Xu et al. *Boundary Value Problems* (2019) 2019:36
https://doi.org/10.1186/s13661-019-1145-9

Boundary Value Problems
a SpringerOpen Journal

RESEARCH

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New results on competition and cooperation model of two enterprises with multiple delays and feedback controls

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Abstract

In this paper, we put up and study a competition and cooperation model of two enterprises with multiple delays and feedback controls. A set of sufficient conditions which ensure the existence of periodic solution of the two-enterprise competition and cooperation model with multiple delays and feedback controls are established by applying the fixed point theorem of strict-set-contraction. An example is given to check the obtained theoretical findings. The derived results are completely new and complement earlier publications.

MSC: 34K20; 34K13; 34C25; 92D25

Keywords: Competition and cooperation model; Periodic solution; Two-enterprise; Feedback control; Delay

1 Introduction

It is well known that the interaction between two enterprises plays an extremely important role in enterprise operation. In order to grasp the internal mechanism, operating law, administrative procedure, risk and efficiency, etc. [1], to improve the management level of enterprises, it is necessary for us to establish the interaction model of two enterprises and analyze their dynamical behavior. In recent decades, some scholars have considered this topic. For example, Tian and Nie [2] proposed the following two-enterprise interaction model:

$$\begin{cases} \dot{u}_1(t) = \gamma_1(t)u_1(t)[1 - \frac{u_1(t)}{\kappa_1} - \frac{a(u_2(t)-\sigma_2)^2}{\kappa_2}], \\ \dot{u}_2(t) = \gamma_2(t)u_2(t)[1 - \frac{u_2(t)}{\kappa_2} - \frac{b(u_1(t)-\sigma_1)^2}{\kappa_1}], \end{cases} \tag{1.1}$$

where $u_1(t)$, $u_2(t)$ denote the output of enterprises A and B, γ_1, γ_2 stand for the intrinsic growth rate, κ_i $(i = 1, 2)$ represents the carrying capacity of mark under natural unlimited conditions, a, b denote the competitive coefficients of two enterprises, σ_1, σ_2 represent the initial production of two enterprises. Let $\alpha_1 = \frac{\gamma_1}{\kappa_1}$, $\alpha_2 = \frac{\gamma_2}{\kappa_2}$, $\beta_1 = \frac{\gamma_1 a}{\kappa_2}$, $\beta_2 = \frac{\gamma_2 b}{\kappa_1}$, then system (1.1) becomes

$$\begin{cases} \dot{u}_1(t) = u_1(t)[\gamma_1 - \alpha_1 u_1(t) - \beta_1(u_2(t) - \sigma_2)^2], \\ \dot{u}_2(t) = u_2(t)[\gamma_2 - \alpha_2 u_2(t) + \beta_2(u_1(t) - \sigma_1)^2]. \end{cases} \tag{1.2}$$

Considering that the parameters of (1.2) vary with time, Li and Zhang [1] established the following modified two-enterprise interaction model with variable coefficients:

$$\begin{cases} \dot{u}_1(t) = u_1(t)[\gamma_1(t) - \alpha_1(t)u_1(t) - \beta_1(t)(u_2(t) - \sigma_2(t))^2], \\ \dot{u}_2(t) = u_2(t)[\gamma_2(t) - \alpha_2(t)u_2(t) + \beta_2(t)(u_1(t) - \sigma_1(t))^2]. \end{cases} \tag{1.3}$$

In 2012, Xu [3] established some sufficient criteria to guarantee the existence of periodic solutions of (1.3) by using the coincidence degree method and differential inequality technique. In many cases, the environment often varies and the output of two enterprises is subjected to rapid change at certain instants in time. Xu and Shao [4] established the following two-enterprise interaction model with impulses:

$$\begin{cases} \dot{u}_1(t) = u_1(t)[\gamma_1(t) - \alpha_1(t)u_1(t) - \beta_1(t)(u_2(t) - \sigma_2(t))^2], \quad t \neq t_k, \\ \dot{u}_2(t) = u_2(t)[\gamma_2(t) - \alpha_2(t)u_2(t) + \beta_2(t)(u_1(t) - \sigma_1(t))^2], \quad t \neq t_k, \\ \Delta_i(t_k) = u_i(t_k^+) - u_i(t_k^-) = -\varrho_{ik}(t_k), \quad i = 1,2, k = 1,2,\ldots,q, \end{cases} \tag{1.4}$$

where $\Delta_i(t_k) = u_i(t_k^+) - u_i(t_k^-)$ are the impulses at moments t_k and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to+\infty} t_k = +\infty$ and q is a positive integer. Using continuation theorem and constructing an appropriate Lyapunov functional, Xu and Shao [4] discussed the periodic solution and global attractivity of (1.4). Considering the different effect of time delay on the two-enterprise interaction, Liao et al. [5] investigated the stability and Hopf bifurcation of the following two-enterprise interaction model with delays:

$$\begin{cases} \dot{u}_1(t) = \gamma_1(t)u_1(t)[1 - \frac{u_1(t-\upsilon)}{\kappa_1} - \frac{a(u_2(t-\upsilon)-\sigma_2)^2}{\kappa_2}], \\ \dot{u}_2(t) = \gamma_2(t)u_2(t)[1 - \frac{u_2(t-\upsilon)}{\kappa_2} - \frac{b(u_1(t-\upsilon)-\sigma_1)^2}{\kappa_1}], \end{cases} \tag{1.5}$$

where υ is the time delay in the interior of enterprises and among different enterprises. In 2014, Liao et al. [6] studied the bifurcation behavior of the following two-enterprise interaction model with two different delays:

$$\begin{cases} \dot{u}_1(t) = \gamma_1(t)u_1(t)[1 - \frac{u_1(t-\upsilon_1)}{\kappa_1} - \frac{a(u_2(t-\upsilon_2)-\sigma_2)^2}{\kappa_2}], \\ \dot{u}_2(t) = \gamma_2(t)u_2(t)[1 - \frac{u_2(t-\upsilon_1)}{\kappa_2} - \frac{b(u_1(t-\upsilon_2)-\sigma_1)^2}{\kappa_1}], \end{cases} \tag{1.6}$$

where υ_i $(i = 1,2)$ is the time delay in the interior of enterprises and among different enterprises. Li et al. [7] considered the stability and bifurcation behavior of the following two-enterprise interaction model with four different delays:

$$\begin{cases} \dot{u}_1(t) = \gamma_1(t)u_1(t)[1 - \frac{u_1(t-\upsilon_1)}{\kappa_1} - \frac{a(u_2(t-\upsilon_2)-\sigma_2)^2}{\kappa_2}], \\ \dot{u}_2(t) = \gamma_2(t)u_2(t)[1 - \frac{u_2(t-\upsilon_3)}{\kappa_2} - \frac{b(u_1(t-\upsilon_4)-\sigma_1)^2}{\kappa_1}], \end{cases} \tag{1.7}$$

where υ_i $(i = 1,2,3,4)$ is the time delay in the interior of enterprises and among different enterprises. For more related works, we refer the readers to [8–11]. Here we would like to point out that the output of enterprises A and B is often affected by unpredictable forces which can be expressed as disturbance functions [12–16]. In addition, there usually exists a time delay in unpredictable forces. Stimulated by this viewpoint, we can establish the following competition and cooperation model of two enterprises with multiple delays and feedback controls:

$$\begin{cases} \dot{u}_1(t) = u_1(t)[\gamma_1(t) - \alpha_1(t)u_1(t) - \beta_1(t)(u_2(t) - \sigma_2(t))^2 - a_1(t)v_1(t - \eta_1(t))], \\ \dot{v}_1(t) = -\delta_1(t)v_1(t) + \varrho_1(t)u_1(t - \zeta_1(t)), \\ \dot{u}_2(t) = u_2(t)[\gamma_2(t) - \alpha_2(t)u_2(t) + \beta_2(t)(u_1(t) - \sigma_1(t))^2 - a_2(t)v_2(t - \eta_2(t))], \\ \dot{v}_2(t) = -\delta_2(t)v_2(t) + v_2(t)u_2(t - \zeta_2(t)), \end{cases} \tag{1.8}$$

where v_1, v_2 are the control variables. The existence of periodic solution of the competition and cooperation model of two enterprises plays an important role in running mechanism, administering process, economic performance of enterprises. Thus the study on the existence of periodic solution of the competition and cooperation model of two enterprises has theoretical significance and practical application [17–36]. The main objective of this paper is to focus on the periodic solutions of model (1.8).

The remainder of the paper is organized as follows: in Sect. 2, basic notations, assumptions, definitions, and lemmas are prepared. In Sect. 3, a set of sufficient conditions which ensure the existence of periodic solution of the system are established. In Sect. 4, an example with its simulations is given to illustrate the correctness of the obtained results in Sect. 3. A simple conclusion is given in Sect. 5.

2 Preliminaries

In this section, we shall first state some notations, assumptions, definitions, and lemmas.

Let R and R^+ denote the sets of all real numbers and nonnegative real numbers, respectively. Let $\bar{l} = \max\{l(t) : t \in [0, \varpi]\}$ and $\underline{l} = \min\{l(t) : t \in [0, \varpi]\}$, where l is a continuous bounded ϖ-periodic function on R.

Throughout this paper, we give the following assumptions:

(H1) For $i = 1, 2$, γ_i, α_i, β_i, σ_i, a_i, η_i, δ_i, ϱ_1, ζ_i are all continuous ϖ-periodic functions and

$$\begin{aligned} &\delta_i^* = e^{\int_0^{\varpi} \delta_i(s)\,ds} > 1, \qquad \gamma_{i*} = e^{-\int_0^{\varpi} \gamma_i(s)\,ds} < 1, \\ &A_i(t,s) = \frac{e^{\int_t^s \delta_i(s)\,ds}}{e^{\int_0^{\varpi} \delta_i(s)\,ds} - 1}, \qquad B_i(t,s) = \frac{e^{-\int_t^s \gamma_i(s)\,ds}}{1 - e^{\int_0^{\varpi} \gamma_i(s)\,ds}}, \\ &G_i(t) = \int_t^{t+\varpi} A_i(t,s)\varrho_i(s)\,ds, \\ &K_i = \int_0^{\varpi} \bigl[\gamma_{i*}\alpha_i(s) + \gamma_{i*}\beta_i(s)\|u\| + \gamma_{i*}a_i(s)G_i\bigl(s - \eta_i(s)\bigr)\bigr]\,ds, \\ &P_i = \int_0^{\varpi} \bigl[\alpha_i(s) + \beta_i(s)\|u\| + a_i(s)G_i\bigl(s - \eta_1(s)\bigr)\bigr]\,ds. \end{aligned}$$

(H2) The following inequalities hold:

$$\begin{aligned} &\alpha_1(t)\gamma_{1*}\|u\| + \beta_1(t)\gamma_{2*}\|u\|^2 + 2\beta_1(t)\sigma_2(t)\gamma_{2*}\|u\| \\ &\quad + \beta_1(t)\sigma_2^2(t) + \gamma_{1*}a_1(t)G_1\bigl(t - \eta_1(t)\bigr)\|u\| \geq 0 \end{aligned}$$

and

$$\alpha_2(t)\gamma_{2*}\|u\| + \gamma_{2*}a_2(t)G_2\bigl(t - \eta_2(t)\bigr)\|u\| \geq 0.$$

(H3) For $i = 1, 2$, the following inequality holds:

$$\frac{(1+\underline{\gamma}_{i*})(\gamma_{i*})^2}{1-\gamma_{i*}} K_i \geq \max\bigl\{\alpha_i(s) + \beta_i(s)\|u\| + a_i(s)G_i\bigl(s-\eta_1(s)\bigr), t \in [0, \varpi]\bigr\}.$$

(H4) For $i = 1, 2$, the following inequality holds:

$$\frac{1-\bar{\gamma}_{i*}}{\gamma_{i*}(1-\gamma_{i*})} P_i \leq \min\bigl\{\gamma_{i*}\alpha_i(t) + \gamma_{i*}\beta_i(t)\|u\| + \gamma_{i*}a_i(t)G_i\bigl(t-\eta_i(t)\bigr), t \in [0, \varpi]\bigr\}.$$

Definition 2.1 Let $\mathbf{B}$ be a Banach space and $\mathbf{C}$ be a closed, nonempty subset of $\mathbf{B}$. We say that $\mathbf{C}$ is a cone if (1) $a\alpha + b\beta \in \mathbf{C}$ for all $\alpha, \beta \in \mathbf{C}$ and all $a, b > 0$, (2) $\alpha, -\alpha \in \mathbf{C}$ imply $\alpha = 0$.

Let $\mathbf{B}$ be a Banach space and $\mathbf{C}$ be a cone in $\mathbf{B}$. The semi-order induced by the cone $\mathbf{C}$ is denoted by "$\leq$", i.e., $\alpha \leq \beta$ if and only if $\beta - \alpha \in \mathbf{C}$. For a bounded subset $\mathbf{E} \subset \mathbf{B}$, let $a_{\mathbf{B}}(\mathbf{E})$ denote the measure (Kuratowski) of non-compactness defined by

$$\begin{aligned} a_{\mathbf{B}}(\mathbf{E}) = \inf\biggl\{&\varrho > \mathbf{0} : \text{there is a finite number of subset } \mathbf{B}_i \subset \mathbf{B} \\ &\text{such that } \mathbf{B} = \bigcup_i \mathbf{B}_i \text{ and } \operatorname{diam}(\mathbf{B}_i) = \varrho\biggr\}, \end{aligned}$$

where $\operatorname{diam}(\mathbf{B}_i)$ represents the diameter of the set $\mathbf{B}_i$.

Definition 2.2 Let $\mathbf{C}$, $\mathbf{D}$ be two Banach spaces and $\mathbf{F} \subset \mathbf{C}$, a continuous and bounded map $\Gamma : \mathbf{F} \to \mathbf{D}$ is called κ-set-contractive if for any bounded set $\mathbf{S} \subset \mathbf{F}$ we get $a_{\mathbf{D}}(\Gamma(\mathbf{S})) \leq \kappa a_{\mathbf{C}}(\mathbf{S})$. Γ is called strict-set-contractive if it is κ-set-contractive for some $0 \leq \kappa \leq 1$.

Lemma 2.1 *Let $\mathbf{Q}$ be a cone of the real Banach space $\mathbf{B}$ and $\Omega_{R_1} = \{x \in \mathbf{B} : \|x\| < R_1\}$, $\Omega_{R_2} = \{x \in \mathbf{B} : \|x\| < R_2\}$, where $R_1 > R_2 > 0$. Assume that $\Gamma : \mathbf{Q} \cap \bar{\Omega}_{R_1} \setminus \Omega_{R_2} \to \mathbf{Q}$ is strict-set-contractive such that one of the following two conditions are satisfied*: (1) *Not $\Gamma x \geq x$ for all $x \in \mathbf{Q} \cap \partial\Omega_{R_2}$*. (2) *Not $\Gamma x \leq x$ for all $x \in \mathbf{Q} \cap \partial\Omega_{R_1}$. Then Γ has at least one fixed point in $\mathbf{Q} \cap \bar{\Omega}_{R_1} \setminus \Omega_{R_2}$.*

3 Existence of periodic solution

In this section, we will consider the existence of at least one positive periodic solution of system (1.8) by applying the fixed point theorem for the strict-set-contraction.

It is easy to see that each ϖ-periodic solution of the equation

$$\dot{v}_1(t) = -\delta_1(t)v_1(t) + \varrho_1(t)u_1\bigl(t - \zeta_1(t)\bigr) \tag{3.1}$$

is equivalent to that of the equation

$$v_1(t) = \int_t^{t+\varpi} A_1(t,s)\varrho_1(s)u_1\bigl(s - \zeta_1(s)\bigr)\, ds = \psi_1(t), \tag{3.2}$$

and each ϖ-periodic solution of the equation

$$\dot{v}_2(t) = -\delta_2(t)v_2(t) + v_2(t)u_2\bigl(t - \zeta_2(t)\bigr) \tag{3.3}$$

is equivalent to that of the equation

$$v_2(t) = \int_t^{t+\varpi} A_2(t,s)v_2(s)u_2\bigl(s-\zeta_2(s)\bigr)\,ds = \psi_2(t). \tag{3.4}$$

Then the existence of ϖ-periodic solution of (1.8) can be equivalent to the existence of ϖ-periodic solution of the following equations:

$$\begin{cases} u_1(t) = \int_t^{t+\varpi} B_1(t,s)u_1(s)[\alpha_1(s)u_1(s) + \beta_1(s)(u_2(s)+\sigma_2(s))^2 \\ \qquad + a_1(s)\psi_1(s-\eta_1(s))]\,ds, \\ \dot{u}_2(t) = \int_t^{t+\varpi} B_2(t,s)u_2(s)[\alpha_2(s)u_2(s) - \beta_2(s)(u_1(s)+\sigma_1(s))^2 \\ \qquad + a_2(s)\psi_2(s-\eta_2(s))]\,ds. \end{cases} \tag{3.5}$$

Let

$$C_\varpi = \left\{ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} : u_i \in C(R,R), u_i(t+\varpi) = u_i(t), i = 1,2, t \in R \right\},$$

with the norm $\|u\| = \max\{|u_i(t)| : t \in [0,\varpi], i = 1,2\}$. Then C_ϖ is a Banach space. Let

$$\mathbf{Q} = \bigl\{u : u \in C_\varpi, u_i(t) \geq \gamma_{i*}\|u\|, i = 1,2, t \in [0,\varpi]\bigr\}.$$

Let

$$\Omega_{R_1} = \bigl\{u \in C_\varpi : \|x\| < R_1\bigr\}, \Omega_{R_2} = \bigl\{u \in C_\varpi : \|x\| < R_2\bigr\},$$

where $R_1 > R_2 > 0$. Define the map Γ as follows:

$$\Gamma u = (\Gamma u_1, \Gamma u_2)^T,$$

where

$$\begin{cases} \Gamma u_1 = \int_t^{t+\varpi} B_1(t,s)u_1(s)[\alpha_1(s)u_1(s) + \beta_1(s)(u_2(s)+\sigma_2(s))^2 \\ \qquad + a_1(s)\psi_1(s-\eta_1(s))]\,ds, \\ \Gamma u_2 = \int_t^{t+\varpi} B_2(t,s)u_2(s)[\alpha_2(s)u_2(s) - \beta_2(s)(u_1(s)+\sigma_1(s))^2 \\ \qquad + a_2(s)\psi_2(s-\eta_2(s))]\,ds. \end{cases} \tag{3.6}$$

Lemma 3.1 *In addition to* (H1)–(H3), *assume that* $\bar{\gamma}_i < 1$ $(i = 1,2)$ *holds, then* $\Gamma : \mathbf{Q} \to \mathbf{Q}$ *is well defined.*

Proof $\forall u \in \mathbf{Q}$ we have $\Gamma u \in C_\varpi$. In view of (H2), for $t \in [0,\varpi]$, we have

$$\begin{cases} \alpha_1(t)u_1(t) + \beta_1(t)(u_2(t)+\sigma_2(t))^2 + a_1(t)\psi_1(t-\eta_1(t)) \\ \quad \geq \alpha_1(t)\gamma_{1*}\|u\| + \beta_1(t)\gamma_{2*}\|u\|^2 + 2\beta_1(t)\sigma_2(t)\gamma_{2*}\|u\| \\ \qquad + \beta_1(t)\sigma_2^2(t) + \gamma_{1*}a_1(t)G_1(t-\eta_1(t))\|u\| \geq 0, \\ \alpha_2(t)u_2(t) - \beta_2(t)(u_1(t)+\sigma_1(t))^2 + a_2(t)\psi_2(t-\eta_2(t)) \\ \quad \geq \alpha_2(t)\gamma_{2*}\|u\| + \gamma_{2*}a_2(t)G_2(t-\eta_2(t))\|u\| \geq 0. \end{cases} \tag{3.7}$$

In addition, for $t \in [0, \varpi]$,

$$
\begin{aligned}
|\Gamma u_1| &\le \frac{1}{1-\gamma_{1*}} \int_t^{t+\varpi} u_1(s)\bigl[\alpha_1(s)u_1(s) + \beta_1(s)\bigl(u_2(s)+\sigma_2(s)\bigr)^2 + a_1(s)\psi_1\bigl(s-\eta_1(s)\bigr)\bigr]\,ds \\
&= \frac{1}{1-\gamma_{1*}} \int_0^{\varpi} u_1(s)\bigl[\alpha_1(s)u_1(s) + \beta_1(s)\bigl(u_2(s)+\sigma_2(s)\bigr)^2 \\
&\quad + a_1(s)\psi_1\bigl(s-\eta_1(s)\bigr)\bigr]\,ds, \qquad (3.8)\\
|\Gamma u_2| &\le \frac{1}{1-\gamma_{2*}} \int_t^{t+\varpi} u_2(s)\bigl[\alpha_2(s)u_2(s) - \beta_2(s)\bigl(u_1(s)+\sigma_1(s)\bigr)^2 + a_2(s)\psi_2\bigl(s-\eta_2(s)\bigr)\bigr]\,ds \\
&= \frac{1}{1-\gamma_{2*}} \int_0^{\varpi} u_2(s)\bigl[\alpha_2(s)u_2(s) - \beta_2(s)\bigl(u_1(s)+\sigma_1(s)\bigr)^2 \\
&\quad + a_2(s)\psi_2\bigl(s-\eta_2(s)\bigr)\bigr]\,ds. \qquad (3.9)
\end{aligned}
$$

Then, for $t \in [0, \varpi]$, we have

$$
\begin{aligned}
\Gamma u_1 &\ge \frac{\gamma_{1*}}{1-\gamma_{1*}} \int_t^{t+\varpi} u_1(s)\bigl[\alpha_1(s)u_1(s) + \beta_1(s)\bigl(u_2(s)+\sigma_2(s)\bigr)^2 + a_1(s)\psi_1\bigl(s-\eta_1(s)\bigr)\bigr]\,ds \\
&= \frac{\gamma_{1*}}{1-\gamma_{1*}} \int_0^{\varpi} u_1(s)\bigl[\alpha_1(s)u_1(s) + \beta_1(s)\bigl(u_2(s)+\sigma_2(s)\bigr)^2 + a_1(s)\psi_1\bigl(s-\eta_1(s)\bigr)\bigr]\,ds \\
&\le \gamma_{1*}\|\Gamma u\|, \qquad (3.10)\\
\Gamma u_2 &\ge \frac{\gamma_{2*}}{1-\gamma_{2*}} \int_t^{t+\varpi} u_2(s)\bigl[\alpha_2(s)u_2(s) - \beta_2(s)\bigl(u_1(s)+\sigma_1(s)\bigr)^2 + a_2(s)\psi_2\bigl(s-\eta_2(s)\bigr)\bigr]\,ds \\
&= \frac{\gamma_{2*}}{1-\gamma_{2*}} \int_0^{\varpi} u_2(s)\bigl[\alpha_2(s)u_2(s) - \beta_2(s)\bigl(u_1(s)+\sigma_1(s)\bigr)^2 + a_2(s)\psi_2\bigl(s-\eta_2(s)\bigr)\bigr]\,ds \\
&\le \gamma_{2*}\|\Gamma u\|. \qquad (3.11)
\end{aligned}
$$

By (3.10) and (3.11), we get $\Gamma u \in \mathbf{Q}$. The proof of Lemma 3.1 is complete. □

Lemma 3.2 *In addition to* (H1)–(H3), *if* $\kappa = \max_{i=1,2}\{\frac{R_1^2}{1-\gamma_{i*}}[2\alpha_1(s) + 3\beta_1(s)R_1]\} < 1$, *then* $\Gamma : \mathbf{Q} \cap \bar{\Omega}_{R_1} \setminus \Omega_{R_2} \to \mathbf{Q}$ *is strict-set-contractive.*

Proof Obviously, Γ is continuous and bounded. Now we prove that $a_{C_\varpi}(\Gamma(\mathbf{S})) \le \kappa a_{\mathbf{C}}(\mathbf{S})$ for any bounded set $\mathbf{S} \subset \mathbf{Q} \cap \bar{\Omega}_{R_1}$. For any $\epsilon > 0$, there is a finite family of subsets $\mathbf{S} \subset \bigcup_i \mathbf{S}_i$ with $\operatorname{diam}(\mathbf{S}_i) \le \epsilon$. Then $\|u - v\| \le \epsilon$ for any $u, v \in \mathbf{S}_i$. For $t \in [0, \varpi]$, we have

$$
\begin{aligned}
&|\Gamma u_1 - \Gamma v_1| \\
&\quad = \int_t^{t+\varpi} B_1(t,s)u_1(s)\bigl[\alpha_1(s)u_1(s) + \beta_1(s)\bigl(u_2(s)+\sigma_2(s)\bigr)^2 + a_1(s)\psi_1\bigl(s-\eta_1(s)\bigr)\bigr]\,ds, \\
&\qquad - \int_t^{t+\varpi} B_1(t,s)v_1(s)\bigl[\alpha_1(s)v_1(s) + \beta_1(s)\bigl(v_2(s)+\sigma_2(s)\bigr)^2 + a_1(s)\psi_1\bigl(s-\eta_1(s)\bigr)\bigr]\,ds \\
&\quad \le \frac{R_1^2}{1-\gamma_{1*}} \int_0^{\varpi} \bigl[2\alpha_1(s)\epsilon + 3\beta_1(s)R_1\epsilon\bigr]\,ds \\
&\quad = \frac{R_1^2}{1-\gamma_{1*}}\bigl[2\alpha_1(s)\epsilon + 3\beta_1(s)R_1\epsilon\bigr]\varpi, \qquad (3.12)
\end{aligned}
$$

$$
\begin{aligned}
&|\Gamma u_2 - \Gamma v_2| \\
&\quad = \int_t^{t+\varpi} B_2(t,s)u_2(s)\big[\alpha_2(s)u_2(s) - \beta_2(s)\big(u_1(s)+\sigma_1(s)\big)^2 + a_2(s)\psi_2\big(s-\eta_2(s)\big)\big]\,ds \\
&\qquad - \int_t^{t+\varpi} B_2(t,s)v_2(s)\big[\alpha_2(s)v_2(s) - \beta_2(s)\big(v_1(s)+\sigma_1(s)\big)^2 + a_2(s)\psi_2\big(s-\eta_2(s)\big)\big]\,ds \\
&\quad \le \frac{R_1^2}{1-\gamma_{2*}} \int_0^{\varpi} \big[2\alpha_2(s)\epsilon + 3\beta_2(s)R_1\epsilon\big]\,ds \\
&\quad = \frac{R_1^2}{1-\gamma_{2*}} \big[2\alpha_2(s)\epsilon + 3\beta_2(s)R_1\epsilon\big]\varpi.
\end{aligned} \tag{3.13}
$$

In view of (3.12) and (3.13), we have

$$
a_{C_\varpi}\big(\Gamma(\mathbf{S})\big) \le \kappa a_{\mathbf{C}}(\mathbf{S}).
$$

Then $\Gamma : \mathbf{Q} \cap \bar{\Omega}_{R_1} \setminus \Omega_{R_2} \to \mathbf{Q}$ is strict-set-contractive. This completes the proof. □

Theorem 3.1 *In addition to* (H1)–(H3), *if* $\kappa = \max_{i=1,2}\{\frac{R_1^2}{1-\gamma_{i*}}[2\alpha_1(s) + 3\beta_1(s)R_1]\} < 1$, *then system* (1.8) *has at least one positive* ϖ*-periodic solution.*

Proof Let

$$
R_1 = \max_{i=1,2}\left\{\frac{1-\gamma_{i*}}{(\gamma_{i*})^2 K_i}\right\}
$$

and

$$
0 < R_2 < \min_{i=1,2}\left\{\frac{\gamma_{i*}(1-\gamma_{i*})}{P_i}\right\}.
$$

Clearly $R_1 > R_2 > 0$. In view of Lemmas 3.1 and 3.2, we get $\Gamma : \mathbf{Q} \cap \bar{\Omega}_{R_1} \setminus \Omega_{R_2} \to \mathbf{Q}$ is strict-set-contractive. If $u^* \in \mathbf{Q} \cap \partial\Omega_{R_2}$ or $u^* \in \mathbf{Q} \cap \partial\Omega_{R_1}$ such that $\Gamma u^* = u^*$, then system (1.8) has at least one positive ϖ-periodic solution. Now we check that (ii) of Lemma 2.1 is true.

If $\Gamma u \ge u$, then $\Gamma u - u \in \mathbf{Q}$. Thus $\Gamma u_1 - u_1 \ge \gamma_{1*}\|\Gamma u_1 - u_1\| \ge 0$ for $t \in [0,\varpi]$. In addition,

$$
\begin{aligned}
\Gamma u_1 &= \int_t^{t+\varpi} B_1(t,s)u_1(s)\big[\alpha_1(s)u_1(s) + \beta_1(s)\big(u_2(s)+\sigma_2(s)\big)^2 + a_1(s)\psi_1\big(s-\eta_1(s)\big)\big]\,ds \\
&\le \frac{1}{1-\gamma_{1*}}\|u\| \int_0^{\varpi} \big[\alpha_i(s) + \beta_i(s)\|x\| + a_i(s)G_i\big(s-\eta_1(s)\big)\big]\,ds \\
&\le \gamma_{1*}\|u\|.
\end{aligned} \tag{3.14}
$$

Then

$$
\|u\| \le \|\Gamma u\| \le \gamma_{1*}\|u\| \le \|u\|,
$$

which is a contradiction. Next we only need to check that for $u \in \mathbf{Q} \cap \partial\Omega_{R_1}$, Not $\Gamma u \le u$. If there exists $u \in \mathbf{Q} \cap \partial\Omega_{R_1}$ such that $\Gamma u > u$, then $\Gamma u - u \in \mathbf{Q} \setminus \{0\}$. For $t \in [0,\varpi]$, one has

$$u_1(t) - \Gamma u_1(t) \geq \gamma_1 \|u - \Gamma u\| \geq 0.$$

Thus

$$\begin{aligned} \Gamma u_1 &= \int_t^{t+\varpi} B_1(t,s)u_1(s)\big[\alpha_1(s)u_1(s) + \beta_1(s)\big(u_2(s) + \sigma_2(s)\big)^2 + a_1(s)\psi_1\big(s - \eta_1(s)\big)\big]\,ds \\ &\geq \frac{\gamma_{1*}}{1-\gamma_{1*}}\gamma_{1*}\|u\|^2 \int_0^{\varpi} \big[\alpha_i(s) + \beta_i(s)\|x\| + a_i(s)G_i\big(s - \eta_1(s)\big)\big]\,ds. \end{aligned} \tag{3.15}$$

Then $\|u\| > \|\Gamma u\| > R_1$, which is a contradiction. By Lemma 3.1, we can conclude that Γ has at least one nonzero fixed point in $\mathbf{Q} \cap \bar{\Omega}_{R_1} \setminus \Omega_{R_2}$. Therefore system (1.8) has at least one positive ϖ-periodic solution. This completes the proof. □

4 Examples

In this section, we give two examples with their numerical simulations to illustrate the feasibility of our results.

Example 4.1 Consider the following system:

$$\begin{cases} \dot{u}_1(t) = u_1(t)[\gamma_1(t) - \alpha_1(t)u_1(t) - \beta_1(t)(u_2(t) - \sigma_2(t))^2 - a_1(t)v_1(t - \eta_1(t))], \\ \dot{v}_1(t) = -\delta_1(t)v_1(t) + \varrho_1(t)u_1(t - \zeta_1(t)), \\ \dot{u}_2(t) = u_2(t)[\gamma_2(t) - \alpha_2(t)u_2(t) + \beta_2(t)(u_1(t) - \sigma_1(t))^2 - a_2(t)v_2(t - \eta_2(t))], \\ \dot{v}_2(t) = -\delta_2(t)v_2(t) + \nu_2(t)u_2(t - \zeta_2(t)), \end{cases} \tag{4.1}$$

where $\gamma_1(t) = 1$, $\gamma_2(t) = 2$, $\delta_1(t) = 0.5$, $\delta_2(t) = 0.8$, $\alpha_1(t) = 0.8|\sin \pi t|$, $\alpha_2(t) = 0.3|\cos \pi t|$, $\beta_1(t) = 0.7|\sin \pi t|$, $\beta_2(t) = 0.9|\sin \pi t|$, $\sigma_1(t) = 0.2|\cos \pi t|$, $\sigma_2(t) = 0.5|\cos \pi t|$, $\eta_1(t) =$

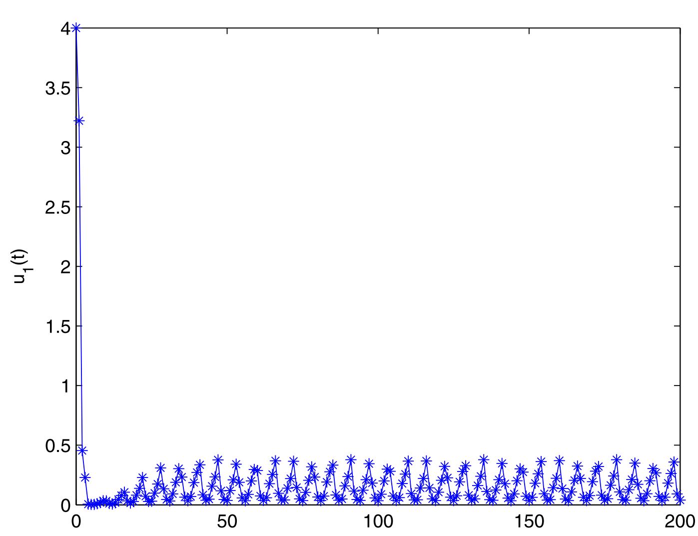

Figure 1 Times series of u_1 of system (4.1)

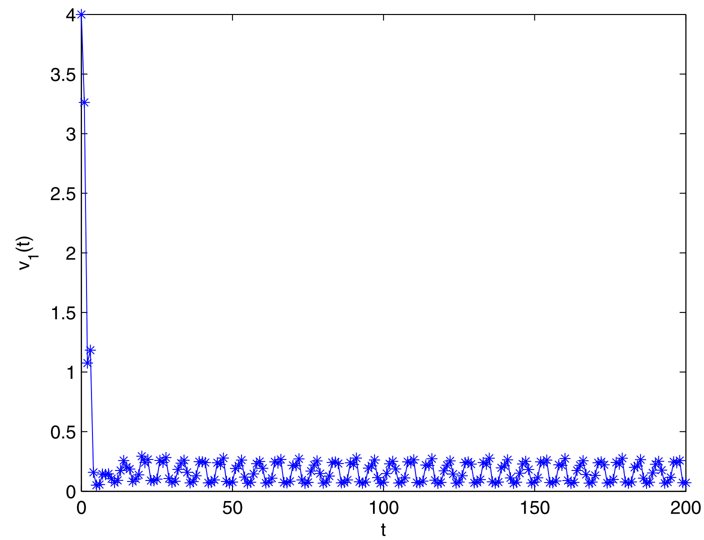

Figure 2 Times series of u_2 of system (4.1)

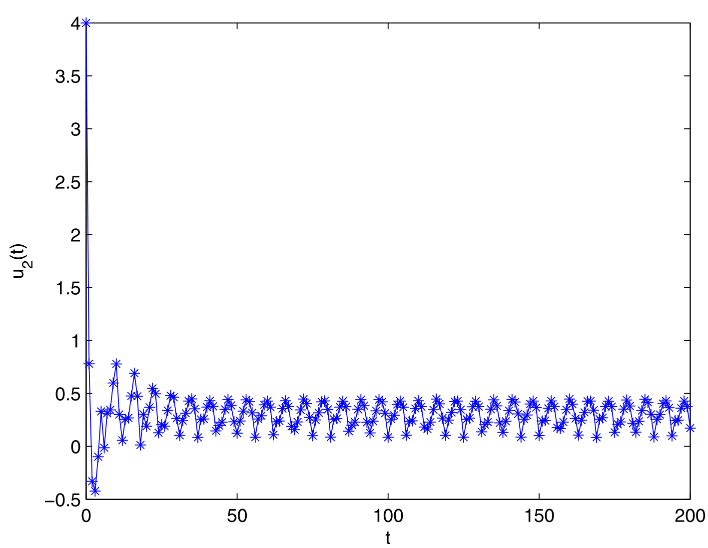

Figure 3 Times series of v_1 of system (4.1)

$0.4|\cos\pi t|$, $\eta_2(t) = 0.4|\cos\pi t|$, $\varrho_1(t) = 0.7|\sin\pi t|$. Thus one can check that all the conditions in Theorem 3.1 are fulfilled. Then we can conclude that system (1.8) has at least one positive 1-periodic solution which is shown in Figs. 1–4.

5 Conclusions

In the present paper, we propose a competition and cooperation model of two enterprises with multiple delays and feedback controls. Applying fixed point theorem of strict-set-contraction, we obtain a sufficient criterion to guarantee the existence of periodic solution of the two-enterprise competition and cooperation model with multiple delays and feedback controls. It is shown that the feedback control terms and time delays have im-

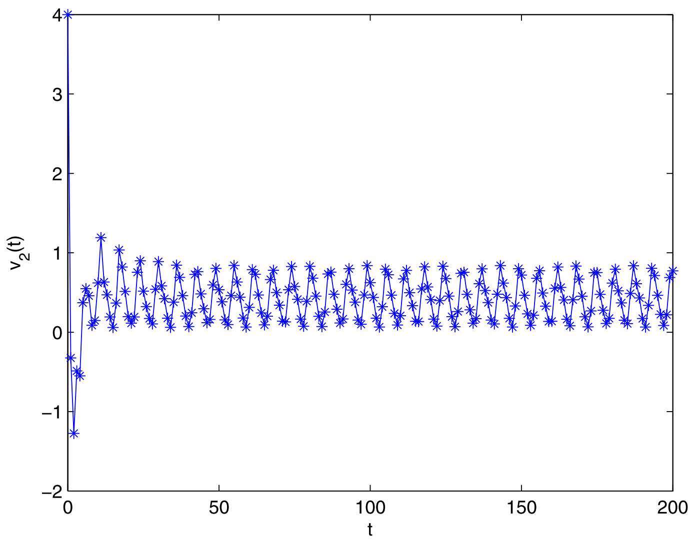

Figure 4 Times series of v_2 of system (4.1)

portant effect on the periodic behavior. The research reveals that under fairish conditions, the competition of two species can remain a periodic vibration. The derived results are new and complement the earlier publications (for example, [1–9]). In recent years, there have been rare reports on the competition and cooperation model of two enterprises with stochastic perturbation, which might be our future investigation topic.

Acknowledgements
The work is supported by the National Natural Science Foundation of China (No. 61673008), Project of High-level Innovative Talents of Guizhou Province ([2016]5651), Major Research Project of the Innovation Group of the Education Department of Guizhou Province ([2017]039), Project of Key Laboratory of Guizhou Province with Financial and Physical Features ([2017]004), and Hunan Provincial Key Laboratory of Mathematical Modeling and Analysis in Engineering (Changsha University of Science & Technology) (2018MMAEZD21). The authors would like to thank the referees and the editor for helpful suggestions incorporated into this paper.

Funding
The work is supported by the National Natural Science Foundation of China (No. 61673008), Project of High-level Innovative Talents of Guizhou Province ([2016]5651), Major Research Project of the Innovation Group of the Education Department of Guizhou Province ([2017]039), Project of Key Laboratory of Guizhou Province with Financial and Physical Features ([2017]004), and Hunan Provincial Key Laboratory of Mathematical Modeling and Analysis in Engineering (Changsha University of Science & Technology) (2018MMAEZD21).

Availability of data and materials
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors have read and approved the final manuscript.

Authors' information
Changjin Xu's research interests are the bifurcation theory of delayed differential equations. Peiluan Li's research topics are nonlinear systems, functional differential equations, boundary value problems. Qimei Xiao's research topics are rough set and formal concept analysis. Shuai Yuan's research topics are theory and application of functional differential equations.

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Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 13 January 2019 Accepted: 30 January 2019 Published online: 15 February 2019

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