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New existence results for coupled delayed differential systems with multi-parameters

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Abstract

In this paper, several novel existence and multiplicity results are established for a coupled functional differential system with multi-parameters. The discussion is based upon fixed point theory, and our main findings enrich and complement those available in the literature.

MSC: 34B15

Keywords: Positive periodic solution; Existence; Multi-parameters; Fixed point

1 Introduction

This paper is mainly concerned with the existence and multiplicity of positive periodic solutions of the nonlinear coupled differential systems

$$\begin{cases} u'(t) + a_1(t)g_1(u(t))u(t) = \lambda b_1(t)f(u(t - \tau_1(t)), v(t - \zeta_1(t))), \\ v'(t) + a_2(t)g_2(v(t))v(t) = \mu b_2(t)g(u(t - \tau_2(t)), v(t - \zeta_2(t))), \end{cases} \quad (1.1)$$

where we always assume that $a_i, b_i, \tau_i, \zeta_i \in C(\mathbb{R}, \mathbb{R})$ are ω -periodic functions, $g_i : [0, \infty) \rightarrow [0, \infty)$ are continuous and positive functions, $i = 1, 2$; The nonlinearities $f, g : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ are continuous, and $\lambda, \mu > 0$ are parameters. We also assume the periodic ω is a positive number to avoid the trivial cases.

By a positive periodic solution of system (1.1), we mean a solution $(u, v) \in E := X^2$ of (1.1) satisfying $u > 0, v > 0$ on $[0, \omega]$, where

$$X = \{u \in C(\mathbb{R}, \mathbb{R}) : u(t + \omega) = u(t)\}$$

is the Banach space with the usual norm $\|u\| = \max_{t \in [0, \omega]} |u(t)|$. In addition, we write $\|(u, v)\| = \|u\| + \|v\|$ for $(u, v) \in E$.

In some special situations, the u -equation of system (1.1) becomes

$$u'(t) + a(t)u(t) = \lambda b(t)f(u(t - \tau(t))), \quad (1.2)$$

and if $\lambda = 0$, then (1.2) reduces to $u'(t) + a(t)u(t) = 0$, which is well known in Malthusian population models. In real world applications, (1.2) has also been viewed as a mathemat-

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ical model to describe a variety of physiological processes, such as the control of testosterone levels in the blood stream, the production of blood cells, cardiac arrhythmias and so on. In view of this, Eq. (1.2) has been widely studied by a number of authors during the past few decades. Among them, one may refer, with references therein, to [1–14]. Besides, researchers have also focused on the existence of positive periodic solutions of differential systems corresponds to (1.2) of the form

$$u'_i(t) + a_i(t)u_i(t) = \lambda b_i(t)f_i(u_1(t), u_2(t), \dots, u_n(t)), \quad i = 1, 2, \dots, n, \quad (1.3)$$

we refer the readers here to Wang [15, 16] and Chen et al. [17] for some related results in this direction.

Obviously, system (1.3) considered in the existing literature contains only one parameter $\lambda > 0$, and therefore it seems to be interesting to study the multi-parameter system (1.1). On the other hand, what is worth mentioning is that Zhang et al. [18] have established the existence of positive periodic solutions of system (1.1) for the special case $g_i \equiv 1$, $i = 1, 2$, where $f(u, v)$ and $g(u, v)$ were assumed to be nondecreasing, and the authors have only dealt with the case $f(0, 0) > 0$, $g(0, 0) > 0$. Therefore, it is interesting to see whether or not (1.1) admits a positive periodic solution under more relaxed assumptions $f(0, 0) = 0$, $g(0, 0) = 0$. In the current paper, we shall apply a fixed point theorem in cones to establish the existence and multiplicity of positive periodic solutions of (1.1), to further generalize and complement those in [16–18]. For other research work on differential equations and systems with multi-parameters, we refer the reader to [19–21] and the references therein.

We conclude this section giving the main tool adopted in the subsequent discussion.

Lemma 1.1 ([22, 23]) *Let E be a Banach space and $K \subseteq E$ a cone. Assume Ω_1, Ω_2 are open subsets of E with $0 \in \Omega_1$, $\bar{\Omega}_1 \subseteq \Omega_2$ and let*

$$T : K \cap (\bar{\Omega}_2 \setminus \Omega_1) \rightarrow K$$

be a completely continuous operator such that either

$$(i) \quad \|Tu\| \leq \|u\|, \quad u \in K \cap \partial\Omega_1, \quad \text{and} \quad \|Tu\| \geq \|u\|, \quad u \in K \cap \partial\Omega_2,$$

or

$$(ii) \quad \|Tu\| \geq \|u\|, \quad u \in K \cap \partial\Omega_1, \quad \text{and} \quad \|Tu\| \leq \|u\|, \quad u \in K \cap \partial\Omega_2.$$

Then T has a fixed point in $K \cap (\bar{\Omega}_2 \setminus \Omega_1)$.

The rest of the paper is arranged as follows. In Sect. 2, we introduce some preliminaries required during our proof. In Sect. 3, we shall state and prove the existence and multiplicity results established for (1.1). And finally in Sect. 4, we give some related results and remarks to demonstrate the feasibility of our main findings.

2 Preliminaries

Suppose that $(u, v) \in E$ is a ω -periodic solution of system (1.1), then by simple calculation we can obtain

$$\begin{aligned} u(t) &= \lambda \int_t^{t+\omega} G_1(t, s) b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds, \\ v(t) &= \mu \int_t^{t+\omega} G_2(t, s) b_2(s) g(u(s - \tau_2(s)), v(s - \zeta_2(s))) ds, \end{aligned}$$

where the Green functions $G_1(t, s)$ and $G_2(t, s)$ can be expressed as

$$G_1(t, s) = \frac{e^{\int_t^s a_1(\theta) g_1(u(\theta)) d\theta}}{e^{\int_0^\omega a_1(\theta) g_1(u(\theta)) d\theta} - 1}, \quad G_2(t, s) = \frac{e^{\int_t^s a_2(\theta) g_2(v(\theta)) d\theta}}{e^{\int_0^\omega a_2(\theta) g_2(v(\theta)) d\theta} - 1},$$

for $t \leq s \leq t + \omega$.

Lemma 2.1 Assume that

(H1) $a_i, b_i \in C(\mathbb{R}, [0, \infty))$ are ω -periodic with $\int_0^\omega a_i(t) dt > 0$ and $\int_0^\omega b_i(t) dt > 0$, $i = 1, 2$.

(H2) There are $l_i, L_i > 0$ such that $0 < l_i \leq g_i(s) \leq L_i$ for all $s \in [0, \infty)$.

Then we have, for $i = 1, 2$,

$$0 < \frac{1}{e^{L_i \int_0^\omega a_i(\theta) d\theta} - 1} \leq G_i(t, s) \leq \frac{e^{L_i \int_0^\omega a_i(\theta) d\theta}}{e^{L_i \int_0^\omega a_i(\theta) d\theta} - 1}, \quad t \leq s \leq t + \omega.$$

Proof Applying the assumptions (H1) and (H2), and by simple estimation, we can easily get the conclusions. $\square$

For $i = 1, 2$, we denote

$$m_i = \frac{1}{e^{L_i \int_0^\omega a_i(\theta) d\theta} - 1}, \quad M_i = \frac{e^{L_i \int_0^\omega a_i(\theta) d\theta}}{e^{L_i \int_0^\omega a_i(\theta) d\theta} - 1}, \quad \sigma_i = \frac{m_i}{M_i}. \quad (2.1)$$

Then it is not difficult to check that $\sigma_i \in (0, 1)$, and accordingly $\sigma := \min\{\sigma_1, \sigma_2\} \in (0, 1)$, which allows us to define some suitable cone in E as below. Indeed, setting

$$P = \{(u, v) \in E : u(t) \geq 0, v(t) \geq 0, t \in [0, \omega]\},$$

$$K = \{(u, v) \in P : u(t) + v(t) \geq \sigma \|(u, v)\|, t \in [0, \omega]\}.$$

Then P and K are cones in E .

Define, for given $(u, v) \in E$,

$$T(u, v)(t) = (A(u, v)(t), B(u, v)(t)),$$

where

$$A(u, v)(t) = \lambda \int_t^{t+\omega} G_1(t, s) b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds$$

and

$$B(u, v)(t) = \mu \int_t^{t+\omega} G_2(t, s) b_2(s) g(u(s - \tau_2(s)), v(s - \zeta_2(s))) ds.$$

Then we have the following.

Lemma 2.2 Assume that (H1) and (H2) hold. Then $T(P) \subseteq K$ and $T : P \rightarrow K$ is compact and continuous.

Proof Choosing $(u, v) \in P$, then

$$\begin{aligned} A(u, v)(t) &= \lambda \int_t^{t+\omega} G_1(t, s) b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds \\ &\leq \lambda M_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds, \end{aligned}$$

which yields

$$\|A(u, v)\| \leq \lambda M_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds.$$

On the other hand, by Lemma 2.1 and (2.1) we can obtain

$$\begin{aligned} A(u, v)(t) &\geq \lambda m_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds \\ &= \sigma_1 \cdot \lambda M_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds \\ &\geq \sigma \|A(u, v)\|. \end{aligned}$$

Proof of $B(u, v)(t) \geq \sigma \|B(u, v)\|$ can be handled in an analogous manner. Consequently, we have $T(P) \subseteq K$. The completely continuity of T is evident. $\square$

Clearly, if (u, v) is a fixed point of the operator equation

$$(u, v) = T(u, v), \quad (2.2)$$

then the original system (1.1) admits a positive ω -periodic solution (u, v) . Therefore, we shall concentrate on Eq. (2.2) in the arguments that follow.

3 Main results

In this section, we shall state and prove our main findings. We first introduce some notations as follows:

$$\begin{aligned} f_0 &= \lim_{(u,v) \rightarrow 0} \frac{f(u, v)}{u + v}, & f_\infty &= \lim_{(u,v) \rightarrow \infty} \frac{f(u, v)}{u + v}, \\ g_0 &= \lim_{(u,v) \rightarrow 0} \frac{g(u, v)}{u + v}, & g_\infty &= \lim_{(u,v) \rightarrow \infty} \frac{g(u, v)}{u + v}. \end{aligned}$$

Theorem 3.1 Assume (H1) and (H2). If $f_0 = g_0 = 0$ and $f_\infty = \infty$, then system (1.1) admits at least one positive ω -periodic solution for all $\lambda > 0$, $\mu > 0$.

Proof Since $f_0 = g_0 = 0$, there exists a $r_1 > 0$ such that, for $0 < u, v \leq r_1$,

$$f(u, v) \leq \varepsilon(u + v), \quad g(u, v) \leq \varepsilon(u + v),$$

where the positive constant ε satisfies

$$2\varepsilon\lambda M_1 \int_0^\omega b_1(s) ds \leq 1, \quad 2\varepsilon\mu M_2 \int_0^\omega b_2(s) ds \leq 1, \quad (3.1)$$

and $M_i > 0$ is given by (2.1). Let

$$\Omega_1 = \{(u, v) \in E : \|(u, v)\| < r_1\}.$$

Then $(0, 0) \in \Omega_1$ and for $(u, v) \in K \cap \partial\Omega_1$, we can get by (3.1) that

$$A(u, v)(t) \leq \varepsilon \lambda M_1 (\|u\| + \|v\|) \cdot \int_0^\omega b_1(s) ds \leq \frac{1}{2} \|(u, v)\|.$$

An analogous estimate also gives $B(u, v)(t) \leq \frac{1}{2} \|(u, v)\|$. Thus,

$$\|T(u, v)\| = \|A(u, v)\| + \|B(u, v)\| \leq \|(u, v)\|, \quad (u, v) \in K \cap \partial\Omega_1.$$

On the other hand, since $f_\infty = \infty$, there exists $\hat{r} > 0$ so that $f(u, v) \geq \delta(u + v)$ for $u + v \geq \hat{r}$, where $\delta > 0$ is large enough such that

$$\delta \lambda m_1 \sigma \cdot \int_0^\omega b_1(s) ds \geq 1, \quad (3.2)$$

and $\sigma = \min\{\sigma_1, \sigma_2\} \in (0, 1)$ is introduced in Sect. 2. Setting $r_2 = \max\{2r_1, \frac{\hat{r}}{\sigma}\}$ and

$$\Omega_2 = \{(u, v) \in E : \|(u, v)\| < r_2\}.$$

Then it is not hard to see $\bar{\Omega}_1 \subseteq \Omega_2$. Moreover for $(u, v) \in K \cap \partial\Omega_2$, we can deduce from (3.2) that

$$A(u, v)(t) \geq \delta \lambda m_1 \sigma (\|u\| + \|v\|) \cdot \int_0^\omega b_1(s) ds \geq \|(u, v)\|,$$

which implies $\|T(u, v)\| \geq \|A(u, v)\| \geq \|(u, v)\|$, $(u, v) \in K \cap \partial\Omega_2$.

Consequently, Lemma 1.1 guarantees T admits a fixed point $(u, v) \in K \cap (\bar{\Omega}_2 \setminus \Omega_1)$ with $r_1 \leq \|(u, v)\| \leq r_2$, and thus system (1.1) has a positive ω -periodic solution. $\square$

Using Lemma 1.1 again, we can establish the following corollary with some obvious modifications in the proof of Theorem 3.1.

Corollary 3.1 *Let (H1) and (H2) hold. If $f_0 = g_0 = 0$ and $g_\infty = \infty$, then system (1.1) has at least one positive ω -periodic solution for all $\lambda > 0$, $\mu > 0$.*

Theorem 3.2 *Assume (H1) and (H2). If $f_\infty = g_\infty = 0$ and $f_0 = \infty$, then system (1.1) admits at least one positive ω -periodic solution for all $\lambda > 0$, $\mu > 0$.*

Proof We shall follow the same strategy and notations as before. Firstly, we show there is a $r_1 > 0$ such that

$$\|T(u, v)\| \geq \|(u, v)\|, \quad (u, v) \in K \cap \partial\Omega_1. \quad (3.3)$$

In fact, since $f_0 = \infty$, a constant $r_1 > 0$ can be chosen so that $f(u, v) \geq \tilde{\delta}(u + v)$ for $0 < u, v \leq r_1$, where $\tilde{\delta} > 0$ is sufficiently large such that $\tilde{\delta} \lambda m_1 \sigma \cdot \int_0^\omega b_1(s) ds \geq 1$. Let $(u, v) \in K \cap \partial\Omega_1$.

Then

$$A(u, v)(t) \geq \tilde{\delta} \lambda m_1 \sigma \| (u, v) \| \cdot \int_0^\omega b_1(s) ds \geq \| (u, v) \|,$$

which means (3.3) is true.

We next show there exists $r_2 > 0$ satisfying

$$\| T(u, v) \| \leq \| (u, v) \|, \quad (u, v) \in K \cap \partial \Omega_2. \quad (3.4)$$

To the end, we shall denote $\bar{f}(\epsilon) = \max_{0 \leq u+v \leq \epsilon} f(u, v)$ and $\bar{g}(\epsilon) = \max_{0 \leq u+v \leq \epsilon} g(u, v)$. Clearly, $\bar{f}(\epsilon)$ and $\bar{g}(\epsilon)$ are nondecreasing, and $\lim_{\epsilon \rightarrow \infty} \frac{\bar{f}(\epsilon)}{\epsilon} = 0 = \lim_{\epsilon \rightarrow \infty} \frac{\bar{g}(\epsilon)}{\epsilon}$ since $f_\infty = g_\infty = 0$. Now we may choose $r_2 > 2r_1$ so that, for $\epsilon \geq r_2$,

$$\bar{f}(\epsilon) \leq \epsilon \epsilon, \quad \bar{g}(\epsilon) \leq \epsilon \epsilon,$$

where ϵ is a positive constant satisfies (3.1). For $(u, v) \in K \cap \partial \Omega_2$, we can obtain

$$A(u, v)(t) \leq \lambda M_1 \int_0^\omega b_1(s) \bar{f}(r_2) ds \leq \lambda M_1 \epsilon r_2 \cdot \int_0^\omega b_1(s) ds \leq \frac{1}{2} \| (u, v) \|.$$

Similarly, $B(u, v)(t) \leq \frac{1}{2} \| (u, v) \|$ for $(u, v) \in K \cap \partial \Omega_2$. Hence (3.4) holds true.

Finally, Lemma 1.1 shows that system (1.1) admits a positive ω -periodic solution. $\square$

The following existence result is an analog of Theorem 3.2, and it can be proved in an analogous way as above.

Corollary 3.2 *Let (H1) and (H2) hold. If $f_\infty = g_\infty = 0$ and $g_0 = \infty$, then system (1.1) has at least one positive ω -periodic solution for all $\lambda > 0$, $\mu > 0$.*

Let us consider the multiplicity of positive periodic solutions in the remainder of the section. Suppose in addition that

(H3) $f(u, v) > 0$ and $g(u, v) > 0$ for $u, v > 0$.

Theorem 3.3 *Assume (H1)–(H3). If $f_0 = f_\infty = g_0 = g_\infty = 0$, then there exists a constant $\rho_1 > 0$ such that (1.1) admits at least two positive ω -periodic solutions for all $\lambda, \mu \geq \rho_1$.*

Proof Let us define several numbers corresponds to $(u, v) \in K$ with $\| (u, v) \| = d$,

$$\gamma_1(d) = m_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds,$$

$$\gamma_2(d) = m_2 \int_0^\omega b_2(s) g(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds,$$

and $\gamma(d) = \min\{\gamma_1(d), \gamma_2(d)\}$. Then it is not difficult to see $\gamma(d) > 0$ for $d > 0$. Choosing $0 < r_3 < r_4$ and setting $\Omega_i = \{(u, v) \in E : \| (u, v) \| < r_i\}$ for $i = 3, 4$. Let $\rho_1 = \max\{\frac{r_3}{2\gamma(r_3)}, \frac{r_4}{2\gamma(r_4)}\}$. Then we can prove for $(u, v) \in K \cap \partial \Omega_3$ and $\lambda, \mu \geq \rho_1$,

$$A(u, v)(t) \geq \lambda m_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds \geq \lambda \gamma(r_3) \geq \frac{r_3}{2} = \frac{1}{2} \| (u, v) \|,$$

then, as above, we obtain $A(u, v)(t) \geq \frac{r_4}{2} = \frac{1}{2} \|(u, v)\|$ for $(u, v) \in K \cap \partial\Omega_4$ and

$$B(u, v)(t) \geq \frac{r_i}{2}, \quad (u, v) \in K \cap \partial\Omega_i, i = 3, 4.$$

Therefore, $\|T(u, v)\| \geq \|(u, v)\|$ for $(u, v) \in K \cap \partial\Omega_i, i = 3, 4$.

Applying the assumption $f_0 = g_0 = 0$ and by an argument similar to the proof of Theorem 3.1, $r_1 > 0$ can be chosen so that $r_1 < \frac{r_3}{2}$ and

$$\|T(u, v)\| \leq \|(u, v)\|, \quad (u, v) \in K \cap \partial\Omega_1,$$

and similarly, we may choose $r_2 > 2r_4$ such that $\|T(u, v)\| \leq \|(u, v)\|$ for $(u, v) \in K \cap \partial\Omega_2$, where Ω_1, Ω_2 are defined as in Theorem 3.1. Obviously, Lemma 1.1 shows that system (1.1) has two distinct positive ω -periodic solutions. $\square$

Theorem 3.4 Assume (H1)–(H3). Then there exists a constant $\rho_2 > 0$ so that (1.1) has at least two positive ω -periodic solutions for all $\lambda, \mu \leq \rho_2$, provided that either $f_0 = \infty$ or $g_0 = \infty$ and either $f_\infty = \infty$ or $g_\infty = \infty$.

Proof Similar to the proof of Theorem 3.3, we can denote

$$\Gamma_1(d) = M_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds,$$

$$\Gamma_2(d) = M_2 \int_0^\omega b_2(s) g(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds,$$

and $\Gamma(d) = \max\{\Gamma_1(d), \Gamma_2(d)\}$ for given $(u, v) \in K$ with $\|(u, v)\| = d$. Clearly, $\Gamma(d) > 0$ for $d > 0$. Let $0 < r_3 < r_4$ and $\rho_2 = \min\{\frac{r_3}{2\Gamma(r_3)}, \frac{r_4}{2\Gamma(r_4)}\}$. Then

$$A(u, v)(t) \leq \lambda M_1 \int_0^\omega b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds \leq \lambda \Gamma(r_3) \leq \frac{r_3}{2} = \frac{1}{2} \|(u, v)\|$$

for $(u, v) \in K \cap \partial\Omega_3$ and $\lambda, \mu \leq \rho_2$. A similar estimation also gives $A(u, v)(t) \leq \frac{r_4}{2} = \frac{1}{2} \|(u, v)\|$ for $(u, v) \in K \cap \partial\Omega_4$ and

$$B(u, v)(t) \leq \frac{r_i}{2}, \quad (u, v) \in K \cap \partial\Omega_i, i = 3, 4,$$

where Ω_i ($i = 3, 4$) are defined by Theorem 3.3. Hence $\|T(u, v)\| \leq \|(u, v)\|$ for $(u, v) \in K \cap \partial\Omega_i, i = 3, 4$. Choosing $r_1 < \frac{r_3}{2}$ and $r_2 > 2r_4$. Then, by the assumptions of the theorem and a same argument as Theorems 3.1 and 3.2, we can show

$$\|T(u, v)\| \geq \|(u, v)\|, \quad (u, v) \in K \cap \partial\Omega_i, i = 1, 2.$$

Consequently, Lemma 1.1 ensures (1.1) has two distinct positive ω -periodic solutions. $\square$

Remark 3.1 We would like to point out that the results obtained in the present paper are distinct from those in [16–18], since we have considered the differential systems in the multi-parameter case without any monotonicity assumptions on nonlinear terms, and we allow for $f(0, 0) = 0, g(0, 0) = 0$.

4 Related results and remarks

Let us consider the following multi-parameter systems

$$\begin{cases} u'(t) = a_1(t)g_1(u(t))u(t) - \lambda b_1(t)f(u(t - \tau_1(t)), v(t - \zeta_1(t))), \\ v'(t) = a_2(t)g_2(v(t))v(t) - \mu b_2(t)g(u(t - \tau_2(t)), v(t - \zeta_2(t))). \end{cases} \quad (4.1)$$

Under the same assumptions as before, one can see (4.1) is equivalent to

$$\begin{aligned} u(t) &= \lambda \int_t^{t+\omega} G_1(t, s) b_1(s) f(u(s - \tau_1(s)), v(s - \zeta_1(s))) ds, \\ v(t) &= \mu \int_t^{t+\omega} G_2(t, s) b_2(s) g(u(s - \tau_2(s)), v(s - \zeta_2(s))) ds, \end{aligned}$$

where

$$G_1(t, s) = \frac{e^{-\int_t^s a_1(\theta)g_1(u(\theta))d\theta}}{1 - e^{-\int_0^\omega a_1(\theta)g_1(u(\theta))d\theta}}, \quad G_2(t, s) = \frac{e^{-\int_t^s a_2(\theta)g_2(v(\theta))d\theta}}{1 - e^{-\int_0^\omega a_2(\theta)g_2(v(\theta))d\theta}}, \quad t \leq s \leq t + \omega.$$

Furthermore, by inspection of the arguments in Sects. 2 and 3, it is not difficult to see our main results remain true for system (4.1).

Remark 4.1 It is worth remarking that under some natural assumptions, the results of the paper are still valid for more general coupled systems

$$u'_i(t) + a_i(t)g_i(u_i(t))u_i(t) = \lambda_i b_i(t)f_i(u_1(t - \tau_{i1}(t)), \dots, u_n(t - \tau_{in}(t))), \quad i = 1, 2, \dots, n,$$

and

$$u'_i(t) = a_i(t)g_i(u_i(t))u_i(t) - \lambda_i b_i(t)f_i(u_1(t - \tau_{i1}(t)), \dots, u_n(t - \tau_{in}(t))), \quad i = 1, 2, \dots, n.$$

Remark 4.2 In this paper, we have not considered the multi-parameter system (1.1) for the case g_i is unbounded as in [7–9]. Nevertheless, but we believe that in the case, systems (1.1) may have positive periodic solutions if some suitable cones can be defined. It will be treated in the forthcoming paper.

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Abbreviations

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Availability of data and materials

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

RC carried out the analysis and proof the main results, and was a major contributor in writing the manuscript. XL participated in checking of the proofs, English grammar as well as typing errors in the full text. All authors read and approved the final manuscript.

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