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A note on singular two-dimensional fractional coupled Burgers' equation and triple Laplace Adomian decomposition method

Hassan Eltayeb^{1*} and Imed Bachar¹

*Correspondence:

hgadain@ksu.edu.sa

¹Mathematics Department, College of Science, King Saud University, P.O. Box 2455, 11451 Riyadh, Saudi Arabia

Abstract

The present article focuses on how to find the exact solutions of the time-fractional regular and singular coupled Burgers' equations by applying a new method that is called triple Laplace and Adomian decomposition method. Furthermore, the proposed method is a strong tool for solving many problems. The accuracy of the method is considered through the use of some examples, and the results obtained are compared with those of the existing methods in the literature.

Keywords: Double and triple Laplace transform; Inverse double and triple Laplace transform; Singular fractional coupled Burgers' equation; Mittag-Leffler function; Single Laplace transform; Decomposition methods

1 Introduction

Fractional Burgers' equation has received significant attention. The solution of this equation becomes very important for mathematical and physical phenomena. This equation has been discovered to explain different kinds of events, for example, it is a mathematical model of turbulence and approximate theory of a flow over a shock wave traveling in a viscous fluid [1, 2]. Local fractional homotopy analysis method has been explained in [3, 4]. The authors in [5] applied the new semianalytical method which is called the homotopy analysis Shehu transform method to solve multidimensional fractional diffusion equations. The researchers in [6, 7] examined the numerical solutions of three-dimensional Burger's equation and Riccati differential equations by applying Laplace decomposition methods. From a recent couple of years, important dedication has been given to Laplace decomposition method and its modifications for studying mathematical model [8, 9]. In the literature several authors have suggested different types of approximation and exact methods for solving fractional Burger's equation [10–12]. The authors in [13] applied the variational iteration method to obtain Burger's equation. In [14] the Laplace decomposition method (LDM) is suggested to solve the two-dimensional nonlinear Burgers' equations. The authors in [15] used the double Laplace decomposition methods to solve sin-

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gular Burgers' equation and coupled Burgers' equations. In this study, we propose a new hybrid triple Laplace Adomian decomposition method to obtain exact solutions of the time-fractional regular and singular coupled Burgers' equations. Finally, two examples are provided to illustrate the suggested approach.

Here we recall some definitions, notation of Laplace transform, and fractional calculus facts which are useful in this article.

Definition 1 ([16]) Let f be a function of three variables x, y , and t , where $x, y, t > 0$. The triple Laplace transform of f is defined by

$$L_x L_y L_t [f(x, y, t)] = F(p, q, s) = \int_0^\infty \int_0^\infty \int_0^\infty e^{-px - qy - st} f(x, y, t) dt dy dx,$$

where p, q, s are Laplace variables. Further, the triple Laplace transform of the partial derivatives is denoted by

$$L_x L_y L_t [u_x(x, y, t)] = pU(p, q, s) - U(0, q, s),$$

$$L_x L_y L_t [u_t(x, y, t)] = sU(p, q, s) - U(p, q, 0).$$

Similarly, the triple Laplace transform for the second partial derivatives with respect to x, y , and t are defined as follows:

$$L_x L_y L_t [u_{tt}(x, y, t)] = p^2 U(p, q, s) - pU(0, q, s) - \frac{\partial U(0, q, s)}{\partial x},$$

$$L_x L_y L_t [u_{yy}(x, y, t)] = q^2 U(p, q, s) - qU(p, 0, s) - \frac{\partial U(p, 0, s)}{\partial y},$$

$$L_x L_y L_t [u_{tt}(x, y, t)] = s^2 U(p, q, s) - sU(p, q, 0) - \frac{\partial U(p, q, 0)}{\partial t}.$$

The inverse triple Laplace transform $L_p^{-1} L_q^{-1} L_s^{-1} [F(p, q, s)] = f(x, y, t)$ is defined as in [16] by the complex triple integral formula

$$f(x, y, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{px} dp \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{qy} dq \frac{1}{2\pi i} \int_{d-i\infty}^{d+i\infty} e^{st} ds.$$

Definition 2 ([17–19]) The Caputo time-fractional derivative operator of order $\alpha > 0$ is defined by

$$D_t^\alpha u(r, t) = \begin{cases} \frac{\partial^m u(r, t)}{\partial t^m}, & \text{for } m = \alpha \in \mathbb{N}; \\ \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m u(r, \tau)}{\partial \tau^m} d\tau, & m-1 < \alpha < m. \end{cases}$$

In the following theorem, we present the triple Laplace transforms of the partial fractional Caputo derivatives.

Theorem 1 ([20]) Let $\alpha, \beta, \gamma > 0$, $n-1 < \alpha \leq n$, $m-1 < \beta \leq m$, $r-1 < \gamma \leq r$, and $n, m, p \in \mathbb{N}$, so that $f \in C^l(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+)$, $l = \max\{n, m, p\}$, $f^{(l)} \in L_1[(0, a) \times (0, b) \times (0, c)]$ for any $a, b, c > 0$, $|f(x, y, t)| \leq we^{x\tau_1 + y\tau_2 + t\tau_3}$, $x > a > 0$, $y > b > 0$, and $t > c > 0$. Then the triple

Laplace transform of Caputo's fractional derivatives $D_t^\alpha u(x, y, t)$, $D_t^\alpha u(x, y, t)$ and $D_t^\alpha u(x, y, t)$ are given by

$$\begin{aligned} & L_x L_y L_t [D_t^\alpha u(x, y, t)] \\ &= s^\alpha U(p, q, s) - \sum_{i=0}^{n-1} s^{\alpha-1-i} L_y L_t [D_t^i u(x, y, 0)], \quad n-1 < \alpha < n, \end{aligned} \quad (1.1)$$

$$\begin{aligned} & L_x L_y L_t [D_y^\beta u(x, y, t)] \\ &= q^\beta U(p, q, s) - \sum_{j=0}^{m-1} q^{\beta-1-j} L_y L_t [D_y^j u(x, 0, t)], \quad m-1 < \beta < m, \end{aligned} \quad (1.2)$$

$$\begin{aligned} & L_x L_y L_t [D_x^\gamma u(x, y, t)] \\ &= p^\gamma U(p, q, s) - \sum_{k=0}^{r-1} p^{\gamma-1-k} L_y L_t [D_x^k u(0, y, t)], \quad r-1 < \gamma < r. \end{aligned} \quad (1.3)$$

Below, we establish the relation between Mittag-Leffler function and Laplace transform, which will be useful in this paper. The Mittag-Leffler function is defined by the following series representation:

$$E_\beta(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta k + 1)}, \quad z \in \mathbb{C}, \Re(\beta) > 0, \quad (1.4)$$

the Mittag-Leffler function with two parameters is defined by

$$E_{\beta, \gamma}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta k + \gamma)}, \quad z \in \mathbb{C}, \Re(\alpha) > 0, \quad (1.5)$$

see [21]. If we put $\beta = 1$ in Eq. (1.5), we obtain Eq. (1.4). It follows from Eq. (1.5) that

$$E_{1,1}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k+1)} = \sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z, \quad (1.6)$$

$$E_{1,2}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k+2)} = \sum_{k=0}^{\infty} \frac{z^k}{(k+1)!} = \frac{1}{z} \sum_{k=0}^{\infty} \frac{z^{k+1}}{(k+1)!} = \frac{e^z - 1}{z}, \quad (1.7)$$

and

$$E_{1,3}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k+3)} = \sum_{k=0}^{\infty} \frac{z^k}{(k+2)!} = \frac{1}{z^2} \sum_{k=0}^{\infty} \frac{z^{k+2}}{(k+2)!} = \frac{e^z - 1 - z}{z^2}, \quad (1.8)$$

in general,

$$E_{1,m}(z) = \frac{1}{z^{m-1}} \left[e^z - \sum_{k=0}^{m-2} \frac{z^k}{k!} \right]. \quad (1.9)$$

In the following, we introduce the Laplace transforms of some Mittag-Leffler functions which are useful in this work:

$$\begin{aligned} L_x L_y L_t [x^2 t^\alpha E_{1,\alpha+1}(t)] &= \frac{2!}{p^3 q s^\alpha (s-1)}, \\ L_x L_y L_t [t^\alpha E_{1,\alpha+1}(t)] &= \frac{1}{p q s^\alpha (s-1)}, \\ L_x L_y L_t [t^{2\alpha} E_{1,2\alpha+1}(t)] &= \frac{1}{p q s^{2\alpha} (s-1)}. \end{aligned}$$

2 Analysis of the triple Laplace decomposition method

The main objective of this section is to address the use of triple Laplace Adomian decomposition method (TLADM) for solving two-dimensional time-fractional coupled Burger's equation. We consider two-dimensional time-fractional coupled Burger's equation in the form:

$$\begin{aligned} D_t^\alpha u + uu_x + vu_y &= \frac{1}{\Re}(u_{xx} + u_{yy}), \quad x, y, t > 0, \\ D_t^\alpha v + uv_x + vv_y &= \frac{1}{\Re}(v_{xx} + v_{yy}), \quad x, y, t > 0, \\ n-1 < \alpha < n; \end{aligned} \quad (2.1)$$

with the initial condition

$$u(x, y, 0) = f_1(x, y), \quad v(x, y, 0) = g_1(x, y),$$

where $D_t^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$ is the fractional Caputo derivative, $\Re$ is the Reynolds number, and the velocity components are given by $u(x, y, t)$ and $v(x, y, t)$ in the x and y direction, respectively. For the purpose of finding the solution of Eq. (2.1), we apply triple Laplace Adomian decomposition method as follows:

Step 1. Taking the triple Laplace transform for Eq. (2.1), we obtain

$$\begin{aligned} s^\alpha L_x L_y L_t [u(x, y, t)] - s^{\alpha-1} U(p, q, 0) &= -L_x L_y L_t (uu_x + vu_y) \\ &\quad + L_x L_y L_t \left(\frac{1}{\Re} (u_{xx} + u_{yy}) \right), \\ s^\alpha L_x L_y L_t [v(x, y, t)] - s^{\alpha-1} V(p, q, 0) &= -L_x L_y L_t (uv_x + vv_y) \\ &\quad + L_x L_y L_t \left(\frac{1}{\Re} (v_{xx} + v_{yy}) \right). \end{aligned} \quad (2.2)$$

Step 2. Now, employing the differentiation property of the Laplace transform, we have

$$\begin{aligned} L_x L_y L_t [u(x, y, t)] &= \frac{1}{s} F_1(p, q) - \frac{1}{s^\alpha} L_x L_y L_t (uu_x + vu_y) \\ &\quad + \frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\Re} (u_{xx} + u_{yy}) \right), \end{aligned} \quad (2.3)$$

$$\begin{aligned} L_x L_y L_t [v(x, y, t)] &= \frac{1}{s} G_1(p, q) - \frac{1}{s^\alpha} L_x L_y L_t (u v_x + v v_y) \\ &\quad + \frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\mathfrak{H}} (v_{xx} + v_{yy}) \right). \end{aligned}$$

Step 3. By implementing the triple inverse Laplace transformation of Eq. (2.3), we obtain

$$\begin{aligned} u(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} F_1(p, q) \right) - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (u u_x + v u_y) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\mathfrak{H}} (u_{xx} + u_{yy}) \right) \right), \\ v(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} G_1(p, q) \right) - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (u v_x + v v_y) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\mathfrak{H}} (v_{xx} + v_{yy}) \right) \right). \end{aligned} \quad (2.4)$$

Step 4. The Laplace Adomian decomposition solution functions $u(x, y, t)$ and $v(x, y, t)$ are given by the infinite series

$$u(x, y, t) = \sum_{n=0}^{\infty} u_n(x, y, t), \quad v(x, y, t) = \sum_{n=0}^{\infty} v_n(x, y, t), \quad (2.5)$$

further, the nonlinear terms $u u_x$, $v \frac{\partial u}{\partial y}$, $u \frac{\partial v}{\partial x}$, and $v \frac{\partial v}{\partial y}$ are given by:

$$u u_x = \sum_{n=0}^{\infty} A_n, \quad v u_y = \sum_{n=0}^{\infty} B_n, \quad u v_x = \sum_{n=0}^{\infty} C_n, \quad v v_y = \sum_{n=0}^{\infty} D_n, \quad (2.6)$$

and, by substituting Eqs. (2.5) and (2.6) into Eq. (2.4), we get

$$\begin{aligned} \sum_{m=0}^{\infty} u_m(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} F_1(p, q) \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\sum_{n=0}^{\infty} (A_n + B_n) \right) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\mathfrak{H}} \left(\sum_{n=0}^{\infty} (u_{xxn} + u_{yy n}) \right) \right) \right) \end{aligned} \quad (2.7)$$

and

$$\begin{aligned} \sum_{n=0}^{\infty} v_n(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} G_1(p, q) \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\sum_{n=0}^{\infty} (C_n + D_n) \right) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\mathfrak{H}} \left(\sum_{n=0}^{\infty} (v_{xxn} + v_{yy n}) \right) \right) \right). \end{aligned} \quad (2.8)$$

Step 5. Using Laplace Adomian decomposition method, we introduce the recursive relations and get:

$$\begin{aligned} u_0(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} F_1(p, q) \right), \\ v_0(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} G_1(p, q) \right), \end{aligned} \quad (2.9)$$

and the remainder components u_{n+1} and v_{n+1} , for $n \geq 0$, are given by

$$\begin{aligned} u_{n+1}(x, y, t) &= -L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t ((A_n + B_n)) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\Re} (u_{xxn} + u_{yy n}) \right) \right) \end{aligned} \quad (2.10)$$

and

$$\begin{aligned} v_{n+1}(x, y, t) &= -L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (C_n + D_n) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{\Re} (v_{xx} + v_{yy}) \right) \right), \end{aligned} \quad (2.11)$$

where a few first terms of the Adomian polynomials A_n , B_n , C_n , and D_n are given by

$$\begin{aligned} A_0 &= u_0 u_{0x}, & A_1 &= u_0 u_{1x} + u_1 u_{0x}, \\ A_2 &= u_0 u_{2x} + u_1 u_{1x} + u_2 u_{0x}, \end{aligned} \quad (2.12)$$

$$\begin{aligned} A_3 &= u_0 u_{3x} + u_1 u_{2x} + u_2 u_{1x} + u_3 u_{0x}, \\ B_0 &= v_0 u_{0y}, & B_1 &= v_0 u_{1y} + v_1 u_{0y}, \\ B_2 &= v_0 u_{2y} + v_1 u_{1y} + v_2 u_{0y}, \end{aligned} \quad (2.13)$$

$$\begin{aligned} B_3 &= v_0 u_{3y} + v_1 u_{2y} + v_2 u_{1y} + v_3 u_{0y}, \\ C_0 &= u_0 v_{0x}, & C_1 &= u_0 v_{1x} + u_1 v_{0x}, \\ C_2 &= u_0 v_{2x} + u_1 v_{1x} + u_2 v_{0x}, \end{aligned} \quad (2.14)$$

$$\begin{aligned} C_3 &= u_0 v_{3x} + u_1 v_{2x} + u_2 v_{1x} + u_3 v_{0x}, \\ D_0 &= v_0 v_{0y}, & D_1 &= v_0 v_{1y} + v_1 v_{0y}, \\ D_2 &= v_0 v_{2y} + v_1 v_{1y} + v_2 v_{0y}, \end{aligned} \quad (2.15)$$

$$D_3 = v_0 v_{3y} + v_1 v_{2y} + v_2 v_{1y} + v_3 v_{0y},$$

and where $L_x L_y L_t$ is the triple Laplace transform with respect to x, y, t and the triple inverse Laplace transform with respect to p, q, s is denoted by $L_p^{-1} L_q^{-1} L_s^{-1}$. We supposed that the triple inverse Laplace transform with respect to p, q , and s exists in Eqs. (2.9), (2.10), and (2.11).

In the next numerical example, the suggested method is applied to a two-dimensional time-fractional coupled Burger's equation when $\Re = 1$ as follows:

Example 1 Consider a two-dimensional nonlinear Burger's differential equation

$$\begin{aligned} D_t^\alpha u + uu_x + v u_y &= u_{xx} + u_{yy}, \quad x, y, t > 0, \\ D_t^\alpha v + u v_x + v v_y &= v_{xx} + v_{yy}, \quad x, y, t > 0, \\ n - 1 < \alpha < n; \end{aligned} \quad (2.16)$$

subject to the condition

$$u(x, y, 0) = x + y, \quad v(x, y, 0) = x - y.$$

As reported by the above steps, we have

$$\begin{aligned} u(x, y, t) &= x + y - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (uu_x + v u_y) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t \left(\frac{1}{2} (u_{xx} + u_{yy}) \right) \right), \\ v(x, y, t) &= x - y - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (u v_x + v v_y) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (v_{xx} + v_{yy}) \right). \end{aligned} \quad (2.17)$$

The zeroth components u_0 and v_0 , recommended by Adomian method, always contain initial condition and the source term, so we set

$$u_0 = x + y, \quad v_0 = x - y.$$

The other components u_{n+1} , v_{n+1} , $n \geq 0$ are given by the relations

$$\begin{aligned} u_{n+1} &= -L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t ((A_n + B_n)) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (u_{xxn} + u_{yy n}) \right) \end{aligned} \quad (2.18)$$

and

$$\begin{aligned} v_{n+1} &= -L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (C_n + D_n) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (v_{xxn} + v_{yy n}) \right). \end{aligned} \quad (2.19)$$

By taking $n = 0$ in Eqs. (2.18) and (2.19), we get

$$\begin{aligned} u_1 &= -L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t ((A_0 + B_0)) \right) \\ &\quad + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} L_x L_y L_t (u_{0xx} + u_{0yy}) \right) \end{aligned}$$

$$\begin{aligned}
 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(2x)\right) \\
 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{2}{p^2s^{\alpha+1}}\right) \\
 &= \frac{-2xt^\alpha}{\Gamma(\alpha+1)}
 \end{aligned}$$

and

$$\begin{aligned}
 v_1 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(C_0 + D_0)\right) \\
 &\quad + L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(v_{0xx} + v_{0yy})\right) \\
 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(2y)\right) \\
 &= \frac{-2yt^\alpha}{\Gamma(\alpha+1)},
 \end{aligned}$$

similarly, when $n = 1$, we have

$$\begin{aligned}
 u_2 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t((u_0u_{1x} + u_1u_{0x} + v_0u_{1y} + v_1u_{0y}))\right) \\
 &\quad + L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\frac{1}{2}(u_{1xx} + u_{1yy})\right)\right) \\
 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\frac{-t^\alpha}{\Gamma(\alpha+1)}(2x + 2y)\right)\right) \\
 &= L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^{2\alpha+1}}\left(\frac{4}{p^2q} + \frac{4}{pq^2}\right)\right) \\
 &= \frac{4(x+y)t^{2\alpha}}{\Gamma(2\alpha+1)}
 \end{aligned}$$

and

$$\begin{aligned}
 v_2 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(D_1 + E_1)\right) \\
 &\quad + L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(v_{1xx} + v_{1yy})\right) \\
 &= L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\frac{-t^\alpha}{\Gamma(\alpha+1)}(-4x + 4y)\right)\right) \\
 &= L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^{2\alpha+1}}\left(\frac{-4}{p^2q} + \frac{4}{pq^2}\right)\right) \\
 &= \frac{4(x-y)t^{2\alpha}}{\Gamma(2\alpha+1)},
 \end{aligned}$$

when $n = 2$, we have

$$\begin{aligned} u_3 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(u_0u_{2x}+u_1u_{1x}+u_2u_{0x}+v_0u_{2y}+v_1u_{1y}+v_2u_{0y})\right) \\ &\quad + L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\frac{1}{2}(u_{2xx}+u_{2yy})\right)\right) \\ &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\left(16x+\frac{4x\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right) \\ &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{16}{p^2qs^{3\alpha+1}}+\frac{4\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2p^2qs^{3\alpha+1}}\right) \\ &= \left(-16x-\frac{4x\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \end{aligned}$$

similarly,

$$\begin{aligned} v_3 &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t(D_2+E_2)\right) \\ &\quad + L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\frac{1}{2}(v_{2xx}+v_{2yy})\right)\right) \\ &= -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}L_xL_yL_t\left(\left(16y+\frac{4y\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right) \\ &= \left(-16y-\frac{4y\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \end{aligned}$$

hence, the solution of Eq. (2.16) is given by

$$\begin{aligned} u(x,y,t) &= \sum_{n=0}^{\infty} u_n = u_0 + u_1 + u_2 + u_3 + \cdots, \\ u(x,y,t) &= x + y - \frac{2xt^\alpha}{\Gamma(\alpha+1)} + \frac{4(x+y)t^{2\alpha}}{\Gamma(2\alpha+1)} - \left(16x + \frac{4x\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots, \end{aligned}$$

and

$$\begin{aligned} v(x,y,t) &= \sum_{n=0}^{\infty} v_n = v_0 + v_1 + v_2 + v_3 + \cdots, \\ v(x,y,t) &= x - y - \frac{2yt^\alpha}{\Gamma(\alpha+1)} + \frac{4(x-y)t^{2\alpha}}{\Gamma(2\alpha+1)} + \left(-16y - \frac{4y\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\right)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots, \end{aligned}$$

at $\alpha = 1$ the solution of above equation becomes

$$\begin{aligned} u(x,y,t) &= x + y - 2xt + 2(x+y)t^2 - 4xt^3 + 4(x+y)t^4 - 8xt^5 \\ &\quad + 8(x+y)t^6 - 16xt^7 + 16(x+y)t^8 + \cdots \\ &= x(1 + 2t^2 + 4t^4 + 8t^6 + \cdots) + y(1 + 2t^2 + 4t^4 + 8t^6 + \cdots) \\ &\quad - 2xt(1 + 2t^2 + 4t^4 + 8t^6 + \cdots), \\ u(x,y,t) &= \frac{(x+y-2xt)}{1-2t^2}, \end{aligned}$$

and

$$\begin{aligned} v(x, y, t) &= x - y - \frac{2yt^\alpha}{\Gamma(\alpha + 1)} + \frac{4(x - y)t^{2\alpha}}{\Gamma(2\alpha + 1)} + \left(-16y - \frac{4y\Gamma(2\alpha + 1)}{(\Gamma(\alpha + 1))^2} \right) \frac{t^{3\alpha}}{\Gamma(3\alpha + 1)} + \cdots \\ &= (x - y) - 2yt + 2(x - y)t^2 - 4yt^3 + 4(x - y)t^4 - 8yt^5 + 8(x - y)t^6 \\ &\quad - 16yt^7 + 16(x - y)t^8 + \cdots \\ &= x(1 + 2t^2 + 4t^4 + 8t^6 + \cdots) - y(1 + 2t^2 + 4t^4 + 8t^6 + \cdots) \\ &\quad - 2yt(1 + 2t^2 + 4t^4 + 8t^6 + \cdots), \\ v(x, y, t) &= \frac{(x - y - 2yt)}{1 - 2t^2}. \end{aligned}$$

We obtained the same results as in [14].

3 Triple Laplace Adomian decomposition method and singular two-dimensional fractional coupled Burgers' equation

The principal algorithm of the triple Laplace decomposition method will be applied to singular two-dimensional fractional coupled Burgers' equation of the form

$$\begin{aligned} D_t^\alpha u + \frac{1}{x}uu_x + \frac{1}{y}vu_y - \frac{1}{x}(xu_x)_x - \frac{1}{y}(yu_y)_y &= f(x, y, t), \\ D_t^\alpha v + \frac{1}{x}uv_x + \frac{1}{y}vv_y - \frac{1}{x}(xv_x)_x - \frac{1}{y}(yv_y)_y &= g(x, y, t), \\ x, y, t > 0, \end{aligned} \quad (3.1)$$

associated with the initial condition

$$u(x, y, 0) = f_1(x, y), \quad v(x, y, 0) = g_1(x, y), \quad (3.2)$$

where $D_t^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$ is the fractional Caputo derivative and $\frac{1}{x}(xu_x)_x$, $\frac{1}{y}(yu_y)_y$ are called Bessel operators, $u(x, y, t)$ and $v(x, y, t)$ are the velocity components to be determined, $f(x, y, t)$; $g(x, y, t)$; $f_1(x, y)$ and $g_1(x, y)$ are known functions. In order to obtain the solution of Eq. (3.1), we use the following steps:

Step 1. Multiplying both sides of Eq. (3.1) by xy , we have

$$\begin{aligned} xyD_t^\alpha u + yuu_x + xvu_y - y(xu_x)_x - x(yu_y)_y &= xyf(x, y, t), \\ xyD_t^\alpha v + yuv_x + xvv_y - y(xv_x)_x - x(yv_y)_y &= xyg(x, y, t), \\ x, y, t > 0, \end{aligned} \quad (3.3)$$

Step 2. Applying the triple Laplace transform to both sides of Eq. (3.3), we obtain

$$\begin{aligned} \frac{\partial^2}{\partial p \partial q} (s^\alpha U(p, q, s) - s^{\alpha-1} U(p, q, 0)) &= L_x L_y L_t (y(xu_x)_x + x(yu_y)_y) \\ &\quad - L_x L_y L_t (yuu_x + xvu_y) \\ &\quad + L_x L_y L_t (xyf(x, y, t)) \end{aligned} \quad (3.4)$$

and

$$\begin{aligned} \frac{\partial^2}{\partial p \partial q} (s^\alpha V(p, q, s) - s^{\alpha-1} V(p, q, 0)) &= L_x L_y L_t (y(xv_x)_x + x(yv_y)_y) \\ &\quad - L_x L_y L_t (yuv_x + xvv_y) \\ &\quad + L_x L_y L_t (xyg(x, y, t)). \end{aligned} \quad (3.5)$$

Now, using the differentiation property of the Laplace transform yields

$$\begin{aligned} \frac{\partial^2}{\partial p \partial q} U(p, q, s) &= \frac{1}{s} F_1(p, q) - \frac{1}{s^\alpha} L_x L_y L_t (yuu_x + xvu_y) \\ &\quad + \frac{1}{s^\alpha} L_x L_y L_t (y(xu_x)_x + x(yu_y)_y) \\ &\quad + \frac{1}{s^\alpha} L_x L_y L_t (xyf(x, y, t)), \\ \frac{\partial^2}{\partial p \partial q} V(p, q, s) &= \frac{1}{s} G_1(p, q) - \frac{1}{s^\alpha} L_x L_y L_t (yuv_x + xvv_y) \\ &\quad + \frac{1}{s^\alpha} L_x L_y L_t (y(xv_x)_x + x(yv_y)_y) \\ &\quad + \frac{1}{s^\alpha} L_x L_y L_t (xyg(x, y, t)). \end{aligned} \quad (3.6)$$

Step 3. Integrating both sides of Eq. (3.6) from 0 to p and from 0 to q with respect to p and q , respectively, we have

$$\begin{aligned} U(p, q, s) &= \frac{1}{s} \int_0^p \int_0^q \left(\frac{\partial^2}{\partial p \partial q} F_1(p, q) \right) dq dp \\ &\quad + \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xu_x)_x + x(yu_y)_y)) dq dp \\ &\quad - \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yuu_x + xvu_y)) dq dp \\ &\quad + \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (xyf(x, y, t))) dq dp \end{aligned} \quad (3.7)$$

and

$$\begin{aligned} V(p, q, s) &= \frac{1}{s} \int_0^p \int_0^q \left(\frac{\partial^2}{\partial p \partial q} G_1(p, q) \right) dq dp \\ &\quad + \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xv_x)_x + x(yv_y)_y)) dq dp \\ &\quad - \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yuv_x + xvv_y)) dq dp \\ &\quad + \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (xyg(x, y, t))) dq dp. \end{aligned} \quad (3.8)$$

Step 4. By taking the triple inverse Laplace transformation of Eqs. (3.7) and (3.8), we obtain

$$\begin{aligned} u(x, y, t) = & L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} \int_0^p \int_0^q \left(\frac{\partial^2}{\partial p \partial q} F_1(p, q) \right) dq dp \right) \\ & + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xu_x)_x + x(yu_y)_y)) dq dp \right) \\ & - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yuu_x + xvu_y)) dq dp \right) \\ & + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (xyf(x, y, t))) dq dp \right) \end{aligned} \quad (3.9)$$

and

$$\begin{aligned} v(x, y, t) = & L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} \int_0^p \int_0^q \left(\frac{\partial^2}{\partial p \partial q} G_1(p, q) \right) dq dp \right) \\ & + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xv_x)_x + x(yv_y)_y)) dq dp \right) \\ & - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yuv_x + xvv_y)) dq dp \right) \\ & + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (xyg(x, y, t))) dq dp \right). \end{aligned} \quad (3.10)$$

Step 5. Substituting Eqs. (2.5) and (2.6) into Eqs. (3.9) and (3.10), we get

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x, y, t) = & L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} \int_0^p \int_0^q \left(\frac{\partial^2}{\partial p \partial q} F_1(p, q) \right) dq dp \right) \\ & + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q \left(L_x L_y L_t \left(y \left(x \sum_{n=0}^{\infty} u_{nx} \right)_x \right. \right. \right. \\ & \left. \left. \left. + x \left(y \sum_{n=0}^{\infty} u_{ny} \right)_y \right) \right) dq dp \right) \\ & - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q \left(L_x L_y L_t \left(y \sum_{n=0}^{\infty} A_n + x \sum_{n=0}^{\infty} B_n \right) \right) dq dp \right) \\ & + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (xyf(x, y, t))) dq dp \right) \end{aligned}$$

and

$$\begin{aligned} \sum_{n=0}^{\infty} v_n(x, y, t) = & L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s} \int_0^p \int_0^q \left(\frac{\partial^2}{\partial p \partial q} G_1(p, q) \right) dq dp \right) \\ & + L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q \left(L_x L_y L_t \left(y \left(x \sum_{n=0}^{\infty} v_{nx} \right)_x \right. \right. \right. \\ & \left. \left. \left. + x \left(y \sum_{n=0}^{\infty} v_{ny} \right)_y \right) \right) dq dp \right) \end{aligned}$$

$$\begin{aligned} & -L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t\left(y\sum_{n=0}^\infty C_n+x\sum_{n=0}^\infty D_n\right)\right)dqdp\right) \\ & +L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t(xy g(x,y,t))\right)dqdp\right). \end{aligned}$$

Step 6. Using Laplace Adomian decomposition method, we introduce the recursive relations and get

$$\begin{aligned} u_0(x,y,t) &= L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s}\int_0^p\int_0^q\left(\frac{\partial^2}{\partial p\partial q}F_1(p,q)\right)dqdp\right) \\ &+ L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t(xy f(x,y,t))\right)dqdp\right) \end{aligned} \quad (3.11)$$

and

$$\begin{aligned} v_0(x,y,t) &= L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s}\int_0^p\int_0^q\left(\frac{\partial^2}{\partial p\partial q}G_1(p,q)\right)dqdp\right) \\ &+ L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t(xy g(x,y,t))\right)dqdp\right), \end{aligned} \quad (3.12)$$

the other components u_{n+1} and v_{n+1} , for $n \geq 0$, are given by

$$\begin{aligned} u_{n+1}(x,y,t) &= L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t\left(y\left(x\sum_{n=0}^\infty u_{nx}\right)_x\right.\right.\right. \\ &\quad \left.\left.+x\left(y\sum_{n=0}^\infty u_{ny}\right)_y\right)\right)dqdp\right) \\ &- L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t\left(y\sum_{n=0}^\infty A_n+x\sum_{n=0}^\infty B_n\right)\right)dqdp\right) \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} v_{n+1}(x,y,t) &= L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t\left(y\left(x\sum_{n=0}^\infty v_{nx}\right)_x\right.\right.\right. \\ &\quad \left.\left.+x\left(y\sum_{n=0}^\infty v_{ny}\right)_y\right)\right)dqdp\right) \\ &- L_p^{-1}L_q^{-1}L_s^{-1}\left(\frac{1}{s^\alpha}\int_0^p\int_0^q\left(L_xL_yL_t\left(y\sum_{n=0}^\infty C_n+x\sum_{n=0}^\infty D_n\right)\right)dqdp\right), \end{aligned} \quad (3.14)$$

where $L_xL_yL_t$ is the triple Laplace transform with respect to x, y, t and the triple inverse Laplace transform with respect to p, q, s is denoted by $L_p^{-1}L_q^{-1}L_s^{-1}$. We assumed that the triple inverse Laplace transform with respect to p, q , and s exists for Eqs. (3.11), (3.12), (3.13), and (3.14). In the following example we apply the triple Laplace Adomian decomposition method to solve singular two-dimensional time-fractional coupled Burgers' equations.

Example 2 Consider singular two-dimensional time-fractional coupled Burgers' equations given by

$$\begin{aligned} D_t^\alpha u + \frac{1}{x}uu_x + \frac{1}{y}vu_y - \frac{1}{x}(xu_x)_x - \frac{1}{y}(yu_y)_y &= (x^2 - y^2)e^t, \\ D_t^\alpha v + \frac{1}{x}uv_x + \frac{1}{y}vv_y - \frac{1}{x}(xv_x)_x - \frac{1}{y}(yv_y)_y &= (x^2 - y^2)e^t, \\ x, y, t &> 0, \end{aligned} \quad (3.15)$$

with the initial condition

$$u(x, y, 0) = x^2 - y^2, \quad v(x, y, 0) = x^2 - y^2.$$

As stated by the above steps, we have

$$\begin{aligned} u(x, y, t) &= x^2 - y^2 + \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xu_x)_x + x(yu_y)_y)) dq dp \\ &\quad - \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yuu_x + xvu_y)) dq dp \\ &\quad + x^2 t^\alpha E_{1,\alpha+1}(t) - y^2 t^\alpha E_{1,\alpha+1}(t) \end{aligned} \quad (3.16)$$

and

$$\begin{aligned} v(x, y, t) &= x^2 - y^2 + \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xv_x)_x + x(yv_y)_y)) dq dp \\ &\quad - \frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yuv_x + xvvy)) dq dp \\ &\quad + x^2 t^\alpha E_{1,\alpha+1}(t) - y^2 t^\alpha E_{1,\alpha+1}(t). \end{aligned} \quad (3.17)$$

By applying Eqs. (3.11), (3.12), (3.13), and (3.14), we obtain

$$\begin{aligned} u_0(x, y, t) &= x^2 - y^2 + x^2 t^\alpha E_{1,\alpha+1}(t) - y^2 t^\alpha E_{1,\alpha+1}(t), \\ v_0(x, y, t) &= x^2 - y^2 + x^2 t^\alpha E_{1,\alpha+1}(t) - y^2 t^\alpha E_{1,\alpha+1}(t), \end{aligned} \quad (3.18)$$

and the other components u_{n+1} and v_{n+1} , for $n \geq 0$, are given by

$$\begin{aligned} u_{n+1}(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xu_{nx})_x + x(yu_{ny})_y)) dq dp \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yA_n + xB_n)) dq dp \right) \end{aligned} \quad (3.19)$$

and

$$\begin{aligned} v_{n+1}(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xv_{nx})_x + x(yv_{ny})_y)) dq dp \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yC_n + xD_n)) dq dp \right), \end{aligned} \quad (3.20)$$

where a few first terms of the Adomian polynomials A_n , B_n , C_n , and D_n are given by Eqs. (2.12), (2.13), (2.14), and (2.15), respectively.

By substituting $n = 0$ into Eqs. (3.19) and (3.20), we get

$$\begin{aligned} u_1(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xu_{0x})_x + x(yu_{0y})_y)) dq dp \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yA_0 + xB_0)) dq dp \right), \\ u_1(x, y, t) &= 0, \end{aligned}$$

and

$$\begin{aligned} v_1(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xv_{0x})_x + x(yv_{0y})_y)) dq dp \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yC_0 + xD_0)) dq dp \right), \\ v_1(x, y, t) &= 0. \end{aligned}$$

In the same manner for $n = 1$, we obtain that

$$\begin{aligned} u_2(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xu_{1x})_x + x(yu_{1y})_y)) dq dp \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yA_1 + xB_1)) dq dp \right), \\ u_2(x, y, t) &= 0, \end{aligned}$$

and

$$\begin{aligned} v_2(x, y, t) &= L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (y(xv_{1x})_x + x(yv_{1y})_y)) dq dp \right) \\ &\quad - L_p^{-1} L_q^{-1} L_s^{-1} \left(\frac{1}{s^\alpha} \int_0^p \int_0^q (L_x L_y L_t (yC_1 + xD_1)) dq dp \right), \\ v_2(x, y, t) &= 0. \end{aligned}$$

The solution of Eq. (3.15) is given by

$$\begin{aligned} u(x, y, t) &= u_0 + u_1 + u_2 + \cdots + u_n, \\ v(x, y, t) &= v_0 + v_1 + v_2 + \cdots + v_n. \end{aligned}$$

Hence, the exact solution is given by

$$\begin{aligned} u(x, y, t) &= x^2 - y^2 + x^2 t^\alpha E_{1, \alpha+1}(t) - y^2 t^\alpha E_{1, \alpha+1}(t), \\ v(x, y, t) &= x^2 - y^2 + x^2 t^\alpha E_{1, \alpha+1}(t) - y^2 t^\alpha E_{1, \alpha+1}(t), \end{aligned}$$

where E denotes Mittag-Leffler function. Setting $\alpha = 1$ in Eq. (3.15), we get that the exact solution of the singular two-dimensional coupled Burgers' equation

$$\begin{aligned} D_t u + \frac{1}{x} u u_x + \frac{1}{y} v u_y - \frac{1}{x} (x u_x)_x - \frac{1}{y} (y u_y)_y &= (x^2 - y^2) e^t, \\ D_t v + \frac{1}{x} u v_x + \frac{1}{y} v v_y - \frac{1}{x} (x v_x)_x - \frac{1}{y} (y v_y)_y &= (x^2 - y^2) e^t, \\ x, y, t &> 0, \end{aligned}$$

with the initial condition

$$u(x, y, 0) = x^2 - y^2, \quad v(x, y, 0) = x^2 - y^2,$$

is given by

$$u(x, y, t) = (x^2 - y^2) e^t, \quad v(x, y, t) = (x^2 - y^2) e^t.$$

4 Conclusion

In this study, the triple Laplace transform and Adomian decomposition have been successfully combined to obtain a new powerful method named a hybrid triple Laplace Adomian decomposition method (TLADM). This method has been used to solve regular and singular coupled Burgers' equations. By applying this method on some examples, we have obtained new efficient relations to solve our problems. It allows more realistic series solution that converges very quickly to the true solution.

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Availability of data and materials

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors read and approved the final manuscript.

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