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Existence and convergence results of meromorphic solutions to the equilibrium system with angular velocity

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Abstract

We study the equilibrium system with angular velocity for the prey. This system is a generalization of the two-species equilibrium model with Neumann type boundary condition. Firstly, we consider the asymptotical stability of equilibrium points to the system of ordinary differential equations type. Then, the existence of meromorphic solutions and the stability of equilibrium points to the system of weakly coupled meromorphic type are discussed. Finally, the existence of nonnegative meromorphic solutions to the system of strongly coupled meromorphic type is investigated, and the asymptotic stability of unique positive equilibrium point of the system is proved by constructing meromorphic functions.

Keywords: Asymptotic stability; Equilibrium system; Meromorphic function

1 Introduction

The equilibrium system with angular velocity is noted for its pattern-forming behavior and has been widely used as a model for the study of obstacle problems involving reservoir simulation. These include the effects of noise on bifurcations, pattern selection, spatiotemporal chaos, and the dynamics of defects; see, for example, [1–8] and the references therein for details. It also has been used to model patterns in simple fluids and in a variety of complex fluids and biological materials, such as neural tissue [3, 7]. These problems are widely studied and very well used in many areas of mathematics and physics, see [3, 5, 6, 9]. Since it was initiated by Paul Dirac in order to get a form of quantum theory compatible with special relativity, the Dirac equation has been playing a critical role in some fields of mathematics and physics, such as quantum mechanics, Clifford analysis, and partial differential equations.

As one of the universal equilibrium systems used in the description of pattern formation in spatially extended dissipative systems, the general equilibrium differential equation can also be found in the study of convective hydrodynamics, plasma confinement in toroidal devices, viscous film flow, and bifurcating solutions of the modified equilibrium differential equation [6, 10, 11]. In recent years, some references such as Sheng et al. [12], Zhai et al. [13], Zhang [14], Wu et al. [15], Sun et al. [16], Li et al. [17], Bai and Sun [18], Wang et al. [19], and so on, introduced many beautiful patterns to satisfy practical requirements of modern computing systems with

multi-processors. There is the potential of considering the linearization characteristics to be further developed for the system of equilibrium boundary value problems. Ardila [20] studied the existence and stability of standing waves solutions of a three-coupled nonlinear Schrödinger system related to the Raman amplification in a plasma. Hu and Yin [21] considered dynamics of the compressible Navier–Stokes equation in one spatial dimension for a viscous fluid with vanishing thermal conductivity. For the case of ideal polytropic gases, it is shown that the rarefaction waves in this medium are stable with regards to sufficiently weak perturbations of the velocity and pressure fields.

Motivated and inspired by the references [18–21], in this paper we further consider the following universal equilibrium equations with nondifferentiable boundary conditions:

$$\begin{aligned}
 -\Delta u + u &= a(|x|)|u|^{p-2}u, \quad x \in B_1 \\
 u &> 0, \quad x \in B_1, \\
 \frac{\partial u}{\partial \nu} &= 0, \quad x \in \partial B_1,
 \end{aligned} \tag{1}$$

where B_1 is the unit ball centered at the origin in \mathbb{R}^n , $n \geq 3$ and $p > 2$ and $a \in L^1(0, 1)$ is increasing, not constant and $a(r) > 0$ a.e. in $[0, 1]$.

Meanwhile, we are also interested in the equilibrium elliptic system given by

$$\begin{aligned}
 -\Delta u + u &= f_u(|x|, u, v), \quad x \in B_1 \\
 -\Delta v + v &= f_v(|x|, u, v), \quad x \in B_1 \\
 \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} &= 0, \quad x \in \partial B_1,
 \end{aligned} \tag{2}$$

under suitable assumptions on f . Our assumptions do allow some supercritical nonlinearities. Problems in this abstract form are often referred to as equilibrium problems. More details on this problem class can be found in [9, 22].

Let $I = \{1, 2, \dots, n\}$ be an index set, H_i be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle_i$ and norm $\| \cdot \|_i$, respectively. Let $A : H_1 \rightarrow H_1$, $B : H_2 \rightarrow H_2$, $F_1 : H_1 \times H_2 \rightarrow H_1$, and $\eta_1 : H_1 \times H_1 \rightarrow H_1$ be mappings. Let $a_i : H_i \times H_i \rightarrow \mathbb{R}$ be a coercive continuous map such that

- (C1) $a_i(\sigma_i, \sigma_i) \geq c_i \|\sigma_i\|_i^2$;
- (C2) $|a_i(\varrho_i, \sigma_i)| \leq d_i \|\varrho_i\|_i \cdot \|\sigma_i\|_i$

for any $\varrho_i, \sigma_i \in H_i$.

Let $b_i : H_i \times H_i \rightarrow \mathbb{R}$ be a map with nondifferentiable terms such that

- (C3) b_i is a linear function for the first variable;
- (C4) b_i is a convex function;
- (C5) There exists a positive constant γ_i satisfying

$$\gamma_i \|\varrho_i\|_i \cdot \|\sigma_i\|_i \geq a_i(\varrho_i, \sigma_i)$$

for any $\varrho_i, \sigma_i \in H_i$.

(C6)

$$b_i(\varrho_i, \sigma_i - w_i) \geq b_i(\varrho_i, \sigma_i) - b_i(\varrho_i, w_i)$$

for any $\varrho_i, \sigma_i, w_i \in H_i$.

Based on the above notations, we then define the proposed system of generalized non-linear variational inequality problems as follows:

Find $(x, y) \in H_1 \times H_2$ such that

$$\begin{cases} \langle F_1(Ax, By) - f_1, \eta_1(\sigma_1, x) \rangle_1 + a_1(x, \sigma_1 - x) + b_1(x, \sigma_1) - b_1(x, x) \geq 0 \\ \quad \forall \sigma_1 \in H_1, \\ \langle F_2(Ax, By) - f_2, \eta_2(\sigma_2, y) \rangle_2 + a_2(y, \sigma_2 - y) + b_2(y, \sigma_2) - b_2(y, y) \geq 0 \\ \quad \forall \sigma_2 \in H_2, \end{cases} \tag{3}$$

where $f_i \in H_i$ is given for each $i \in I$.

Remark 1 There are some special cases for the model problem (3) (see [23]):

(1) If $A = B = I, f_i = 0$, and $a_i(\varrho_i, \sigma_i) = 0$, then (3) is equivalent to

$$\langle F_1(x, y), \eta_1(\sigma_1, x) \rangle_1 + b_1(x, \sigma_1) - b_1(x, x) \geq 0,$$

where $\sigma_1 \in H_1$.

$$\langle F_2(x, y), \eta_2(\sigma_2, y) \rangle_2 + b_2(y, \sigma_2) - b_2(y, y) \geq 0,$$

where $\sigma_2 \in H_2$.

(2) If $H_1 = H_2 = H, f_i = f_2 = f, \eta_1 = \eta_2 = \eta, a_1 = a_2 = a, b_1 = b_2 = b$, then (3) is reduced to

$$\langle F(Ax, Bx) - f, \eta(v, x) \rangle + a(x, v - x) + b(x, v) - b(x, x) \geq 0,$$

where $v \in H$.

2 Preliminaries

Let us recall some basic definitions and lemmas that we need in the forthcoming analysis.

Definition 1 We say that the functional Φ satisfies the Palais–Smale condition (see [15, 24–26]) if any sequence $\{u_n\}_{n \in \mathbb{N}} \subset X$ has a convergent subsequence provided $\{\Phi(u_n)\}_{n \in \mathbb{N}}$ is bounded and $\Phi'(u_n) \rightarrow 0$ as $n \rightarrow +\infty$.

Definition 2 For any $\lambda > 0$, we define the subfunctions associated with the universal equilibrium operator (1) by (see [11])

$$\mathbb{I}_+^{\tau, \lambda} f(x) = \frac{1}{\Gamma(\tau)} \int_{-\infty}^x f(\xi)(x - \xi)^{\tau-1} e^{-\lambda(x-\xi)} d\xi$$

and the subfunctions associated with the universal equilibrium operator (2) by

$$\mathbb{I}_-^{\tau, \lambda} f(x) = \frac{1}{\Gamma(\tau)} \int_x^{+\infty} f(\xi)(\xi - x)^{\tau-1} e^{-\lambda(\xi-x)} d\xi.$$

Definition 3 The positive and negative tempered equilibrium derivatives of order $0 < \tau < 1$ are defined as follows (see [27]):

$$\begin{aligned} \mathbb{D}_+^{\tau,\lambda} f(x) &= \lambda^\tau f(x) + \frac{\tau}{\Gamma(1-\tau)} \int_{-\infty}^x \frac{f(x) - f(\xi)}{(x-\xi)^{\tau+1}} e^{-\lambda(x-\xi)} d\xi, \\ \mathbb{D}_-^{\tau,\lambda} f(x) &= \lambda^\tau f(x) + \frac{\tau}{\Gamma(1-\tau)} \int_x^{+\infty} \frac{f(x) - f(\xi)}{(\xi-x)^{\tau+1}} e^{-\lambda(\xi-x)} d\xi, \end{aligned}$$

for any $\lambda > 0$, respectively, where $f : \mathbb{R} \rightarrow \mathbb{R}$.

Define the Banach space

$$W_\lambda^{\tau,2}(\mathbb{R}) = \left\{ f \in L^2(\mathbb{R}) : \int_{\mathbb{R}} (\lambda^2 + \omega^2)^\tau |\widehat{f}(\omega)|^2 d\omega < \infty \right\}$$

with the norm

$$\|f\|_{\tau,\lambda} = \left(\int_{\mathbb{R}} (\lambda^2 + \omega^2)^\tau |\widehat{f}(\omega)|^2 d\omega \right)^{1/2}.$$

For any $f \in W_\lambda^{\tau,2}(\mathbb{R})$, let $\mathbb{D}_\pm^{\tau,\lambda} f(x)$ denote the subfunctions associated with the universal equilibrium operator (1) with Fourier transform $(\lambda \pm i\omega)^\tau \widehat{f}(\omega)$ (see [23]), where the Fourier transform of $u(x)$ is defined as follows:

$$\mathcal{F}(u)(\xi) = \int_{-\infty}^{\infty} e^{-ix\xi} u(x) dx.$$

Now we state the following known results.

Lemma 1 (see [23])

(i)

$$\mathbb{D}_\pm^{\tau,\lambda} \mathbb{I}_\pm^{\tau,\lambda} f(x) = f(x)$$

for any $\tau, \lambda > 0$ and $f \in L^2(\mathbb{R})$ and

$$\mathbb{I}_\pm^{\tau,\lambda} \mathbb{D}_\pm^{\tau,\lambda} f(x) = f(x)$$

for any $f \in W_\lambda^{\tau,2}(\mathbb{R})$.

(ii)

$$\langle f, \mathbb{D}_+^{\tau,\lambda} g \rangle_{L^2(\mathbb{R})} = \langle \mathbb{D}_-^{\tau,\lambda} f, g \rangle_{L^2(\mathbb{R})}$$

for any $\tau, \lambda > 0$ and $f, g \in W_\lambda^{\tau,2}(\mathbb{R})$.

Lemma 2 (see [2])

(i) For any $\tau, \lambda > 0$ and $p \geq 1$, $\mathbb{I}_\pm^{\tau,\lambda} : L^p(\mathbb{R}) \rightarrow L^p(\mathbb{R})$ are bounded equilibrium operators with

$$\|\mathbb{I}_\pm^{\tau,\lambda} f\|_{L^p(\mathbb{R})} \leq \lambda^{-\tau} \|f\|_{L^p(\mathbb{R})}.$$

(ii)

$$\mathbb{I}_{\pm}^{\tau,\lambda} \mathbb{I}_{\pm}^{\beta,\lambda} f(x) = \mathbb{I}_{\pm}^{\tau+\beta,\lambda} f(x)$$

for any $\tau, \beta, \lambda > 0$ and $f \in L^p(\mathbb{R})$.

(iii)

$$\langle f, \mathbb{I}_{+}^{\tau,\lambda} g \rangle_{L^2(\mathbb{R})} = \langle \mathbb{I}_{-}^{\tau,\lambda} f, g \rangle_{L^2(\mathbb{R})}$$

for any $\tau, \lambda > 0$ and $f, g \in L^2(\mathbb{R})$.

Next, for $0 < \tau < 1$, we define a fractional Sobolev space $H^\tau(\mathbb{R})$ as follows:

$$H^\tau(\mathbb{R}) = \overline{C_0^\infty(\mathbb{R})}^{\|\cdot\|_\tau}$$

endowed with

$$\|u\|_\tau = \left(\int_{\mathbb{R}} |u(t)|^2 dt + \int_{\mathbb{R}} |\omega|^{2\tau} |\widehat{u}(\omega)|^2 d\omega \right)^{1/2}. \tag{4}$$

It follows that

$$2^{\frac{\tau-1}{2}} \|u\|_\tau \leq \|u\|_{\tau,1} \leq \|u\|_\tau, \tag{5}$$

$$\|u\|_{\tau,1} \leq \|u\|_{\tau,\lambda} \leq \lambda^\tau \|u\|_{\tau,1}, \tag{6}$$

$$\|u\|_{\tau,\lambda} < \|u\|_{\tau,1} < \lambda^{-\tau} \|u\|_{\tau,\lambda} \tag{7}$$

for $0 < \tau < 1$, where $\|u\|_{\tau,1}$ is the norm on $W_1^{\tau,2}(\mathbb{R})$, and so $W_1^{\tau,2}(\mathbb{R}) = H^\tau(\mathbb{R})$ with equivalent norms.

Lemma 3 (see [5, 18]) *Let $\tau > 1/2$. Then any $u \in W_\lambda^{\tau,2}(\mathbb{R})$ is uniformly continuous, bounded and there exists a constant $C = C_\tau$ such that*

$$\sup_{t \in \mathbb{R}} |u(t)| \leq C \|u\|_{\tau,\lambda}. \tag{8}$$

Remark 2 From Lemma 3 and (5)–(7), we have the following implication: if $u \in W_\lambda^{\tau,2}$ with $\frac{1}{2} < \tau < 1$, then $u \in L^q(\mathbb{R})$ for all $q \in [2, \infty)$ as

$$\int_{\mathbb{R}} |u(t)|^q dt \leq \|u\|_\infty^{q-2} \|u\|_{L^2(\mathbb{R})}^2 \leq 2^{1-\tau} C^{q-2} \|u\|_{\tau,\lambda}^q.$$

Remark 3 The imbedding of $W_\lambda^{\tau,2}$ in $L^q(-T, T)$ is compact for $q \in (2, \infty)$ and any $T > 0$ (see [3]).

3 Existence and convergence

In this section, we prove the existence of solution of (3) and discuss the convergence of the sequence.

Theorem 1 *Assume that u is a critical point of*

$$I(w) := \psi(w) - \frac{1}{p} \int_{B_1} a(|x|)|w|^p dx. \tag{9}$$

If there exists $v \in \text{Dom}(\psi)$ satisfying the equilibrium equation

$$\begin{aligned} -\Delta v + v &= a(|x|)|u|^{p-2}u, \quad x \in B_1 \\ \frac{\partial v}{\partial \nu} &= 0, \quad x \in \partial B_1, \end{aligned} \tag{10}$$

then u is a meromorphic solution of (3).

Proof Assume that $\{\varrho_n\}_{n \in \mathbb{N}} \subset W_\lambda^{\tau,2}(\mathbb{R})$ is a sequence such that $\{\Phi(\varrho_n)\}_{n \in \mathbb{N}}$ is bounded and $\Phi'(\varrho_n) \rightarrow 0$ as $n \rightarrow \infty$. Then there exists a positive constant D such that

$$|\Phi(\varrho_n)| \leq D \quad \text{and} \quad \|\Phi'(\varrho_n)\|_{(W_\lambda^{\tau,2}(\mathbb{R}))^*} \leq D \tag{11}$$

for any $n \in \mathbb{N}$, where $(W_\lambda^{\tau,2}(\mathbb{R}))^*$ is the dual space of $W_\lambda^{\tau,2}(\mathbb{R})$.

Firstly, we show that $\{\varrho_n\}_{n \in \mathbb{N}}$ is bounded. Without loss of generality, we assume that

$$\inf_n \|\varrho_n\|_{\tau,\lambda} = \eta > 0,$$

denote by $\varrho = \varrho(\eta)$ the number corresponding to $\delta = \eta^2$ in (C1) such that

$$M(\|\varrho_n\|_{\tau,\lambda}^2) \geq \varrho \tag{12}$$

for all n .

In view of (C2) and (12), one gets

$$\begin{aligned} D + D\|\varrho_n\|_{\tau,\lambda} &\geq \Phi(\varrho_n) - \frac{1}{\mu} \Phi'(\varrho_n)\varrho_n \\ &= \frac{1}{2} \widehat{M}(\|\varrho_n\|_{\tau,\lambda}^2) - \frac{1}{\mu} M(\|\varrho_n\|_{\tau,\lambda}^2) \|\varrho_n\|_{\tau,\lambda}^2 \\ &\quad - \frac{1}{\mu} \int_{\mathbb{R}} (\mu F(t, \varrho_n(t)) - f(t, \varrho_n(t))\varrho_n(t)) dt \\ &\geq \left(\frac{1}{2\gamma} - \frac{1}{\mu}\right) M(\|\varrho_n\|_{\tau,\lambda}^2) \|\varrho_n\|_{\tau,\lambda}^2 \\ &\geq \varrho \left(\frac{1}{2\gamma} - \frac{1}{\mu}\right) \|\varrho_n\|_{\tau,\lambda}^2. \end{aligned}$$

Since $\mu > 2\gamma$, the boundedness of $\{\varrho_n\}_{n \in \mathbb{N}}$ follows directly. So there exists a subsequence $\{\varrho_n\}_{n \in \mathbb{N}}$ and $u \in W_\lambda^{\tau,2}$ such that

$$\varrho_n \rightharpoonup u \quad \text{weakly in } W_\lambda^{\tau,2}(\mathbb{R}), \tag{13}$$

which yields

$$\begin{aligned} \Phi'(\varrho_n)(\varrho_n - \varrho) &= M(\|\varrho_n\|_{\tau,\lambda}^2) \int_{\mathbb{R}} (\mathbb{D}_+^{\tau,\lambda} \varrho_n \mathbb{D}_+^{\tau,\lambda} (\varrho_n - \varrho)) \, dt \\ &\quad - \int_{\mathbb{R}} f(t, \varrho_n)(\varrho_n - \varrho) \, dt \rightarrow 0 \end{aligned} \tag{14}$$

as $n \rightarrow \infty$.

Now we show that

$$\lim_{n \rightarrow \infty} \int_{\mathbb{R}} f(t, \varrho_n)(\varrho_n - \varrho) \, dt = 0.$$

To this end, by (13), there exists some positive constant d such that

$$\begin{aligned} \|\varrho_n\|_{\tau,\lambda} < d \quad \text{and} \quad \|u\|_{\tau,\lambda} < d, \quad \text{for } n \in \mathbb{N}, \\ \varrho_n \rightarrow u \quad \text{strongly in } L^q(\mathbb{R}) \quad \text{and a.e. in } \mathbb{R}. \end{aligned}$$

Moreover, (C4) implies that there exists a positive constant T such that

$$f(t, \varrho_n) \leq \varepsilon |\varrho_n|^{q-1} \tag{15}$$

for any $\varepsilon > 0$ and $|t| > T$.

Then, by using Remark 2 and Young's inequality, we obtain

$$\begin{aligned} &\left| \int_{\mathbb{R}} f(t, \varrho_n)(\varrho_n - \varrho) \, dt \right| \\ &\leq \int_{\mathbb{R}} |f(t, \varrho_n)| |\varrho_n - u| \, dt \\ &\leq \int_{-T}^T |f(t, \varrho_n)| |\varrho_n - u| \, dt + \int_{|t|>T} |f(t, \varrho_n)| |\varrho_n - u| \, dt \\ &\leq \varepsilon \|\varrho_n\|_{\infty} + \varepsilon \int_{|t|>T} |\varrho_n|^{q-1} |\varrho_n - u| \, dt \\ &\leq \varepsilon C \|\varrho_n\|_{\tau,\lambda} + \varepsilon \int_{|t|>T} \left(\frac{q-1}{q} |\varrho_n|^q + \frac{1}{\mu} |\varrho_n - u|^q \right) \, dt \\ &\leq \varepsilon C \|\varrho_n\|_{\tau,\lambda} + \frac{q-1}{q} \varepsilon 2^{1-\tau} C^{q-2} \|\varrho_n\|_{\tau,\lambda}^q + \varepsilon \frac{1}{\mu} 2^{1-\tau} C^{q-2} \|\varrho_n - u\|_{\tau,\lambda}^{q-2} \\ &\leq \varepsilon C d + \frac{q-1}{q} \varepsilon 2^{1-\tau} C^{q-2} d^q + \varepsilon \frac{1}{\mu} 2^{1-\tau} C^{q-2} \|\varrho_n - u\|_{\tau,\lambda}^{q-2} \end{aligned}$$

for large enough n from (8).

Then

$$\lim_{n \rightarrow \infty} \int_{\mathbb{R}} f(t, \varrho_n)(\varrho_n - \varrho) \, dt = 0.$$

Therefore, we have

$$M(\|\varrho_n\|_{\tau,\lambda}^2) \int_{\mathbb{R}} (\mathbb{D}_+^{\tau,\lambda} \varrho_n \mathbb{D}_+^{\tau,\lambda} (\varrho_n - \varrho)) dt \rightarrow 0$$

from (14) as $n \rightarrow \infty$.

Thus, by the boundedness of $M(\|\varrho_n\|_{\tau,\lambda}^2)$, one can get

$$\int_{\mathbb{R}} (\mathbb{D}_+^{\tau,\lambda} \varrho_n \mathbb{D}_+^{\tau,\lambda} (\varrho_n - \varrho)) dt \rightarrow 0 \tag{16}$$

from (12) as $n \rightarrow \infty$.

In a similar manner, we can get

$$\int_{\mathbb{R}} (\mathbb{D}_+^{\tau,\lambda} u \mathbb{D}_+^{\tau,\lambda} (\varrho_n - \varrho)) dt \rightarrow 0 \tag{17}$$

as $n \rightarrow \infty$.

Combining (16) and (17), we obtain that

$$\int_{\mathbb{R}} (\mathbb{D}_+^{\tau,\lambda} (\varrho_n - \varrho) \mathbb{D}_+^{\tau,\lambda} (\varrho_n - \varrho)) dt \rightarrow 0$$

as $n \rightarrow \infty$.

Hence,

$$\|\varrho_n - u\|_{\tau,\lambda} \rightarrow 0$$

as $n \rightarrow \infty$, and then Φ satisfies the Palais–Smale condition. □

Theorem 2 *Let the following two conditions hold:*

- (1) $\forall \sigma_i, \varrho_i \in H_i, \eta_i(\sigma_i, \varrho_i) = -\eta_i(\varrho_i, \sigma_i)$;
- (2) $a_i : H_i \times H_i \rightarrow \mathbb{R}$ satisfies (C1) and (C2), $b_i : H_i \times H_i \rightarrow \mathbb{R}$ with (C3)–(C6).

Moreover, we have the following conditions:

$$0 < \frac{1}{\sigma_1 + \rho_1 c_1} [\delta_1 \sqrt{1 - 2\rho_1 \tau_1 + \rho_1^2 \beta_1^2} + \rho_1 \gamma_1] + \frac{\rho_2 \delta_2 \xi_2}{\sigma_2 + \rho_2 c_2} < 1, \tag{18}$$

$$0 < \frac{1}{\sigma_2 + \rho_2 c_2} [\delta_2 \sqrt{1 - 2\rho_2 \tau_2 + \rho_2^2 \beta_2^2} + \rho_2 \gamma_2] + \frac{\rho_1 \delta_1 \xi_1}{\sigma_1 + \rho_1 c_1} < 1. \tag{19}$$

Then the sequence $\{(\varrho_n, \sigma_n)\}_{n \geq 0}$ converges to (ϱ^*, σ^*) , where (ϱ^*, σ^*) is the meromorphic solution of (3).

Proof It follows from (18) and (19) that

$$\begin{aligned} & \langle \varrho_n - \varrho_{n-1}, \eta_1(\sigma_1, \varrho_n) \rangle_1 + \rho_1 \langle F_1(A\varrho_{n-1}, B\sigma_{n-1}) - f_1, \eta_1(\sigma_1, \varrho_n) \rangle_1 \\ & + \rho_1 [a_1(\varrho_n, \sigma_1 - \varrho_n)] + \rho_1 [b_1(\varrho_{n-1}, \sigma_1) - b_1(\varrho_{n-1}, \varrho_n)] \geq 0, \end{aligned} \tag{20}$$

$$\begin{aligned} & \langle \sigma_n - \sigma_{n-1}, \eta_2(\sigma_2, \sigma_n) \rangle_2 + \rho_2 \langle F_2(A\varrho_{n-1}, B\sigma_{n-1}) - f_2, \eta_2(\sigma_2, \sigma_n) \rangle_2 \\ & + \rho_2 [a_2(\sigma_n, \sigma_2 - \sigma_n)] + \rho_2 [b_2(\sigma_{n-1}, \sigma_2) - b_1(\sigma_{n-1}, \sigma_n)] \geq 0 \end{aligned} \tag{21}$$

for any $(\sigma_1, \sigma_2) \in H_1 \times H_2$.

If we take $\sigma_1 = \varrho_{n+1}$ in (20) and $\sigma_1 = \varrho_n$ in (21), respectively, then

$$\begin{aligned} & \langle \varrho_n - \varrho_{n-1}, \eta_1(\varrho_{n+1}, \varrho_n) \rangle_1 + \rho_1 \langle F_1(A\varrho_{n-1}, B\sigma_{n-1}) - f_1, \eta_1(\varrho_{n+1}, \varrho_n) \rangle_1 \\ & + \rho_1 [a_1(\varrho_n, \varrho_{n+1} - \varrho_n)] + \rho_1 [b_1(\varrho_{n-1}, \varrho_{n+1}) - b_1(\varrho_{n-1}, \varrho_n)] \geq 0, \end{aligned} \tag{22}$$

$$\begin{aligned} & \langle \varrho_{n+1} - \varrho_n, \eta_1(\varrho_n, \varrho_{n+1}) \rangle_1 + \rho_1 \langle F_1(A\varrho_n, B\sigma_n) - f_1, \eta_1(\varrho_n, \varrho_{n+1}) \rangle_1 \\ & + \rho_1 [a_1(\varrho_{n+1}, \varrho_n - \varrho_{n+1})] + \rho_1 [b_1(\varrho_n, \varrho_n) - b_1(\varrho_n, \varrho_{n+1})] \geq 0. \end{aligned} \tag{23}$$

Adding (22) and (23), we obtain that

$$\begin{aligned} & \langle \varrho_n - \varrho_{n+1}, \eta_1(\varrho_n, \varrho_{n+1}) \rangle_1 \\ & \leq \langle \varrho_{n-1} - \varrho_n, \eta_1(\varrho_n, \varrho_{n+1}) \rangle_1 - \rho_1 \langle F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_n), \eta_1(\varrho_n, \varrho_{n+1}) \rangle_1 \\ & \quad - \rho_1 [a_1(\varrho_n - \varrho_{n+1}, \varrho_n - \varrho_{n+1})] + \rho_1 [b_1(\varrho_{n-1} - \varrho_n, \varrho_{n+1}) + b_1(\varrho_n - \varrho_{n-1}, \varrho_n)] \\ & \leq \langle \varrho_{n-1} - \varrho_n - \rho_1 [F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_n)], \eta_1(\varrho_n, \varrho_{n+1}) \rangle_1 \\ & \quad - \rho_1 [a_1(\varrho_n - \varrho_{n+1}, \varrho_n - \varrho_{n+1})] + \rho_1 [b_1(\varrho_n - \varrho_{n-1}, \varrho_n - \varrho_{n+1})]. \end{aligned}$$

Since η_1 is σ_1 -strongly monotone and δ_1 -Lipschitz continuous and a_1 satisfies (C1), we have

$$\begin{aligned} & \sigma_1 \|\varrho_n - \varrho_{n+1}\|_1^2 \\ & \leq \|\varrho_{n-1} - \varrho_n - \rho_1 [F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_n)]\|_1 \|\eta_1(\varrho_n, \varrho_{n+1})\|_1 \\ & \quad - \rho_1 c_1 \|\varrho_n - \varrho_{n+1}\|_1^2 + \rho_1 \gamma_1 \|\varrho_n - \varrho_{n-1}\|_1 \|\varrho_n - \varrho_{n+1}\|_1 \\ & \leq \delta_1 \|\varrho_{n-1} - \varrho_n - \rho_1 [F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_n)]\|_1 \|\varrho_n - \varrho_{n+1}\|_1 \\ & \quad - \rho_1 c_1 \|\varrho_n - \varrho_{n+1}\|_1^2 + \rho_1 \gamma_1 \|\varrho_n - \varrho_{n-1}\|_1 \|\varrho_n - \varrho_{n+1}\|_1, \end{aligned}$$

which implies that

$$\begin{aligned} & \|\varrho_n - \varrho_{n+1}\|_1 \\ & \leq \frac{1}{\sigma_1 + \rho_1 c_1} (\delta_1 \|\varrho_{n-1} - \varrho_n - \rho_1 [F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_{n-1})]\|_1 \\ & \quad + \rho_1 \delta_1 \|F_1(A\varrho_n, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_n)\|_1 + \rho_1 \gamma_1 \|\varrho_n - \varrho_{n-1}\|_1). \end{aligned} \tag{24}$$

So

$$\begin{aligned} & \|\varrho_{n-1} - \varrho_n - \rho_1 [F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_{n-1})]\|_1^2 \\ & = \|\varrho_{n-1} - \varrho_n\|_1^2 - 2\rho_1 \langle F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_{n-1}), \varrho_{n-1} - \varrho_n \rangle_1 \\ & \quad + \rho_1^2 \|F_1(A\varrho_{n-1}, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_{n-1})\|_1^2 \\ & \leq (1 - 2\rho_1 \tau_1 + \rho_1^2 \beta_1^2) \|\varrho_{n-1} - \varrho_n\|_1^2 \end{aligned} \tag{25}$$

and

$$\|F_1(A\varrho_n, B\sigma_{n-1}) - F_1(A\varrho_n, B\sigma_n)\|_1 \leq \xi_1 \|\sigma_{n-1} - \sigma_n\|_2. \tag{26}$$

It follows from (24), (25), and (26) that

$$\begin{aligned} \|\varrho_n - \varrho_{n+1}\|_1 &\leq \frac{1}{\sigma_1 + \rho_1 c_1} \left[\delta_1 \sqrt{1 - 2\rho_1 \tau_1 + \rho_1^2 \beta_1^2} + \rho_1 \gamma_1 \right] \|\varrho_{n-1} - \varrho_n\|_1 \\ &\quad + \frac{\rho_1 \delta_1 \xi_1}{\sigma_1 + \rho_1 c_1} \|\sigma_{n-1} - \sigma_n\|_2, \end{aligned} \tag{27}$$

taking $\sigma_2 = \sigma_{n+1}$ in (25) and $\sigma_2 = \sigma_n$ in (26), respectively.

Similarly, we have

$$\begin{aligned} \|\sigma_n - \sigma_{n+1}\|_2 &\leq \frac{1}{\sigma_2 + \rho_2 c_2} \left[\delta_2 \sqrt{1 - 2\rho_2 \tau_2 + \rho_2^2 \beta_2^2} + \rho_2 \gamma_2 \right] \|\sigma_{n-1} - \sigma_n\|_2 \\ &\quad + \frac{\rho_2 \delta_2 \xi_2}{\sigma_2 + \rho_2 c_2} \|\varrho_{n-1} - \varrho_n\|_1. \end{aligned} \tag{28}$$

From (27) and (28), we obtain that

$$\begin{aligned} &\|\varrho_n - \varrho_{n+1}\|_1 + \|\sigma_n - \sigma_{n+1}\|_2 \\ &\leq \left(\frac{1}{\sigma_1 + \rho_1 c_1} \left[\delta_1 \sqrt{1 - 2\rho_1 \tau_1 + \rho_1^2 \beta_1^2} + \rho_1 \gamma_1 \right] + \frac{\rho_2 \delta_2 \xi_2}{\sigma_2 + \rho_2 c_2} \right) \|\varrho_{n-1} - \varrho_n\|_1 \\ &\quad + \left(\frac{1}{\sigma_2 + \rho_2 c_2} \left[\delta_2 \sqrt{1 - 2\rho_2 \tau_2 + \rho_2^2 \beta_2^2} + \rho_2 \gamma_2 \right] + \frac{\rho_1 \delta_1 \xi_1}{\sigma_1 + \rho_1 c_1} \right) \|\sigma_{n-1} - \sigma_n\|_2 \\ &\leq \max\{\theta_1, \theta_2\} (\|\varrho_{n-1} - \varrho_n\|_1 + \|\sigma_{n-1} - \sigma_n\|_2), \end{aligned} \tag{29}$$

where

$$\begin{aligned} \theta_1 &:= \frac{1}{\sigma_1 + \rho_1 c_1} \left[\delta_1 \sqrt{1 - 2\rho_1 \tau_1 + \rho_1^2 \beta_1^2} + \rho_1 \gamma_1 \right] + \frac{\rho_2 \delta_2 \xi_2}{\sigma_2 + \rho_2 c_2}, \\ \theta_2 &:= \frac{1}{\sigma_2 + \rho_2 c_2} \left[\delta_2 \sqrt{1 - 2\rho_2 \tau_2 + \rho_2^2 \beta_2^2} + \rho_2 \gamma_2 \right] + \frac{\rho_1 \delta_1 \xi_1}{\sigma_1 + \rho_1 c_1}. \end{aligned}$$

Now, if we define the norm $\|\cdot\|_*$ on $H_1 \times H_2$ by

$$\|(u, v)\|_* = \|u\|_1 + \|v\|_2$$

for any $(u, v) \in H_1 \times H_2$, then we have

$$\|(\varrho_n, \sigma_n) - (\varrho_{n+1}, \sigma_{n+1})\|_* \leq \max\{\theta_1, \theta_2\} \|(\varrho_{n-1}, \sigma_{n-1}) - (\varrho_n, \sigma_n)\|_*. \tag{30}$$

By using (18) and (19), it follows that $\theta_1, \theta_2 \in (0, 1)$. Hence, (30) implies that $\{(\varrho_n, \sigma_n)\}$ is a Halton sequence in $H_1 \times H_2$.

Let $(\varrho_n, \sigma_n) \rightarrow (\varrho^*, \sigma^*)$ in $H_1 \times H_2$ as $n \rightarrow \infty$. Therefore,

$$\begin{aligned} &\langle F_1(A\varrho^*, B\sigma^*) - f_1, \eta_1(\sigma_1, \varrho^*) \rangle_1 + a_1(\varrho^*, \sigma_1 - \varrho^*) + b_1(\varrho^*, \sigma_1) - b_1(\varrho^*, \varrho^*) \geq 0 \\ &\quad \forall \sigma_1 \in H_1, \\ &\langle F_2(A\varrho^*, B\sigma^*) - f_2, \eta_2(\sigma_2, \sigma^*) \rangle_2 + a_2(\sigma^*, \sigma_2 - \sigma^*) + b_2(\sigma^*, \sigma_2) - b_2(\sigma^*, \sigma^*) \geq 0 \\ &\quad \forall \sigma_2 \in H_2. \end{aligned}$$

Thus, (ϱ^*, σ^*) is a meromorphic solution of the model problem (3), which implies the required conclusion. \square

4 Conclusions

In this paper, we studied the equilibrium system with angular velocity for the prey. This system was a generalization of the two-species equilibrium model with Neumann type boundary condition. Firstly, we considered the asymptotical stability of equilibrium points to the system of ordinary differential equations type. Then, the existence of meromorphic solutions and the stability of equilibrium points to the system of weakly coupled meromorphic type were discussed. Finally, the existence of nonnegative meromorphic solutions to the system of strongly coupled meromorphic type was investigated, and the asymptotic stability of unique positive equilibrium point of the system was proved by constructing meromorphic functions.

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Abbreviations

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