr\|_{\rho+2}^{\rho+2} + E\bigl\|\nabla v(t)\bigr\|^2 \\ &\quad + E\bigl\|u(t)\bigr\|_p^p + E(h\circ\nabla u)(t) + E\bigl\|\nabla u(t)\bigr\|^2\bigr]. \end{aligned} \tag{3.37}$$

Combining (3.33) and (3.37)

$$L'(t) \ge KL^{\frac{1}{1-\alpha}}(t), \quad \forall t \ge 0$$

with a positive constant K depending on C and $\delta\gamma$, it follows that

$$L^{\frac{\alpha}{1-\alpha}}(t) \ge \frac{1-\alpha}{(1-\alpha)L^{-\frac{\alpha}{1-\alpha}}(0) - \alpha Kt}.$$

Let

$$T_0 = \frac{1-\alpha}{\alpha KL^{\frac{\alpha}{1-\alpha}}(0)}.$$

Then $L(t) \to \infty$ as $t \to T_0$. This means that there exists a positive time $T^* \in (0, T_0]$ such that

$$\lim_{t\to T^*} E\bigl[F(t)\bigr] = +\infty.$$

As for the case when $P(\tau_\infty = +\infty) < 1$ (i.e., $P(\tau_\infty < +\infty) > 0$), then $\|\nabla u(t)\|$ blows up in finite time $T^* \in (0, \tau_\infty)$ with positive probability. Thus, the proof of Theorem 3.1 is completed. □

4 Global existence

In this section, we show that the solution of (1.1) is global if $q \ge p$. We use the Borel-Cantelli lemma to prove the existence of a global solution. For this goal, we introduce an energy function

$$e\bigl(u(t)\bigr) = \bigl\|u_t(t)\bigr\|_{\rho+2}^{\rho+2} + \bigl\|\nabla u(t)\bigr\|^2 + \bigl\|\nabla u_t(t)\bigr\|^2 + \bigl\|u(t)\bigr\|_p^p + (h\circ\nabla u)(t). \tag{4.1}$$

Theorem 4.1 *Assume that* $(u_0, u_1) \in H_0^1(D) \times L^2(D)$, $E\int_0^T \|\sigma(t)\|^2\,dt < \infty$, *and condition* (2.1) *holds. If* $q \ge p$, $u(t)$ *is a solution of* (1.1) *with the initial data* $(u_0, u_1) \in H_0^1(D) \times L^2(D)$ *according to Definition* 2.1 *on the interval* $[0, T]$, *then for any* $T > 0$, *we have*

$$E\sup_{0\le t\le T} e\bigl(u(t)\bigr) < \infty. \tag{4.2}$$

Proof For any $T > 0$, we will show that $u_N(t) = u(t \wedge \tau_N) \to u(t)$ (a.e.) as $N \to \infty$ for any $t \le T$, so that the local solution becomes a global solution where τ_N is a stopping time which is defined in Section 3. Similarly to Theorem 12 of [23], we can derive the proof of the theorem. □

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