Cui Boundary Value Problems (2022) 2022:31
https://doi.org/10.1186/s13661-022-01612-2

Boundary Value Problems
a SpringerOpen Journal

RESEARCH

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Existence of entire radial solutions to Hessian type system

Jixian Cui[1*]

*Correspondence:
jixiancui@126.com
[1]College of Science, Qiqihar University, Qiqihar, P.R. China

Abstract

In this paper, the Dirichlet problem of Hessian type system is studied. After converting the existence of an entire solution to the existence of a fixed point of a continuous mapping, the existence of entire radial solutions is established by Schaefer fixed point theorem.

Keywords: k-convex radial solution; Existence; Hessian type system

1 Introduction

In this paper, we study the existence of entire radial solutions to the following Dirichlet problem of a Hessian type system:

$$\begin{cases} \sigma_{k_j}(\lambda(D^2 u_j + \mu_j|\nabla u_j|I)) = f_j(|x|, u), & 1 \le j \le m, x \in B_1(0), \\ u = (0, 0, \dots, 0), & x \in \partial B_1(0), \end{cases} \tag{1.1}$$

where $u = (u_1, u_2, \dots, u_m)$. For each integer j satisfying $1 \le j \le m \le N$, where m is a fixed integer, the integer k_j satisfies $1 \le k_j \le N$, μ_j is a nonnegative constant. $B_1(0)$ is the unit ball in $\mathbb{R}^N$. For any $N \times N$ real symmetric matrix A, $\lambda(A)$ denotes the eigenvalues of A. $D^2 w(x) = (\frac{\partial^2 u(x)}{\partial x_i \partial x_j})_{1 \le i,j \le N}$ denotes the Hessian matrix of the function $w \in C^2(\overline{B_1(0)})$, ∇w denotes the gradient of w, and $\sigma_{k_j}(\lambda) = \sum_{1 \le i_1 < \dots < i_{k_j} \le N} \lambda_{i_1} \cdots \lambda_{i_{k_j}}$ denotes the k_jth elementary symmetric function of $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_N) \in \mathbb{R}^N$.

For f_j, $1 \le j \le m$, we introduce the following conditions:

(H1) $f_j \in C([0,1] \times (-\infty, 0]^m, [0, +\infty))$ and $f(r, w) = 0$ if and only if $w = (0, 0, \dots, 0)$.

(H2) $|f_j(r, w)| \le L(r)|w|^\alpha$, where $r \in [0,1]$, $w \in (-\infty, 0]^m$, $L \in C[0,1]$ and the constant α satisfies

$$0 \le \alpha < \min_{1 \le j \le m} \{k_j\}.$$

Denote

$$\Gamma_k := \{\lambda \in \mathbb{R}^N : \sigma_j(\lambda) > 0, 1 \le j \le k\}.$$

We say that a function $u \in C^2(\overline{B_1(0)})$ is k-convex in $B_1(0)$ if $\lambda(D^2 u(x)) \in \Gamma_k$ for all $x \in B_1(0)$.

In problem (1.1), if $m = 1$, the equation becomes the following k-Hessian type equation:

$$\sigma_k\big(\lambda\big(D^2u + \mu|\nabla u|I\big)\big) = f\big(|x|, u\big),$$

which is a generalization of the k-Hessian equation

$$\sigma_k\big(\lambda\big(D^2u\big)\big) = f\big(|x|, u\big), \tag{1.2}$$

but a special case of the following fully nonlinear Hessian equation:

$$F\big(\lambda\big(D^2u + A(x, u, \nabla u)\big)\big) = f(x, u, \nabla u). \tag{1.3}$$

We refer to Guan and Jiao [8] and Jiang and Trudinger [10] and the references therein for research on fully nonlinear Hessian equation (1.3). See also Dai [4] for a similar study. For k-Hessian equation (1.2), it is well known that if $k = 1$, it becomes the famous Laplacian equation. Laplacian equations attract a great deal of attention, and until now there have been a vast number of research papers on this kind of equations. Here we specially mention Keller [12] and Osserman [16] for Laplacian equations. If $k = N$, k-Hessian equation (1.2) becomes the well-known Monge–Ampère equation. Nowadays, the research on Monge–Ampère equations is still a hot topic. Here we want to mention Cheng and Yau [1] and Laser and McKenna [14] for Monge–Ampère equations. For general k-Hessian equation (1.2), when $f(|x|, u) = f(u)$, Ji and Bao [9] gave necessary and sufficient conditions on the existence of entire positive k-convex radial solutions; when $f(|x|, u) = u^{\gamma k}$, $\gamma > 1$, Jin, Li, and Xu [11] showed the nonexistence of entire k-convex positive solutions. Because of the work of Jin, Li, and Xu, to establish the existence of solutions for problem (1.1) we study under condition (H2), which can be treated as a sublinear condition. By a similar method, we can also treat the linear case but under a bit more complex condition than condition (H2). If we generalize $f(|x|, u)$ to $f(x, u)$, de Oliveira, do Ó, and Ubilla obtained the existence of k-convex radial solutions in the case of supercritical nonlinearity by means of variational techniques (see [5] and the references therein for research in this direction).

In problem (1.1), if $m > 1$, the system becomes a coupling k-Hessian type system. There are not so many research papers on coupling k-Hessian type systems. Here we only mention Lair and Wood [13] and Cîrstea and Rădulescu [2] for coupling Laplacian systems (i.e., in the case $k = 1$, $m = 2$, and $\mu_1 = \mu_2 = 0$) and Wang and An [18] and Zhang and Qi [19] for coupling Monge–Ampère systems (i.e., in the case $k = N$, $m = 2$, and $\mu_1 = \mu_2 = 0$). For a general coupling k-Hessian system, if $m = 2$ and $f_1(|x|, u, v) = p(|x|)f(v)$, $f_2(|x|, u, v) = q(|x|)g(u)$, the system becomes

$$\begin{cases} \sigma_k(\lambda(D^2u + \mu|\nabla u|I)) = p(|x|)f(v), \\ \sigma_l(\lambda(D^2v + \nu|\nabla u|I)) = q(|x|)g(u). \end{cases} \tag{1.4}$$

We refer to the papers of Feng and Zhang [6] and Gao, He, and Ran [7] and the references therein for research on coupling k-Hessian system (1.4) when $\mu = \nu = 0$. By the way, on general k-Hessian equation (1.2) and general coupling k-Hessian system (1.4) when $\mu = \nu = 0$, Zhang and Zhou [20] obtained several results on the existence of entire positive

k-convex radial solutions. In [3], we studied the coupling k-Hessian system (1.4) and obtained the existence and nonexistence of entire k-convex radial solutions. In the process of obtaining the existence of entire k-convex radial solutions, we utilized the monotone iterative method, and so we require the monotonicity of f and g. In the present paper, we want to remove the requirement for monotonicity and utilize a method different from the monotone iterative method. As a cost, we only treat the so called sublinear case. We study the Dirichlet problem of k-Hessian type system (1.1) and obtain the following result in this paper.

Theorem 1.1 *Under conditions* (H1) *and* (H2), *problem* (1.1) *admits an entire radial solution* $u \in (C^2(\overline{B_1(0)}))^m$ *and the jth component of* u *is* k_j*-convex.*

The paper is organized as follows. In Sect. 2, we introduce basic concepts and results. In Sect. 3, we prove the existence of entire radial solutions of problem (1.1), which is the main part of this paper.

2 Preliminaries

In this section, we give some preliminary results which will be used to prove the main results in the next section.

Lemma 2.1 *Let the integer* j *be fixed and satisfy* $1 \le j \le m$. *Assume* $\varphi_j(r) \in C^2[0,1]$ *with* $\varphi_j'(0) = 0$. *Then, for* $u_j(x) = \varphi_j(r)$, *there holds that* $u_j \in C^2(\overline{B_1(0)})$ *and*

$$\lambda\left(D^2 u_j + \mu_j |\nabla u_j| I\right) = \begin{cases} (\varphi_j''(r) + \mu_j \varphi_j'(r), (\frac{1}{r} + \mu_j)\varphi_j'(r), \ldots, (\frac{1}{r} + \mu_j)\varphi_j'(r)), \\ \quad r \in (0,1], \\ (\varphi_j''(0), \varphi_j''(0), \ldots, \varphi_j''(0)), \quad r = 0, \end{cases}$$

and further

$$\begin{aligned} &\sigma_{k_j}\left(\lambda\left(D^2 u_j + \mu_j |\nabla u_j| I\right)\right) \\ &\quad = \begin{cases} C_{N-1}^{k_j-1}(\varphi_j''(r) + \mu_j \varphi_j'(r))((\frac{1}{r} + \mu_j)\varphi_j'(r))^{k_j-1} + C_{N-1}^{k_j}((\frac{1}{r} + \mu_j)\varphi_j'(r))^{k_j}, \\ \quad r \in (0,1], \\ C_N^{k_j}(\varphi_j''(0))^{k_j}, \quad r = 0, \end{cases} \end{aligned}$$

where $\mu_j \ge 0$ *is a constant and* $C_N^{k_j} = \frac{N!}{(k_j)!(N-k_j)!}$.

See [3] for the proof.

Lemma 2.2 *Denote* $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_m)$. *Assume that* $\varphi \in (C[0,1])^m \cap (C^1(0,1])^m$ *is a solution of the Cauchy problem*

$$\begin{cases} \varphi_j'(r) = (\frac{k_j}{C_{N-1}^{k_j-1}} e^{-\psi_{k_j,\mu_j}(r)} \int_0^r e^{\psi_{k_j,\mu_j}(s)} \frac{s^{k_j-1} f_j(s,\varphi(s))}{(1+\mu_j s)^{k-1}} \, ds)^{\frac{1}{k_j}}, \quad 0 < r < 1, \\ \varphi(1) = (0,0,\ldots,0), \end{cases}$$

where

$$\psi_{k_j,\mu_j}(r) = \frac{k_j}{C_{N-1}^{k_j-1}}\left(C_N^{k_j}\mu_j r + C_{N-1}^{k_j}\ln r\right).$$

Then $\varphi \in (C^2[0,1])^m$ *and it satisfies the problem*

$$\begin{cases} C_{N-1}^{k_j-1}\varphi_j''(r)(\varphi_j'(r))^{k_j-1}r + (C_N^{k_j}\mu_j r + C_{N-1}^{k_j})(\varphi_j'(r))^{k_j} = \frac{r^{k_j}f_j(r,\varphi(r))}{(1+\mu_j r)^{k_j-1}}, \\ \quad 1 \le j \le m, 0 < r < 1, \\ \varphi'(0) = (0,0,\dots,0). \end{cases}$$

Furthermore, if φ_j *is nontrivial, i.e.,* $\varphi_j(r) < 0$ *for* $0 \le r < 1$, *then*

$$\lambda_r := \left(\varphi_j''(r) + \mu_j\varphi_j'(r), \left(\frac{1}{r} + \mu_j\right)\varphi_j'(r), \dots, \left(\frac{1}{r} + \mu_j\right)\varphi_j'(r)\right) \in \Gamma_{k_j}$$

for $0 \le r < 1$.

Proof It is immediate that $\varphi(r) \in (C^2[0,1])^m$.

From

$$\varphi_j'(r) = \left(\frac{k_j}{C_{N-1}^{k_j-1}} e^{-\psi_{k_j,\mu_j}(r)} \int_0^r e^{\psi_{k_j,\mu_j}(s)} \frac{s^{k_j-1}f_j(s,\varphi(s))}{(1+\mu_j s)^{k_j-1}}\,ds\right)^{\frac{1}{k_j}},$$

we can get

$$\left(\varphi_j'(r)\right)^{k_j} = \frac{k_j}{C_{N-1}^{k_j-1}} e^{-\psi_{k_j,\mu_j}(r)} \int_0^r e^{\psi_{k_j,\mu_j}(s)} \frac{s^{k_j-1}f_j(s,\varphi(s))}{(1+\mu_j s)^{k_j-1}}\,ds,$$

and further, by differentiating with respect to r, we have

$$C_{N-1}^{k_j-1}\varphi_j''(r)\left(\varphi_j'(r)\right)^{k_j-1}r + \left(C_N^{k_j}\mu_j r + C_{N-1}^{k_j}\right)\left(\varphi_j'(r)\right)^{k_j} = \frac{r^{k_j}f_j(r,\varphi(r))}{(1+\mu_j r)^{k_j-1}}.$$

If φ_j is nontrivial, for $0 \le r < 1$, we conclude that $\varphi_j(r) < \varphi_j(1) = 0$, $f_j(r,\varphi(r)) > 0$ and further

$$\sigma_{k_j}(\lambda_r) = f_j\left(r,\varphi(r)\right) > 0 \quad \text{for } 0 \le r < 1.$$

By the properties of k_jth elementary symmetric functions (see for example [15]), we know $\sigma_i(\lambda_r) > 0$ for $1 \le i < k_j$ and $0 \le r < 1$, from which we conclude the result. □

3 Proof of the main result

In this section, we show the existence of entire k-convex radial solutions for problem (1.1) by means of Schaefer fixed point theorem (see [17]).

Theorem 3.1 (Schaefer fixed point theorem) *Let* X *be a Banach space and* $T: X \to X$ *be a compact operator. If the set* $E = \{u \text{ in } X : u = \lambda Tu \text{ for some } 0 \le \lambda \le 1\}$ *is bounded, then* T *has at least a fixed point in* X.

For each j satisfying $1 \leq j \leq m$, from

$$C_{N-1}^{k_j-1}u_j''(r)\big(u_j'(r)\big)^{k_j-1}r + \big(C_N^{k_j}\mu_j r + C_{N-1}^{k_j}\big)\big(u_j'(r)\big)^{k_j} = \frac{r^{k_j}f_j(r,u(r))}{(1+\mu_j r)^{k_j-1}},$$

we get

$$u_j'(r) = \left(\frac{k_j}{C_{N-1}^{k_j-1}}\mathrm{e}^{-\psi_{k_j,\mu_j}(r)}\int_0^r \mathrm{e}^{\psi_{k_j,\mu_j}(s)}\frac{s^{k_j-1}f_j(s,u(s))}{(1+\mu_j s)^{k_j-1}}\,ds\right)^{\frac{1}{k_j}},$$

and further

$$u_j(r) = \int_1^r \left(\frac{k_j}{C_{N-1}^{k_j-1}}\mathrm{e}^{-\psi_{k_j,\mu_j}(t)}\int_0^t \mathrm{e}^{\psi_{k_j,\mu_j}(s)}\frac{s^{k_j-1}f_j(s,u(s))}{(1+\mu_j s)^{k_j-1}}\,ds\right)^{\frac{1}{k_j}}dt.$$

Define an operator $\mathcal{L} = ((\mathcal{L})_1, (\mathcal{L})_2, \dots, (\mathcal{L})_m)$ by

$$\big(\mathcal{L}(u)\big)_j(r) = \int_1^r \left(\frac{k_j}{C_{N-1}^{k_j-1}}\mathrm{e}^{-\psi_{k_j,\mu_j}(t)}\int_0^t \mathrm{e}^{\psi_{k_j,\mu_j}(s)}\frac{s^{k_j-1}f_j(s,u(s))}{(1+\mu_j s)^{k_j-1}}\,ds\right)^{\frac{1}{k_j}}dt,$$

then we need only to find a fixed point of $\mathcal{L}$. As we want to use Schaefer fixed point theorem to find such a fixed point, we should first check that the conditions of Schaefer fixed point theorem are satisfied.

It is easy to show that $\mathcal{L}$ is a mapping from $(C^2[0,1])^m$ to $(C^2[0,1])^m$ and it is continuous on $(C[0,1])^m$.

Proposition 3.1 *Under conditions* (H1) *and* (H2), *$\mathcal{L}$ is compact on* $(C[0,1])^m$.

Proof Let A be a bounded subset of $(C[0,1])^m$, i.e.,

$$\max_{1\leq j\leq m}\max_{0\leq r\leq 1}\big|u_j(r)\big| \leq M$$

for some constant $M > 0$. As f_j is continuous on $[0,1]\times[-M,0]^m$, there exists a constant $\bar{M} > 0$ such that

$$\big|f_j(r,w)\big| \leq \bar{M}, \quad \forall (r,w)\in[0,1]\times[-M,0]^m, 1\leq j\leq m.$$

Then, for any $u = (u_1, u_2, \dots, u_m) \in A$, we have

$$\begin{aligned}
0 &< \big(\mathcal{L}(u)\big)_j'(r) \\
&= \left(\frac{k_j}{C_{N-1}^{k_j-1}}\mathrm{e}^{-\psi_{k_j,\mu_j}(r)}\int_0^r \mathrm{e}^{\psi_{k_j,\mu_j}(s)}\frac{s^{k_j-1}f_j(s,u(s))}{(1+\mu_j s)^{k_j-1}}\,ds\right)^{\frac{1}{k_j}} \\
&\leq \left(\frac{\bar{M}k_j}{C_{N-1}^{k_j-1}}\right)^{\frac{1}{k_j}} \\
&= C(m,M,N,k_j,p_j,f_j),
\end{aligned}$$

where $C(m,M,N,k_j,p_j,f_j)$ is a constant dependent on m, MN, k_j, p_j, f_j. Therefore $\mathcal{L}(A)$ is equicontinuous on $[0,1]^m$.

On the other hand, we have

$$\begin{aligned}\left|\big(\mathcal{L}(u)\big)_j(r)\right| &= \left|\int_1^r \big(\mathcal{L}(u)\big)_j'(s)\,ds\right| \\ &\le \int_r^1 \left|\big(\mathcal{L}(u)\big)_j'(s)\right|ds \\ &\le C(m,M,N,k_j,p_j,f_j),\end{aligned}$$

from which we conclude that $\mathcal{L}(A)(r)$ is uniformly bounded on $[0,1]^m$, i.e., $\mathcal{L}(A)$ is bounded in $(C[0,1])^m$.

By Arzela–Ascoli theorem, $\mathcal{L}(A)$ is a sequentially compact subset of $(C[0,1])^m$ and further $\mathcal{L}$ is compact on $(C[0,1])^m$. □

Denote

$$E = \left\{u \in \big(C[0,1]\big)^m : u = \lambda\mathcal{L}(u) \text{ for some } 0 \le \lambda \le 1\right\}.$$

Proposition 3.2 *Under conditions* (H1) *and* (H2), *the set* E *is bounded in* $(C[0,1])^m$.

Proof Assume that $u = \lambda\mathcal{L}(u)$ for some $0 \le \lambda \le 1$. By condition (H2), for $(r,w) \in [0,1] \times (-\infty,0]^m$, we get

$$\left|f_j(r,w)\right| \le L(r)|w|^\alpha,$$

then

$$\begin{aligned}\left|u_j(r)\right| &= \left|\lambda\big(\mathcal{L}(u)\big)_j(r)\right| \\ &= \left|\lambda\int_1^r \left(\frac{k_j}{C_{N-1}^{k_j-1}}\mathrm{e}^{-\psi_{k_j,\mu_j}(t)}\int_0^t \mathrm{e}^{\psi_{k_j,\mu_j}(s)}\frac{s^{k_j-1}f_j(s,u(s))}{(1+\mu_j s)^{k_j-1}}\,ds\right)^{\frac{1}{k_j}}dt\right| \\ &\le C(m,L,N,k_j,p_j)\max_{0\le r\le 1}\left|u(r)\right|^{\frac{\alpha}{k_j}}.\end{aligned}$$

Suppose that $k_{j0} = \min_{1\le j\le m}\{k_j\}$. Taking the maximum on $[0,1]$, we have

$$\max_{0\le r\le 1}\left|u_j(r)\right|^{k_j} \le C(m,L,N,k_j,p_j)\max_{0\le r\le 1}\left|u(r)\right|^{\alpha},$$

and further

$$\begin{aligned}\max_{0\le r\le 1}\left|u(r)\right|^{k_{j0}} &= \max_{0\le r\le 1}\left(\sum_{j=1}^m\left|u_j(r)\right|^2\right)^{\frac{k_{j0}}{2}} \\ &\le \left(\sum_{j=1}^m\max_{0\le r\le 1}\left|u_j(r)\right|\right)^{k_{j0}}\end{aligned}$$

$$
\begin{aligned}
&\leq C(m,k_j)\left(\sum_{j=1}^{m}\max_{0\leq r\leq 1}\left|u_j(r)\right|^{k_{j_0}}\right)\\
&\leq C(m,k_j)\left(1+\left(\sum_{j=1}^{m}\max_{0\leq r\leq 1}\left|u_j(r)\right|^{k_j}\right)\right)\\
&\leq C(m,L,N,k_j,p_j)\left(1+\left(\max_{0\leq r\leq 1}\left|u(r)\right|^{\alpha}\right)\right)\\
&\leq C(\alpha,\varepsilon,m,L,N,k_j,p_j)+\varepsilon\max_{0\leq r\leq 1}\left|u(r)\right|^{k_{j_0}}
\end{aligned}
$$

in view of Young's inequality. Taking $\varepsilon = \frac{1}{2}$, we can get

$$\max_{0\leq r\leq 1}\left|u(r)\right| \leq C(\alpha,m,L,N,k_j,p_j),$$

from which we conclude that E is bounded in $(C[0,1])^m$. □

At last we prove the main result of this paper.

Proof of Theorem 1.1 First, in view of Propositions 3.1 and 3.2, we conclude that $\mathcal{L}$ admits a fixed point in $(C[0,1])^m$ by Schaefer fixed point theorem. Second, by Lemmas 2.1 and 2.2, the fixed point u of $\mathcal{L}$ in $(C[0,1])^m$ is in fact an entire radial solution in $(C^2(\overline{B_1(0)}))^m$ of problem (1.1) and the jth component of u is k_j-convex. □

At the end of this section, we give some examples for the sake of clearly understanding the results in this paper.

Example 3.1 If $0 \leq \alpha < N$ in condition (H2), then the following problem admits an entire convex radial solution $(u,v) \in C^2(\overline{B_1(0)}) \times C^2(\overline{B_1(0)})$:

$$
\begin{cases}
\det(\lambda(D^2u+\mu|\nabla u|I)) = f(|x|,u,v), & x\in B_1(0),\\
\det(\lambda(D^2v+\nu|\nabla v|I)) = g(|x|,u,v), & x\in B_1(0),\\
u=v=0, & x\in \partial B_1(0).
\end{cases}
$$

Example 3.2 If $0 \leq \alpha < 1$ in condition (H2), then the following problem admits an entire k-convex radial solution $u = (u_1,u_2,\ldots,u_N) \in (C^2(\overline{B_1(0)}))^N$:

$$
\begin{cases}
\operatorname{trace}(\lambda(D^2u_1+\mu_1|\nabla u_1|I)) = f_1(|x|,u), & x\in B_1(0),\\
\sigma_2(\lambda(D^2u_2+\mu_2|\nabla u_2|I)) = f_2(|x|,u), & x\in B_1(0),\\
\cdots & \cdots\\
\sigma_{N-1}(\lambda(D^2u_{N-1}+\mu_{N-1}|\nabla u_{N-1}|I)) = f_{N-1}(|x|,u), & x\in B_1(0),\\
\det(\lambda(D^2u_N+\mu_N|\nabla u_N|I)) = f_N(|x|,u), & x\in B_1(0),\\
u=(0,0,\ldots,0), & x\in \partial B_1(0).
\end{cases}
$$

Acknowledgements
The author is grateful to the referees for their helpful remarks and suggestions.

Funding
The author is supported by the National Natural Science Foundation of China (Grant No. U2031142).

Availability of data and materials
No data were used to support this study. It is not applicable for our paper.

Declarations

Competing interests
The author declares that he has no competing interests.

Authors' contributions
The author contributed independently to the manuscript and read and approved the final manuscript.

Authors' information
College of Science, Qiqihar University, Qiqihar 161006, P.R. China.

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Received: 24 January 2022 Accepted: 22 April 2022 Published online: 05 May 2022

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